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THE EFFECT OF PROBLEM SOLVING INSTRUCTION ON MATHEMATICAL THINKING

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Abstract. This research aims to examine the effect of problem solving instruction on mathematical thinking. The participants of the study include 213 university students (prospective teachers). The participants were required to answer 10 problems before they took "Mathematics Teaching" and "Irregular Problem Solving" courses. After the students successfully completed these courses they were required to answer 10 similar problems. During the data analysis t-test for related sampling and t-test for unrelated sampling were used. The difference in problem solving techniques has also been examined in terms of gender. The study findings show that the courses that the participants took resulted in mathematical thinking changes. The prospective teachers in the study learned the irregular problem solving techniques and successfully applied them in problem solving.

Keywords: Mathematical thinking, reasoning, problem solving strategies, mathematics education

Introduction

The studies conducted on mathematical thinking have shown that before they receive formal education students have preconceptions about problem solving that can be hardly transformed in the following educational years. For example, Confrey (1990) stated that teachers often experience difficulties when they attempt to alter students' background knowledge. Sierpinska (1994) stressed that educational interventions should be done at the right time or else students will pursue to adhere to the existing preconceptions they have. Bracey's (1998) research has shown that although the teacher in his study guided and assisted the students in terms of familiarizing with the new strategies, the students often resisted and used the problem solving strategies that they formerly found effective. In contrast with these findings, several studies have achieved to transform students' preconceptions about problem solving and mathematical thinking. Leinhardt (1998) found that students have changed their intuitive knowledge as a result of the education they received. Moore (2002) achieved to create a student group who eagerly used the new problem solving strategies taught by their teacher.

Considering these contradictory findings in the literature, this study will examine whether the particular university courses that address problems solving skills will transform the participants' preconceptions of mathematical thinking. Another point that the study will focus on will be gender. Thus, the study will examine which gender is more open to adjust to new problem solving skills and transform mathematical thinking preconceptions. The motive to examine the relationship between mathematical thinking transformations and the gender variable stems from the contradictory findings in the literature (see Lubienski, 2002 and Forgasz et al. 2004 for a detailed examinationof the issue).

The research questions of the study are as follows:

1.

• What are the problem solving techniques that the students used before they took "Mathematics Teaching" and "Irregular Problem Solving" courses?

• What are the problem solving techniques that the students used after they took "Mathematics Teaching" and "Irregular Problem Solving" courses?

2.

• What are the problem solving techniques that the female students preferred to use?

• What are the problem solving techniques that the male students preferred to use?

3.

• Are there any similarities and differences between the female students' problem solving techniques and the male students' problem solving techniques before the students take "Mathematics Teaching" and "Irregular Problem Solving" courses?

• Are there any similarities and differences between the female students' problem solving techniques and the male students' problem solving techniques after the students take "Mathematics Teaching" and "Irregular Problem Solving" courses?

4.

• Do the students' problem solving achievement display a meaningful difference according to the techniques that they use?

Method

This research is a study in a descriptive structure which aims to examine the effects of the education that primary school class teaching students in Trakya University on mathematical thinking structures. ''Mathematics Education'' course and ''Irregular Problem solving'' the unit of the same course which the students take at third class are the teaching materials which are expected to influence in this research. The book ''Mathematics Education'' written by Altun (2005) has been used in the mentioned courses. Irregular problems and solution strategies have been mentioned in problem solving section and the students have been made to solve problems using these strategies. These strategies have

been given as 1-) Systematic listing 2-) Estimation and check 3-) Drawing diagram 4-) Finding relationship 5-) Writing open proposition 6-) Estimation 7-) Getting benefit from the solution of similar problems 8-) Studying backward 9-) Making a chart 10-) Judging strategies. It has been stated that the research results done about the problem solving strategies show that these strategies can be learned and they can be used by the students. (Altun,2005).It has been planned here to research the analysis of the students' problem solving strategies before and after education, how they think as mathematical and whether the given education has any effect on the students' approaches towards mathematics problems and their mathematical thinking structures. A similar study has been carried out with the teachers by Dooren,Verschaffel and Onghena (2002) and findings of the research in which the teachers' arithmetical and algebraic problem solving strategies have been examined have shown that the teachers who take part have preferred the algebraic procedure mostly in solving verbal problems. It has been considered in this study that similar results will be got at the beginning of the term and arithmetical problem solving procedure will be preferred at the end of the term. The effects of gender and education type have been also researched in this study.

The students, who have participated in the research, have been wanted to answer 10 sums chosen from class materials at the beginning of education and on the first day of irregular problem solving application. No information about the research to be done has been given to the students and the solutions of the problems by arithmetical or algebraic ways have been left to the students' own initiatives. The aim here is to fix which method the students used to solve the problems before education objectively and whether education has any effects on problem solving approaches or not. 10 similar sums chosen from teaching materials have been given to the same students at the end of the term and whether the education which they get has an effect on their approaches towards problems or not has been examined with the same logic. The students haven't been informed about the research as it is done at the beginning of the term and problems' solutions by arithmetical or algebraic ways have been left to the students' own initiatives.

Universe and sampling

300 third class students who have been getting education in class teaching department, Faculty of Education at Trakya University in 2005-2006 school year have formed the universe of the research. It has been tried to reach the whole universe, the sampling of the research has been determined as 213 students with data loss. 4 groups have been worked together in this research. Class teaching department female students (N=131), class teaching department male students (N=82), class teaching department day time students (N=144), class teaching department night time students (N=69)

Data collection

The data collection devices have been composed of 10 sums which have been prepared by getting help from teaching methods and demanded to be solved by the participants before education. Different sums which have similar content have been chosen at the end of the education by considering that the students will deal with the solution of the problems given first, although the sums solved before and after education are thought to be the same.

Application

The unit ‘‘Problem Solving’’ has been studied nearly 14 weeks (14x6=84 lesson hours) during one term such as ‘‘What is the problem which takes place in the unit? What are the problem solving steps? What is irregular problem and how is it solved? What are the problem solving strategies and how are they used?’ ’Subjects have been studied on and the students have been made to solve irregular problems which demand the usage of each strategy. The students have tried to solve the problems in 10 groups consisted of nearly 5 people in each lesson and organized at the beginning of the term and later they have made comments on solution by carrying out discussions about strategy and its usage with their teachers. Problems which demand usage of different strategies during nearly two weeks when the end of the term is nearer and deficiencies have been completed. The students have been prepared with the strategy they want without any restrictions in the last week.

Data analysis

Students’ problem solutions have been classified as arithmetical and algebraic by being examined one by one. The answers which have been left empty have been named as ‘‘Unsolvable’’. Whether the solutions are right or not has been examined in order to determine the influences of problem solving methods on success. The mistakes which originated from being careless or unimportant calculation errors have been ignored as the research’s aim is to determine the preferred method I problem solving and to determine the influences of strategy usage.

Data provided in the research done have been analyzed by getting help from SPSS 13.0 program in the computer concordant with IBM. Firstly, descriptive analyses of data have been carried out and x (arithmetical average) and S.S. (Standard Deviation) values have been examined. Related samplings t test has been applied in the comparisons of female students pre-test x last-test, male students pre-test x last-test, day time students pre-test x last-test.

Findings

1.

• What are the problem solving techniques which the students have used before and after education?

Table 1. Problem solving techniques used by students before and after teaching

	n	x (ss)
t1 algebra	213	6,14(1,39)
t1arith.	213	2,85(1,32)
t2 algebra	223	3,41(1,74)
t2arith.	223	6,15(1,84)

$p<0.001$

Arithmetical averages and standard deviations which belong to the problem solving techniques preferred by the students before and after education have been given in Table 1.

2.

• Is there any meaningful difference among the female students' problem solving techniques preferences before and after education?

Table 2. The comparison of pre and post tests for famale students

female	n	x (ss)	t
t1 algebra	131	1,398(6,27)	14,397*
t2 algebra	131	1,764(3,47)	
t1arith	131	1,309(2,75)	19,377*
t2arith	131	1,882(6,15)	

*p<0.05 **p<0.01

An increase has happened in female students' algebraic problem solving technique usages after education in meaningful ratio as it can be seen in the table.

• Is there any meaningful difference among the male students' problem solving techniques preferences before and after education?

Table 3. The comparison of pre and post tests for male students

male	n	x(ss)	t	p
t1 algebra	82	5,93(1,359)	10,632	0,00**
t2 algebra	82	3,40(1,818)		
t1arith	82	3,00(1,343)	13,548	0,00**
t2arith	82	6,10(1,890)		

p<0.05 **p<0.01

Male students have preferred the algebraic problem solving technique before education with a meaningful difference in ratio with their preferences education as it can be seen in the table.

3.

• Is there any meaningful difference among the students' problem solving techniques before and after education?

Table 4. The comparison of pre and post tests for all students who participated in the study

Paramet.	n	x(ss)	t	p
t1 algebra	213	6,14(1,390)	17,903	0,00**
t2 algebra	213	3,45(1,781)		
t1 arith	213	2,85(1,324)	23,574	0,00**
t2 arith	213	6,13(1,880)		

*P<0.05 **P<0.01

Meaningful differences among the students' problem solving techniques before and after education have been found. It has been seen that the students have preferred algebraic method at the beginning of term and they have preferred arithmetical method at the end of the term.

• Are there any similarities and differences among the female and male students' problem solving techniques?

Table 5. T-test results based on gender

	gender	n	x(ss)	sd	t	p
t1 algebra	female	131	6,27(1,40)	211	1,787	,075
	male	82	5,93(1,36)			
t1arith.	female	131	2,75(1,31)	211	1,353	,177
	male	82	3,00(1,34)			
t2 algebra	female	134	3,48(1,73)	221	,683	,495
	male	89	3,31(1,76)			
t2 arith.	female	134	6,14(1,87)	221	,106	,915
	male	89	6,17(1,80)			

Preference of algebraic one [$t_{(211)}$ =1.787, p<.01] and preferences of arithmetical one [$t_{(211)}$ =1.353, p<.01] in problem solving techniques before education haven't shown a meaningful difference in terms of gender.

• Is there any meaningful difference among the 1st education students' problem solving techniques before and after education?

4.

Have the students' success in problem solving shown any meaningful difference according to the techniques which they used?

Table 6. The comparison of effects of problem solving techniques on success

Paramet.	n	x(ss)	t	p
t1 algebra	213	4,08(1,262)	10,776	0,00**
t2 algebra	213	2,57(1,527)		
t1arithd	213	1,29(0,926)	20,954	0,00**
t2arithd	213	3,86(1,710)		

*p<0.05 **p<0.01

The results of the comparison that has been made between the solution numbers reached by algebraic and arithmetical methods at the beginning and at the end of the term have shown that more correct results have been reached in problems solved with algebra before education and arithmetical method have brought more correct results with a higher average after education.

Table 7. The comparison of effects of problem solving techniques on success for girls

Paramet.	n	x (ss)	t	p
t1 algebra	131	1,239(4,14)	9,472	0,00**
t2 algebra	131	1,415(2,56)		
t1arith.	131	0,792(1,18)	17,847	0,00**
t2arith.	131	1,651(3,81)		

*p<0.05 **p<0.01

This has shown that the education given during the term has been useful for female students' solving the problems correctly.

Table 8. The comparison of effects of problem solving techniques on success for boys

Paramet.	n	x(ss)	t	p
t1 algebra	82	3,99(1,300)	5,613	0,00**
t2 algebra	82	2,60(1,699)		
t1arith.	82	1,46(1,091)	11,488	0,00**
t2arith.	82	3,94(1,808)		

*p<0.05 **p<0.01

According to this, it has been seen that male students have reached more correct solutions with algebra at the beginning of the term and they have increased their correct numbers with the usage of arithmetical method with a high t value at the end of the term.

Discussion and conclusion

Class teacher candidates' problem solving methods and the influence of education on problem solving have been examined in this study. Whether there can be a change on mathematical thinking structures with the given education or not has been dealt with.

The results of the study have shown that it is possible to change the students' present knowledge with education and it has been possible to change the T.U. class teacher candidates' mathematical thinking structures with education in the wanted direction on the contrary of various research findings (Confrey, 1990; Sierpinska, 1994; Bracey, 1998). The importance of their owning thinking structure at this stage will be realized when it is thought that these candidates will give education at the stage of primary school.

The research Findings have shown that class teaching department students have followed algebraic method by being influenced by their previous education in solving sums before the mathematics education given at the stage of primary school at the university .This preference has shown a change in favor of arithmetic method after education. The students who have been successful in solving problem with algebraic method before education have solved more problems correctly with the usage of arithmetical method after education in the same way. The point to be underlined here is that the recorded number of the problems solved correctly has been more than it is before education. This situation has shown that an increase has been observed in the students' success with the usage of arithmetic method at the end of theory and application lessons have increased in a meaningful way before application course.

A meaningful difference hasn't been come across with when the female and male students' problem solving techniques preferences before and after education as the gender differences have been examined. It has been observed that the female students haven't been able to get benefit from the education in the wanted direction as far as the male students haven't been able to get benefit. The female students who have preferred algebra method in problem solving mostly have continued to use algebra method together with arithmetic method although the usage of arithmetic method has shown an increase after education. On the other hand, female students' success in problem solving as to the situation before education have pointed out that the problem solving ratio has increased as the result of education. Male students have solved problems with algebraic method before education mostly although it hasn't been as the same as female students'.

They have preferred arithmetic method more than female students after education. It has been seen that there has been an increase in the number of correct problem solution after education when success ratio has been examined. This has shown that the education given during the term has provided benefit for female students in correct problem solution. It has been reported that male students have achieved more correct solution with algebra at the beginning of the term and the number of correct answers has increased with the usage of arithmetical method at the end of the term. This has shown that the education given during the term has also provided benefit for male students.

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