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"THE SOURCE OF LIFE" IN BISHOP'S BASILICA OF PHILIPPOPOLIS IN THE CONTEXT OF STEAM

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Abstract. The main task considered in this material is related to the restoration of the mosaic "Source of Life" in the Bishop's Basilica of Philippopolis. Ideas for creation and use of computer models of the geometric scheme and of separate figures in the considered mosaic are presented. Technologies such as 360-degree photography, 3D printing, specialized software for creating computer models, etc. are shared to provide conditions for research work.

STEAM training, especially in the context of STEAM centers and innovative schools, creates good opportunities for the widespread deployment of such resources. Most of the described educational resources are provided with free access in the Virtual Mathematics Laboratory, developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

Keywords: STEAM; geometric constructions; computer models; GeoGebra; 3D printing; symmetry; mathematics and art; IBL

1. Introduction

The study of mathematics through art and information technology, as well as the application of mathematical knowledge in the fine arts, including the use of information technology, is supported by a number of educational resources. Some of them are related to the study or application of the style of specific artists or trends, for example M. Escher (Sendova & Grkovska 2005; Chehlarova, Sendova & Stefanova 2012; Fong & Dunham 2019; Fathauer 2021; Mehta 2021), A. Warhol (Chehlarova & Chehlarova 2013), A. Dürer (Kostov 2020) and others. For other educational resources, ideas for compiling mathematical problems are used (Chehlarova 2013; Chehlarova 2019), games (Chehlarova, T. 2019; Chehlarova, N. 2021; Chehlarova & Valkov 2021; Chehlarova & Chehlarova 2021; Triznya & Katona 2020) and works of art (Chehlarova & Chehlarova 2014; Chehlarova, K. 2021; Zalaya & Barrallo 2018).

STEAM education, especially in the context of STEAM centers, innovative schools with new programs and other iniciatives in Bulgaria, creates good opportunities for dissemination of such resources.

The main task considered in this material is related to the reconstruction of the mosaic "The source of Life" in the Bishop's Basilica of Philippopolis.

2. “The source of life” in the Bishop’s Basilica of Philippopolis

Philippopolis is an old name of the city of Plovdiv. The Bishop’s Basilica of Philippopolis was discovered in 1982 during the reconstruction of the road network of Plovdiv. After its long-term conservation, with the support of the America for Bulgaria Foundation, the Municipality of Plovdiv and the Ministry of Culture, a restoration was made and a visitor center was built. Its official opening for visitors was on April 18, 2021. It is the largest early Christian church in the country and among the most impressive monuments from the Roman era in the Bulgarian lands. The mosaics are in two layers with a total area of 2000 square meters (Kantareva-Decheva & Decheva 2017). They are exposed on two levels – the first layer on site, and much of the second layer – on the second level.

The Bishop’s Basilica of Philippopolis is three-naved. In the two side naves – north and south, there are two identical mosaics “Source of Life”. The photos on Fig. 1 are of the mosaic “The Source of Life” presented on the first floor. It is in this part that the two layers of mosaics from the northern nave of the Bishop’s Basilica of Philippopolis are left. It can be seen that the integrity of the mosaic has been violated and we raise the issue of forming an idea for its restoration.

Figure 1. “The source of life” from the northern nave of the Bishop’s Basilica of Philippopolis

On-site visits and self-collection of data is the most appropriate way to start working with students on the topic in the context of STEAM. For those for whom a visit cannot be provided, the following auxiliary materials have been made:

– 360 degree photos and videos
– Pictures of the mosaic in both naves
– Panoramic photos.

The comparison of the two mosaics – from the north and from the south nave, is an opportunity for orientation in their restoration. We will present several ideas for creating computer models of the geometric composition and individual figures in the considered mosaic.

3. Computer models on the mosaic "The source of Life" in the Bishop's Basilica of Philippopolis

In the Virtual Mathematics Laboratory[1)], developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences, a topic[2)] is provided with free access, which contains some of the described resources. The dynamic files in the theme can be used both in a browser and in *Geogebra* format (Hohenwarter, Hohenwarter & Lavicza 2009).

3.1. Computer model with a photo and its image in axial symmetry

An image of the mosaic under consideration for restoration is placed in the Geogebra file, using a photo from the southern nave (Fig. 2).

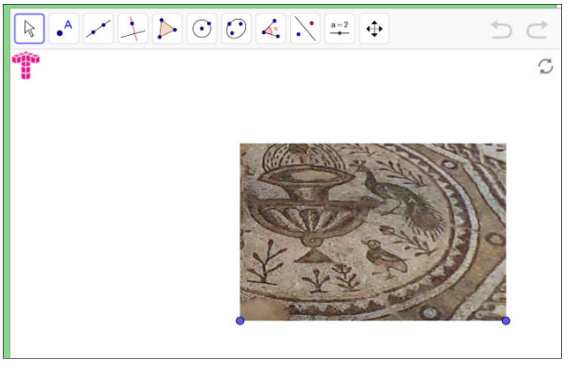

Figure 2. Research through photography and dynamic software

To obtain a general idea of the composition, it is appropriate to use axial symmetry. The image of the picture at axial symmetry is constructed and such a position of the axis of symmetry is sought, at which an idea of the overall composition can be obtained (Fig. 3). 50% transparency was used to support the observation of the image .

Figure 3. Axial symmetry for the exploration of an object for restoration

It is appropriate for students' attention to be focused on both symmetry and fragments in which symmetry is broken.

3.2. Computer model of the geometric composition of the mosaic

The geometric composition of the mosaic (Fig. 5) includes two identical squares with a common center, which can be considered as a prototype and an image in rotation of 45 degrees; octagon; circles, one of which is described around the squares. We use dynamic *GeoGebra* software to create a computer model of the geometric composition. The construction can be done by: using the capabilities of the software (for example, the buttons in *GeoGebra*); as a construction task – with virtual ruler and compass; as well as using analytical geometry. Most often, a combination of elements from the three variants is applied, according to the knowledge of the decision-maker and the desire for lightness, small memory and/or brevity of the description.

There are many options for building a geometric composition using commands and buttons in *GeoGebra*. The reason for this is the multitude of properties of the figures involved in the composition and the different options for choosing an algorithm.

If the construction starts from the "big" circle, the center of the coordinate system can be chosen for its center, while the intersection points of this circle with coordinate axes can serve as vertices of the square.

If the construction starts from a square, the command *Polygon(<Point>, <Point>, <Number of Vertices>* can be used. As input it requires two vertices of the regular polygon and the number of its sides. The center of this square can be built in different ways:

– as its diagonal midpoint (via the Midpoint or Center button  or midpoint command Midpoint (<Point>, <Point>)

– using the Centroid (<Polygon>) command to find the center of a regular polygon

– as an intersection of the perpendicular bisectors of two adjacent sides, etc.

3.2.1. Computer model using buttons in GeoGebra

The step by step construction of the figure can be followed in the so-called *Construction Protocol* (right side of Fig. 4), as well as by means of the Navigation Bar (bottom line of Fig. 4).

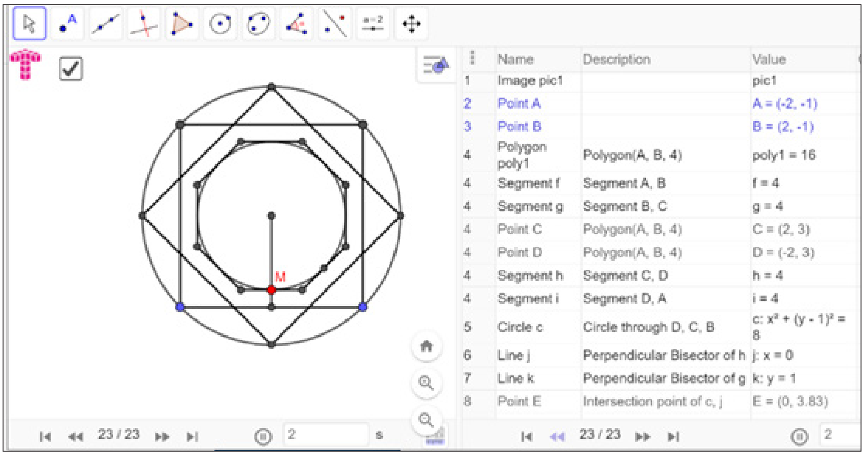

Figure 4. Observation of the construction using the Navigation Bar

Here is a description of a construction version. Step by step are constructed:

– Two points A and B
– Square as a regular polygon with side AB and number of sides 4
– A circle passing through three points – three of the vertices of the constructed square
– Two perpendicular bisectors of two adjacent sides of the square and their intersection point which will be the center of the composition
– The 4 intersections of these perpendicular bisectors with the circle (they serve as vertices of a new square)
– A segment from which a point M will be selected, which will define the "inner circle" centered at the center of the construction and passing through M. One end of this segment is the center of the composition, and the other is the midpoint of the segment AB
– One vertex of the octagon circumscribed around the inner circle is the intersection of two tangents of the inner circle – a tangent at the point M and a tangent at the intersection of this circle with a diagonal of the initial square
– The second vertex of the octagon circumscribed around the inner circle is the image under symmetry with respect to point M of the vertex already obtained.

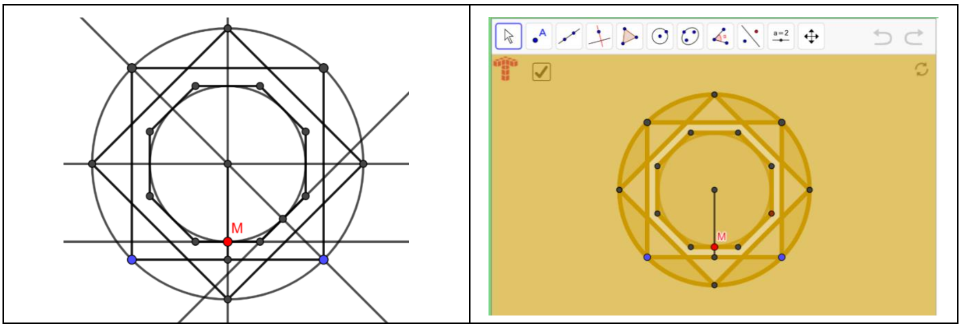

Figure 5. Computer model using buttons in *GeoGebra*

3.2.2. Computer model with a combination of analytical geometry and buttons in GeoGebra

Here we will present a variant for constructing the geometric composition in which some facts from analytical geometry and some *GeoGebra* buttons will be used. In Fig. 6 the first steps of such a file are shown. The circle is introduced by its equation. Some of the points – by their coordinates. A button for a regular polygon in *GeoGebra* is used to construct the square.

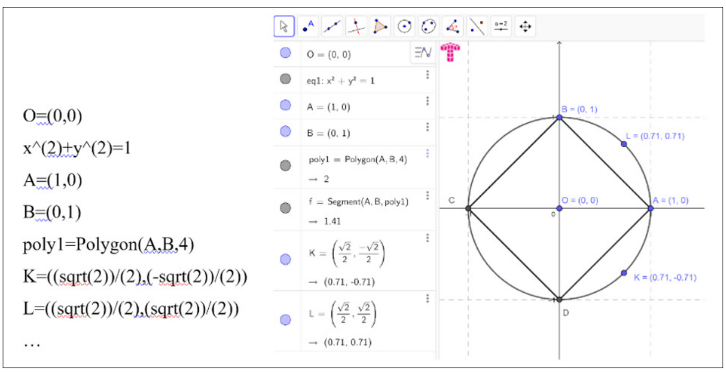

Figure 6. Combined variant for construction of the geometric composition

3.3. Computer model of the fountain from which water gushes

A central element from the mosaic which catches the attention is a fountain from which water gushes (Fig. 2). It can be modeled as a rotational figure.

To obtain a solid of revolution with Geogebra one can use:

– Parametric *Curve: a = Spline ({A, B, C, D, E, F, G}, 3)* (this command draws a smooth curve passing through the points A, B, C, D, E, F, G (as in Fig.7))
– *Surface: b = Surface (a, α, g)* (this command shows the result or rotating the smooth curve around the line g by angle α).

The specific solid of revolution is modeled through the points defining the parametric curve (fig. 7).

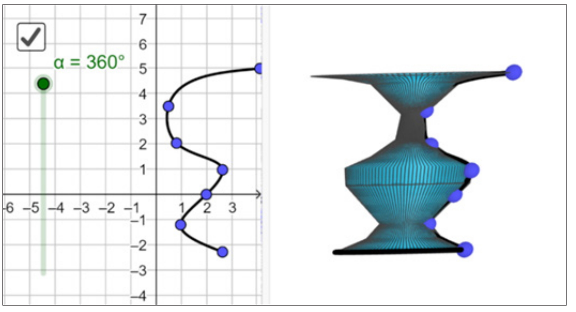

Figure 7. Computer model of a fountain from which water gushes

Using such a computer model, the 3D printed model of fig. 8 was made.

Figure 8. 3D printed model of a fountain from which water gushes

To create a model of the rotating body, it is appropriate to use one of the applications *Bottle Design* or *Potter's Weel* by *DALEST-Elica* (Sendova, Chehlarova & Boytchev 2007). In them, too, the solid of revolution is modeled by points or by positioning a concrete curve (Fig. 9).

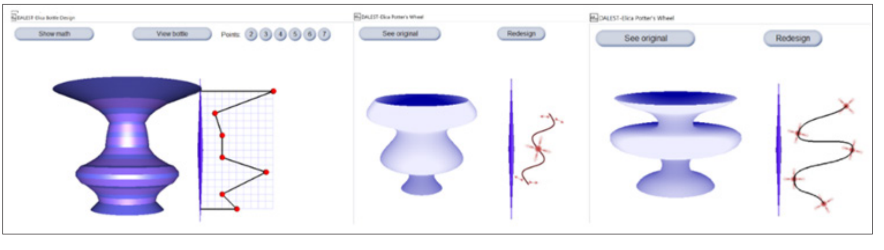

Figure 9. Solid of revolution with *DALEST-Elica* applications

When visiting the Bishop's Basilica of Philippopolis, one can see a model of restoration with a design on the original restoration[3)].

4. Results of professional development (PD) courses with teachers

The ideas described above were used in two qualification courses "Source of Life at the Bishop's Basilica of Philippopolis in the context of STEAM education", and "Technologies for STEAM Education". The first mentioned course was supported by the Scientix project (Nistor et al. 2018). The second was conducted within the frame of "Education with Science" program of the Bulgarian Ministry of Education and Science and the Bulgarian Academy of Sciences. The level of satisfaction of the teachers participating in the courses was very high. Over 90% of the teachers involved in the courses noted that they could use the ideas and resources discussed above when working with their students. Given the effectiveness of competitions related to mathematics and art (Chehlarova & Gachev 2021), it is appropriate to organize competitions related to reconstructions or the creation of works of art in the style of the mosaic discussed above.

Conclusion

The use of research approaches such as those discussed here and obtaining a final product during qualification courses with teachers helps the quick implementation of the considered ideas and resources. The first reviews of teacher's work with students confirmed our expectation that such use of mathematical ideas increases the motivation for learning of students.

The emphasis so far is on teaching mathematics, information technology and art, but it is natural to include areas such as history, technology, science. The latter can be related to research of the depicted birds and flowers in the considered mosaic, the technology of creation and restoration of the mosaics, research of the used materials.

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NOTES

1. http://cabinet.bg/
2. http://cabinet.bg/index.php?contenttype=viewarticle&id=327
3. https://www.plovdivmosaics.org/

REFERENCES

Chehlarova, K., 2021. The glassworks of planet "Cuboctahedron". *Symmetry: Culture and Science*, **32**(1), 083 – 090.

Chehlarova, N., 2021. Axes of symmetry dominoes. *Symmetry: Culture and Science*, **32**(1), 103 – 111.

Chehlarova, N. & Chehlarova, T. 2021. Computer model for self-preparation for playing with dominoes "Axes of Symmetry". *Symmetry: Culture and Science*, **32**(2), 281 – 284.

Chehlarova, N. & Gachev, G., 2021. Online contest "Mathematics and Art" for the development of key competencies. *Pedagogika-Pedagogy*, **93**(1), 87 – 99.

Chehlarova, T., 2013. Tasks with fractions in the style of Escher. In: P. Kenderov & E. Sendova (Eds.). *Inquiry Based Mathematics Education*. Sofia: Regalia 6. 127 – 132. [In Bulgarian] http://www.math.bas.bg/omi/Fibonacci/docs/book/23.pdf.

Chehlarova, T., 2019. Domino Ordinary Fractions Mathematical Tournament as an event at STEM Discovery Week, *Pedagogika-Pedagogy*, **91**(3), [In Bulgarian].

Chehlarova, T. & Chehlarova, K., 2014. Photo-pictures and dynamic software or about the motivation of the art-oriented students, *International Journal for Technology in Mathematics Education*, **21**(1), 27 – 31.

Chehlarova, T. & Chehlarova, N., 2013. Dynamic compositions in the style of Andy Warhol, *Pedagogical forum*, **1** (2), 56 – 62. [In Bulgarian].

Chehlarova, T. & Valkov, M., 2021. Game with vertical axis of symmetry in a rectangular board. Symmetry*: Culture and Science*, **32**(2), 285 – 288.

Chehlarova, T., Sendova, E. & Stefanova, E., 2012. Dynamic tessellations in support of the inquiry-based learning of mathematics and arts. In: Kynigos, C., Clayson, J. & Yiannoutsou, N. (Eds.). *Theory, Practice and Impact - Proceedings of Constructionism 2012*, Athens: The Educational Technology Lab, 570 – 574.

Fathauer, R. 2021., Logarithmic Spiral Tilings of Triangles. In: *Proceedings of Bridges 2021: Mathematics, Art, Music, Architecture, Culture.* USA: Tessellations Publishing, 55 – 62.

Fong, C. & Dunham, D., 2019. Hyperbolization on a Squircular Continuum. In: *Proceedings of Bridges 2019: Mathematics, Art, Music, Architecture, Education, Culture*. USA: Tessellations Publishing, 83 – 90. https://archive.bridgesmathart.org/2019/bridges2019-83.pdf

Hohenwarter, J., Hohenwarter, M. & Lavicza, Z., 2009. Introducing dynamic mathematics software to secondary school teachers: The case of GeoGebra, *Journal of Computers in Mathematics and Science Teaching*, **28**(2), 135 – 146.

Kostov, R., 2020. The enigma of the polyhedron and five-fold symmetry in Dürer's "Melencolia I". *Symmetry: Culture and Science*, **31**(4), 417 – 424. Available from: https://doi.org/10.26830/symmetry_2020_4_417.

Kantareva-Decheva, E., Decheva, R., 2017. Episcopal Basilica of Philippopolis (Plovdiv, Bulgaria) Conservation of the Mosaic Floors. *Proceedings of the XIII Conference of the International Committee for the Conservation of Mosaics*, Barcelona.

Mehta, C., 2021. Quadrilateral Spiral Tilings and Escheresque Art. In: *Proceedings of Bridges 2021: Mathematics, Art, Music, Architecture, Culture*. USA: Tessellations Publishing, 135 – 140.

Nistor, A., Gras-Velazquez, A., Billon, N. & Mihai, G., 2018. *Science, Technology, Engineering and Mathematics Education Practices in Europe.* Scientix Observatory report. Brussels: European Schoolnet.

Sendova, E., Chehlarova, T. & Boytchev, P., 2007. Words are Silver, Mouse-Clicks are Gold? (or how to optimize the level of language formalization of young students in a Logo-based cubics world), *Informatics in Education.* **6**(2), 411 – 428. Available from: https://doi.org/10.15388/infedu.2007.27.

Sendova, E., & Grkovska, S., 2005. Visual modeling as a motivation for studying mathematics and art. *Educational Media International*, **42**(2), 173 – 180.

Triznya, A. & Katona, V. (2020) Search for a computer aided solution of the 7×7 square Poly-Universe – an algorithmic approach. *Symmetry: Culture and Science,* **31**(1), 77 – 90. https://doi.org/10.26830/symmetry_2020_1_077.

Zalaya, R. & Barrallo, J., 2018. A Classification of Mathematical Sculpture. *Recreational Mathematics Magazine*, **5**(9), 71 – 94. https://doi.org/10.2478/rmm-2018-0004.

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