doi.org/10.53656/ped2024-2s.10

Innovative Educational Technologies
Иновативни образователни технологии

TUTORING IN MATHEMATICS AND EDUCATION AND EVERYTHING IN BETWEEN

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Abstract. In this period, we witness the importance of education and teaching as a means of shaping people, if unfortunately for the purposes of inciting hatred or terrorism, and on the other hand, shaping a better person through a change of direction. As we have seen, education and teaching have great power in influencing the shaping of people: value-wise, morally, proportionately, culturally and educationally. When I turn to many people and ask: "What do you think will happen if you talk to one child at home and the rest are passive?". Everyone's reaction is shock and then: "What happened, you've gone crazy, it will be terrible". In light of this, it is clear to everyone that the continuation of teaching in the frontal/traditional approach in mathematics in general, in which about 85% of the students are passive and the teacher presents the solutions and not the way of thinking and guiding the solution process, must stop. The article emphasizes the need to analyze one's own thoughts when applying different methods and when solving practical tasks of a different nature.

Keywords: education; teaching; teaching and education; guiding teaching; frontal teaching; traditional teaching; guiding teaching in mathematics; shaping a better person

Introduction

The development of dialogic learning has been studied by many authors, including contemporary ones. We will note the analysis of Chinese authors who reach the conclusion "Secondary school mathematics teachers should take every possible measure, such as applying the student-lecturing mode to implement student-centered education and enhance student attitudes toward mathematics" (Minglin, Haizhen & Kwan 2023). They follow the way of dialogic approach proposed by Socrates (Wilberding 2021) as the beginning of Mathematical communication, which is the capacity to utilize mathematical language to accurately communicate mathematical ideas to friends, teachers, and others as well as to critique and evaluate the mathematical thinking and methods of others (Rohid & Rusmawati 2019). Since mathematical communication was initially suggested as a goal of mathematics instruction in schools in 1989 by the National

Council of Teachers of Mathematics (Frye 1989), math classrooms around the world have undergone modifications. One of the three components of fostering students' core literacy, for instance, is the requirement that students be able to communicate mathematically, or describe the actual world in the language of mathematics, according to the most recent edition of the Compulsory Mathematics Curriculum Standards.

Didactic

The world has been full of examples of solutions for almost every exercise for many years. Solutions are invested in all kinds of ways: colorful presentations, animations, videos, apps, and if that's not enough, the ChatGPT is added to its types. In recent years, the application of a competency-based approach and the implementation of STEM-learning has been starting from kindergarten (Pavlova & Toncheva 2022). From an early age, students are used to working with technology and dealing with practical tasks, but along with the use of innovations, it is important to stimulate the student to think independently and analyze the ways in which he reaches one or another conclusion.

Against the background of all innovative technologies, we believe it is important to use classical methods, such as the Socratic discussion. It should be noted that even the methods established over the years can be combined with innovative methods and techniques, if desired by the teacher.

In the given article, we follow an example of implementing a dialog for finding a solution to a task. The chosen problem is elementary, but we believe that it is important at the beginning of their learning to solve geometric problems that students are able to follow the ideas for solving the problem. Carrying out the talk under the guidance of the teacher is directly related to the qualitative intellectual reflection of the student.

Below is an example of presenting a guiding instruction for a geometry task:

It is given a trapezoid ABCD (AB||DC, AD╫BC)

AB=DE, $\measuredangle AED = \measuredangle AEB \measuredangle AED = \measuredangle AEB$ (AE – bisecting)

It is necessary to prove:

– ABED-rhombus.

– Given: ABEC-rhombus (mark AB=BC=CE=AE=x)

– Need to calculate: the angles of the trapezoid.

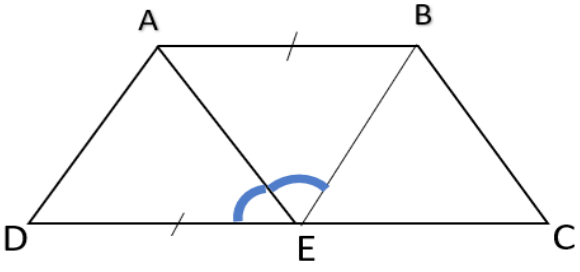

Figure 1. Drawing to the task

Solution: Instruction: First, to solve a task in geometry and mathematics, we will go through the data and translate each data into the basic mathematical meaning that is also useful (definition of the concept). After that, all the theorems are scanned, a definition of the concept and what is appropriate must be illustrated by marking in the drawing.

Question: What does it mean that: ∢BEA=∢AED

Answer: AE intersects an angle.

After handling the data, we will turn to the goal.

Question: What does it mean to prove that: ABED is a rhombus? What are the tools at our disposal to prove this?

Answer: definitions, auxiliary construction, theorems, proofs. To prove it we will look for theorems that deal with the question: What is needed to show that the square is a rhombus?

I recommended to the students (as a start and later they lower support until independence): go and search in the collection of theorems from textbook and this with the intention that during the search they will go over additional theorems and know to decide what is appropriate for the task and while doing so they will go back and work on the other theorems. After the search the students came to the rhombus theorems.

Question: Do we know that the square ABED is a rhombus?

Answer: No.

Question: So, what are we looking for?

Answer: Theorems that ask: What needs to be proven for the square to be a rhombus and below are the theorems (after the students have reached them)

How can it be proved that a square is a rhombus? If we prove that:

– The four sides of the quadrilateral are equal to each other.

– The quadrilateral is a parallelogram and has two equal adjacent sides.

– The quadrilateral is a parallelogram in which one of the diagonals intersects an angle.

– The quadrilateral is a parallelogram in which the diagonals are perpendicular to each other.

It is important to note: in any geometric shape the definition is used both as a feature of the shape and for the purpose of proving that if the definition is true then the shape is true (meaning that the shape is indeed a rhombus or any shape).

We went through the first theorem (definition) and asked: Is it possible to prove according to the theorem?

Answer: At the moment it does not seem that there are no 4 equal sides.

And after checking the second theorem, the students decide that there is currently no information about a pair of equal adjacent sides. After checking the third theorem, the students decide that this is the theorem: the quadrilateral is a parallelogram in which one of the diagonals is an angle bisector.

Question: So, what should be done now?

Answer: To prove that the quadrilateral is a parallelogram.

Now the students responded faster and they came to the theorems with which they prove a parallelogram.

How can it be proved that a quadrilateral is a parallelogram? If we prove that in that quadrilateral there is:

– Two pairs of opposite sides are parallel to each other.
– A pair of equal and parallel opposite sides.
– Two pairs of opposite sides are equal to each other.
– Two pairs of opposite angles are equal to each other.
– Diagonals cross each other.

On the definition (the first theorem) the students decided that it doesn't fit because we don't have a pair of parallel opposite sides and then the students decided on the sentence:

A quadrilateral in which there is a pair of parallel and equal opposite sides is a parallelogram. So, the students said:

AB||DC (given) Þ AB||DE (the students said intuitively: if lines are parallel then their parts and their continuations are also parallel).

I presented the students with an explanation of why AB||DC.

Question: When are two straight lines parallel?

1. In Analytic geometry when their slopes are equal and the slope of the straight line is constant (at this time we also dealt with straight slopes and non-straight graphs in a differential and derivative framework). This is how the topics integrate and reinforce each other and understanding. It also creates interest and reveals the beauty of mathematics and thinking.

2. Two parallel lines are lines that do not intersect; therefore, their parts and continuations will not intersect and hence they are parallel.

So, we may reason in the following synthetic scheme:

$$\begin{vmatrix} AB||DE \\ AB = DE\ (given) \end{vmatrix} \Rightarrow \begin{vmatrix} ABDE - parallelogram \\ AE - bisecting\,(given) \end{vmatrix} \Rightarrow ABED - rhombus$$

Another way of solving section A assuming that we have already proved that ABED is parallelogram.

Try to think of another way to prove that: ABED will be a rhombus.

Answer:

$$AB||DE \Rightarrow \begin{vmatrix} \measuredangle BAE = \measuredangle AED\ (alternate) \\ \measuredangle AEB = \measuredangle AED\ (Given). \end{vmatrix} \Rightarrow \Delta ABE - Isosceles \Rightarrow \begin{vmatrix} ABDE - parallelogram \\ AB = BE \end{vmatrix} \Rightarrow ABED - rhombus$$

Question: Why is it necessary to present another solution? Isn't it a waste of time? Won't that confuse you?

Answer:

A. One of the most important things that you, the students, and we as learners, need very much during a test, is the ability to generate ideas.

B. Different ways allow me to test my way of thinking and solving.

C. Different ways allow me to be exposed to other theorems and ideas that I will likely need in other tasks.

D. Additional ways help to reach the efficiency of the solution and reveal to us the beauty of thinking and the beauty of mathematics.

E. Different ways help us understand that math is not as difficult as we think, which strengthens our confidence and motivation to learn and succeed.

F. In addition, it develops understanding and thinking, which enables the building of knowledge and its stability for many years.

G. Different ways develop the creativity and inventive thinking which in general is important and, in our age, much more[2,3].

Question: What is needed to find an angle?

Answer:

1. Make up an equation with an angle because an equation is the main tool to find the missing (this idea has already been discussed during the previous lessons)[2]. If we reach a special triangle where the angle is known:

– In Equilateral triangle It is known that its angles are 60°.

– A right-angled and isosceles triangle where the base angles are 45°.

– A triangle where the perpendicular is equal to half of the other, the angle opposite the perpendicular is 30° and the other is 60° according to the sum of the angles in the triangle is 180°.

And now we will check what we have in the mission. Let's mark: AB=BC=CE=AE=AD=DE=BE =x (The equality was proved above).

Therefore, we can conclude that ΔAEC and ΔADE are equilateral and: $\measuredangle D = \measuredangle C = 60^0; \measuredangle D = \measuredangle C = 60^0;$ $\measuredangle DAB = \measuredangle ABC = 120^0$ $\measuredangle DAB = \measuredangle ABC = 120^0$

Pedagogical research

These are 10th grade students preparing for 5 units of mathematics. I teach them starting 1.9.23 for 3 weeks and we left for the holidays when, unfortunately, on 7.10.23 war broke out following the terrorist attack by Hamas. I teach while participating students by vote and I came across with considerable trepidation, fear of participating so as not to reveal the lack of knowledge and a natural fear of change.

With the help of encouragement, empowerment, clarification of the teaching approach and the logic in it consistently, pleasantly and persistently, trust is built, confidence to be exposed and participate. Treatment is done in class during learning and in personal conversations as needed.

The teaching approach is to explain the steps, address gaps while learning, write down rules and illustrate their use as they secure and guarantee the student's learning process.

The purpose of teaching is to guide the students and ask questions that will help them reach a solution while giving them time to think and while giving an opportunity

not only to students who already know. During learning, I make it clear to them the importance of copying, repeating the material at home as close as possible to the lesson, preparing homework, while facing challenges and coming to class with questions from homework.

Creating an atmosphere that talks about the error as part of the process, there is nothing to be ashamed of our difficulties but to deal with them, otherwise one should be ashamed of not dealing with the difficulties and leaving them to pat.

I create trust, allow personal assistance, create an atmosphere where students will not laugh or mock when a student speaks. Many conversations around the goal of reaching 100% group and personal success and at the level of 5 study units.

In this way, when done consistently and persistently, the lesson was conducted as presented. It is made clear to them that in order to present a solution as beautiful as it may be, there is no need for a teacher and the role of the teacher is to teach: learning habits, ways to solve tasks, using the existing tools to solve tasks, asking questions to develop higher order thinking as well as a way to self-direct and develop an independent, critical and creative learner.

A. 70 – 80 graduates who sent videos and described their learning with me and its results and effects after 20 – 30 years in the fields of mathematics and computer science in three high schools. Table 1 presents the profile of the respondents in relation to their attitude towards mathematics education.

Table 1. Respondents' profile

CATEGORY	PARTIALITY IN PERCENTAGES
Eligibility for matriculation	100
College degree	95
Master's degree or higher	38
Employee	100
A positive memory	100
Teacher for life	100

B. Results of a second-degree study[3].

C. 7 years of teaching in challenging B groups in 7th and 8th grades that led to 50% – 100% of the students who succeeded in the levels 4 – 5 math units (2015 – 2022 at Yeshiva Beit Yehuda). According to a large study by the Ministry of Education, only 3.5% of such groups Succeed at the levels of 4 units and not 5 units[5].

The combination of didactics and pedagogy enable and reinforce each other and enable the integration of education as part of the learning and empowerment process for the benefit of: tolerance, patience, restraint, delaying gratification, accepting the other, listening to the other, mutual learning, breaking social and academic stigmas.

In conclusion, the teacher with this approach: manages the learning and teaching and not only transfers knowledge, combines knowledge and education. It shapes a better person and a better society for many years to come and realizes the goal of education that is thousands of years old (Aloni 1997).

Discussion

Conducting dialogue and algorithmizing certain actions is the subject of various studies. So, for example, we can consider the need to break down the main task into smaller tasks when solving a particular case study, such as converting numeral text into digit number (Borisova & Karashtranova 2022). In contrast are approaches that emphasize visual examples, such as the application of anaglyph images (Harizanov & Ivanova 2020). Here, technology is given to create and view 3D examples, but to reveal the interrelationships of individual elements, it is again necessary to analyze and discuss them with students. The possibility of generating dialogue with artificial intelligence is discussed in "Flipped dialogic learning method with ChatGPT" (Pavlova 2024). Here, the student communicates with the ChatGPT, thus taking the lead in directing the dialogue.

Conclusion

In conclusion, we should note that it is important that modern educational technologies are based on established practices. The main task of teaching mathematics is to stimulate students to think logically. The ability to maintain logical relationships is important not only in mathematics, but also in any other direction in the life of every person. The training should be based on the competence approach and the stimulation of independent reasoning and dealing with the tasks.

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