https://doi.org/10.53656/math2022-1-1-the

THE INFLUENCE OF COLLABORATIVE LEARNING ON STUDENTS' ACHIEVEMENT IN EXAMINING FUNCTIONS WITH PARAMETERS IN DYNAMIC SOFTWARE ENVIRONMENT

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Abstract. In this paper, the influence of collaborative learning of functions, algebraically represented as functions with parameters, in *GeoGebra* environment is analyzed. The research presented in this paper is the continuation of the research in Bozic and Takaci (2021), conducted at the University of Novi Sad, Serbia, with two groups of students, the experimental and the control one. The students in the experimental group learned in small, four member groups and the students in the control group learned individually. The students from both the groups learned in *GeoGebra* environment. The students' learning achievements in examining properties of functions with parameters is analyzed and compared. It is proved that it is better when the students work in collaborative groups than when they learn individually.

Keywords: Calculus; Collaborative learning; Dynamic software; Functions with parameters

1. Introduction

Numerous research showed that students have difficulties in working with functions with variable parameters and transformations of functions and that some of these difficulties can be overcome by using modern technology in teaching and learning, but some of the students have difficulties even with using technology (Anabousy et al. 2014; Borba & Confrey 1996; Daher & Anabousy 2015). In particular, according to Anabousy et al. (2014), "It could be seen that the students preferred to draw the resulting function, but not to draw manually and graphically the transformations performed on the original function (i.e. they drew the transformations virtually)." Also, it is shown that collaborative learning, applied together with modern technology, contributes to better quality of the learning process (Lipponen 2002; Weinberger & Fischer 2006). Therefore, it is decided to be conducted a research in order to examine the students' approach in examining the properties of related families of functions in a dynamic environment during collaborative learning and to examine whether the application of collaborative learning, combined with modern technology, contribute to better achievement in examining the functions with parameters.

Also, the mentioned studies (Anabousy et al. 2014; Borba & Confrey 1996; Daher & Anabousy 2015) have dealt with several types of functions, such as quadratic and polynomial, also considering the absolute values of these functions, but they haven't deal with functions like rational, trigonometric, exponential or logarithmic, which are, usually, more difficult for students. Taking into account the above, as well as the importance of all these functions for further application, this research covered more types of functions. In the research Takači et al. (Takači et al. 2015), all mentioned types of functions are examined with fixed parameters, hence the students did not consider the process of transformation of functions, but only its result (Anabousy et al. 2014). On the other side, in this research, functions are considered with variable parameters, which enabled observing the process of transformation of functions, a more general overview of different types of functions and their properties, as well as the connections between different types of functions.

2. Theoretical background

2.1. Collaborative learning

Collaborative learning is a common term for different approaches in education that involve collaborative efforts of the students, or collaborative efforts of teacher and students. It is a teaching method that implies that students work together, to achieve a common goal, or to solve a posed problem (Gokhale 1995). During collaborative learning, students' activity is most pronounced. This kind of learning is focused precisely on students' activities, not on the teacher's lecture, or on the teaching unit. The mentioned activities of the students are not always the same, but they are adapted to the needs of concrete collaborative work (Goodsell et al. 1992).

Golub (1988) presents collaborative learning as a form of indirect lecture where the teacher poses a problem and organizes the students who will jointly solve the problem. During collaborative learning, students' interaction is necessary in order to better implement joint activities. In these discussions, learning and creating new knowledge are being realized. With this approach, the student develops the ability to solve problems, but also to understand complex relationships, as well as the ability to make decisions in complex situations.

Benefits of collaborative learning do not come to the fore if the students are expected to solve routine mathematical tasks. Therefore, tasks for collaborative learning should be carefully designed. Open-ended tasks, which require problem solving, are appropriate for collaborative learning (Lotan 2003).

Although students' activity is in the first place, the role of teachers in collaborative learning is also very important. At first, students should be trained to work in groups, because the process of collaborative learning, due to its complexity, can, if not organized properly, be very confusing and disoriented. It is necessary that the teacher, at least initially, leads the students through the process of collaborative learning, until they become trained for this kind of learning and understand what they need to achieve (Romer & Whipple 1990).

2.2. Multiple representations

The opinions of the researchers about the kinds of representations differ, but they agree that it is necessary to use several different types of representations for the successful presentation of certain concepts. Different external representations, which offer the same information in different forms, are called multiple representations. Multiple representations provide information about the observed object in different ways, so the observer can analyze its properties from different points of view (Ozgun-Koca 1998). They provide a convenient environment for abstracting and understanding key concepts in mathematics by students, and therefore there are many researchers in the field of mathematics education which deal with multiple representations (Borba & Confrey 1996; Hwang & Hu 2013; Ozmantar et al. 2010).

A quality of the multiple representations is significantly improved by using modern technology. The use of technology to work within multiple representations and to link them has a great attention of the contemporary researchers (Rau, et al. 2015; Ozgun-Koca 2008; Sever & Yerushalmy 2007).

There are several dynamic software packages. One of the more commonly used, because of its availability, simplicity and performances, is *GeoGebra*. This software is used more often in teaching mathematics, from elementary school to university level. The application of *GeoGebra* was the topic of many researches in the last few years (Abu Bakar et al. 2010; Arzarello et al. 2012; Doruk, Aktumen & Aytekin 2013; Takači et al. 2015). All of the mentioned studies have shown that the application of *GeoGebra* in teaching and learning mathematics contributed to better students' achievements.

2.3. Multiple representations of functions

Multiple representations of functions are, in the researches above, usually considered in order to improve students' achievements and to help them to overcome difficulties they have with functions' examining and graphing. Earlier researches have shown that students, when it comes to functions' examining, have the most difficulties in working with functions with variable parameters, as well as with the transformations of functions (Borba & Confrey 1996; Bozic, Takaci & Stankov 2019; Daher & Anabousy 2015; Dede & Soybas 2011; Tall 1992).

Transformations are connected with parameters. They can be considered as a consequence of the parameter's value changing. The use of multiple representations helps students to analyze properties of the functions from different points of view (Anabousy et al. 2014; Borba & Confrey 1996; Daher & Anabousy 2015).

The students have to work with different representations, because each representation enables an adequate overview of some properties of the functions, but no one representation provides a complete overview of functions' properties (Doorman et al. 2012). By analyzing connections between different representations, students can note some characteristics which they wouldn't note by considering each representation separately, because they can observe the dependence between the properties of functions. By observing different representations simultaneously, the students are enabled to choose the most appropriate representation for each case separately (Borba & Confrey 1996).

GeoGebra is one of the software packages which enables connecting of different representations and work within multiple representations of functions. It also enables forming the dynamic multiple representations of the functions, which are being formed by creating sliders, by which are defined variable parameters. The moving of the slider causes immediate changes of the parameter's value and, consequently, causes changes in algebraic and graphical representation, simultaneously.

3. Research questions

The aim of this research is to examine how the students' collaborative learning in *GeoGebra* environment contributes to better understanding the properties of functions with parameters.

Due to the aims of the research, the main research question is:

1. Does the collaborative learning in *GeoGebra* environment contribute to better achievement in examining functions with parameters?

4. Methodology

4.1. General background

In this research, experimental approach was applied and the experiment is conducted with parallel groups: the experimental and the control group. In the experimental group, collaborative learning was applied, and in the control group students worked individually. The benefits of work within collaborative groups were examined. Students of both the groups had possibility of using dynamic software package *GeoGebra* for examining the properties of related families of functions.

All students in the experimental group worked in the four member heterogeneous collaborative groups, in the manner applied in research Takači et al. (2015).

4.2. Participants

The research is conducted with 120 undergraduate students, during their calculus course, at Faculty of Sciences, University of Novi Sad, Serbia, in 2017.

4.3. Instruments and procedures

At the beginning of the research the experimental and control group of students are formed (with 60 students in each group), based on the pretest results (Bozic & Takaci 2021), such that the difference between these groups was not statistically significant at the level of significance 0.01. The pre-test results were used only to ensure that the difference between the experimental and the control group will not be significant, as well as for creating collaborative groups (Bozic & Takaci 2021). Later, during the research, the pre-test results were not used. Instead, the research is conducted similar to the Campbell and Stanley's "design 6" (Campbell & Stanley 1963).

After forming the experimental and the control group, students' work on examining properties of families of functions in GeoGebra environment. The learning process and the analysis of students' tasks is presented in Bozic and Takaci (2021).

Two weeks after their exercises, students of both the groups, experimental and control one, solved the test, without the computer. The results of this test are analyzed and compared.

4.4. Data analysis

The mean score and standard deviation of the test are determined. In accordance with Campbell & Stanley's (1963) instructions for this type of the research, Student's t-test of difference between arithmetic means of two large independent samples was applied for testing difference between students' test results of the experimental and the control group. The effect size is estimated by Cohan's d.

5. Analysis of results of the test

About two weeks after exercises described in (Bozic & Takaci 2021) the students' knowledge about the properties of functions was tested. The test contained two tasks and each task carried 10 points. Time for solving was 120 minutes. The students did not have the possibility to use a computer during the test. The tasks of the test are given in Appendix A.

In the first task, the students were required to work within algebraic and graphical representation simultaneously. This task consisted requirements which enable students' knowledge about the influence of parameters on the properties of functions to be checked. For correctly determined value of the parameter a, the students got 2 points, while examining the properties of the function carried 8 points, distributed in the following way: correctly discussed domain and determined zeros – 2 points, correctly examined asymptotes and period – 2 points, correctly determined derivative – 2 points and correctly examined monotonicity and the extreme values – 2 points. In the second task, students had to work firstly within algebraic representation and examine properties of a given function, and then to sketch the graph of the function. Points in the second task were distributed in the following way: correctly discussed domain and determined zeros – 2 points, correctly examined asymptotes and period – 2 points, correctly determined derivative – 2 points, correctly examined monotonicity and the extreme values – 2 points and correctly sketched graph of the function – 2 points. The average number of points for each task and for complete test is given in Table 1.

Table 1. Average number of points scored on the test (for each task and total)

Group\Task	1st task	2nd task	Total number of points
Experimental group	7.62	5.58	13.20
Control group	5.97	4.85	10.82

By analyzing results given in Table 1, we can notice that students of both the groups had better results in the first task than in the second. In our opinion this is due to the fact that graphical representation of observed function was given.

Maximum number of points on the test was 20 (10 points for each task). The average number of points scored by the students of the experimental group was 13.20 (66.00%), and the average number of points in the control group was 10.82 (54.08%). In the control group there were students with zero points and in both the groups there were students who achieved the maximum number of points. The highest frequencies were about 16 points and about 10 points in the experimental and control group, respectively.

Distributions of points in the experimental and the control group for the test are presented with graphs in Figure 1. The number of points (from 0 to 20, in the intervals of 4) is presented on x-axis, and the number of students who achieved the corresponding points, is presented on y-axis.

Looking at Figure 1 it can be remarked that the black line (the experimental group) is below the gray line (the control group), under 13 points, and the gray line is under the black one in the intervals from 13 to 20 points. In the experimental group 3 students got maximum points, did everything correctly, and in the control group 1 student got maximum points. The statistical results obtained for the test are shown in Table 2.

It can be concluded that the difference between the results of the test of the experimental group and the control group is statistically significant at the level of significance of 0.01($t(118) = 2.9211$; $p = 0.00418$).

It may also be noted that the effect size of the experimental factors is medium (Cohan's $d = 0.533$), meaning that the obtained difference enables the practical advantage of the experimental group compared to the control group. In fact, we proved that the students' learning achievement of the properties of functions is better when they practice in collaborative groups.

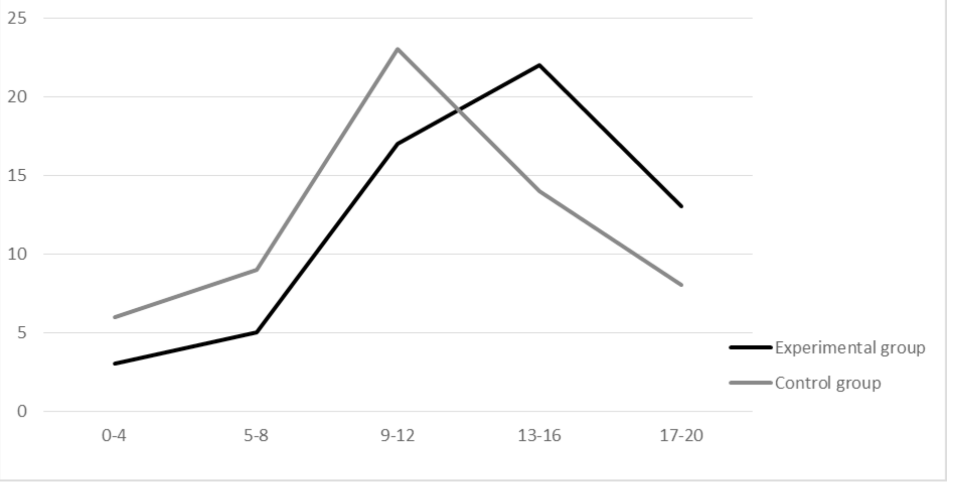

Figure 1. Distributions of the number of students according to the number of points
(scored on the test)

Table 2. Statistical results of the test

Group	Number of students	Arithmetic means	Standard deviation	Test of difference between arithmetic means		Effect size
	n	*M*	*SD*	*t*	*p*(2-tailed)	Cohan's *d*
Experimental	60	13.200	4.505	2.9211	0.00418	0.533
Control	60	10.817	4.432			

6. Discussion

The objective of this research is to show the benefits of collaborative learning of functions with parameters. The dynamic software *GeoGebra*, which enable simultaneous work within dynamic multiple representations, is used within the learning process, in both, the experimental and the control group.

By observing the students' work within the small (collaborative) groups, it is noted that they negotiate and harmonize their attitudes about the internal organization of the collaborative groups. They managed their work in order to finish their job successfully in a short time. When they saw that their organization does not function properly, they were ready to change their way of working and adjust it to current needs, in every moment of their work similarly as in the previous researches (Bozic et al. 2019; Gokhale 1995; Laal & Ghodsi 2012; Takaci et al. 2015).

According to Bozic and Takaci (2021) the students' solutions of the tasks, written works and electronic material were reviewed by the teachers. All students, working in collaborative groups, prepared their final solutions in a form of electronic worksheets, and more than a half of them also attached hand written explanations. It is interesting to note that there were no students in the experimental group who used only classical hand written tasks' solutions. By analyzing students' conclusions and explanations in attached solutions, it can be concluded that they constructed their knowledge in different and interesting ways. Despite these differences, most of the students' solutions were correct and explained in detail (Bozic & Takaci 2021).

In order to check the efficacy of collaborative work, the students of both the groups got the test with appropriate tasks. The results of the test were analyzed statistically. It was shown that the students of the experimental group, working in small collaborative groups, had significantly better results than the students of the control group, working individually.

Finally, we can say that the application of new didactical approach, created for students' exercises of examining the properties of families of functions, in *GeoGebra* collaborative environment, proved to be successful, in terms of students' achievements.

7. Conclusions

In this research, collaborative learning process was applied, unlike the earlier (Anabousy et al. 2014; Borba & Confrey 1996; Daher & Anabousy 2015), where students' worked on examining the functions with variable parameters individually. In the research Takači et al. (2015), collaborative learning was applied, but functions with parameters were not observed. The collaboration between students contributes to overcoming difficulties with understanding some of the most important functions' properties (Bozic & Takaci 2021). From their discussion, we can conclude that the students are able to manage their work within collaborative groups independently, so that the organization of the group contributes to successful solving their tasks.

Dynamic multiple representations of the functions with parameters are appropriate for examining the properties of such functions, because the students can visualize the changes and connections between the algebraic and graphical representation. In this research, students examined more general families of functions, than in previous researches (Anabousy et al. 2014; Borba & Confrey 1996; Daher & Anabousy 2015). By reviewing and comparing the students' solutions of the tasks for learning (Bozic & Takaci 2021) and for the test (Appendix A) the benefits of using dynamic multiple representations in collaborative learning environment are shown. This is, in fact, the positive answer to the research question.

The test results showed that the experimental group students better understand the properties of functions than the control group students and that they are able to apply previously learned in the tasks which are new for them. At the end, we can conclude that, during this research, many benefits of the collaborative learning functions and their properties are confirmed.

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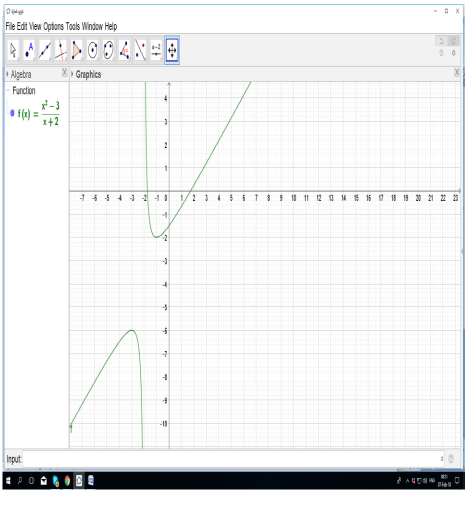
Appendix A. *The tasks for the test*.

1. The graph of the function $f(x) = \frac{x^2-3}{x+a}$ is given.

a) Determine the value of parameter a, so that the graph below corresponds to the function f.

b) Examine the function for the determined parameter and show its properties on the graph.



2. Examine the properties and sketch the graph of the function $f(x) = \frac{x-2}{x-6} e^{\frac{x}{8}}$.

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