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THE COSMOS VERSUS THE INFINITE – ON THE DOUBLE STANDARD TOWARDS INFINITY AND ITS AESTHETIC ROOTS

Latchezar Tomov
New Bulgarian University

Abstract. The concept of cosmos (κόσμος), introduced by Pythagoras, conveys beauty and order, integrating infinity through symmetry but rejecting it via proportion. For the Pythagoreans, beauty lies in the commensurable—what can be measured with finite actions—while infinity appears in irrational, incommensurable quantities, which are unknowable to humans. They believed that the cosmos' beauty stems from numerical order, particularly the ratios of whole numbers, or rational numbers. Beauty, in this context, is tied to what can be understood. However, symmetry in its highest form, like the sphere, embodies the infinite. This duality leads to a mixed attitude toward infinity, accepting it when it pleases and rejecting it when it causes cognitive discomfort. This distinction between actual and potential infinity is aesthetically driven and reflected in both mathematics and philosophy, particularly in works by Plato, Euclid, and Aristotle.

Keywords: Plato; Aristotle; Pythagoreanism; infinity; cosmos; symmetry; beauty; aesthetics

Introduction

In literature, there exists the mistaken belief that among Greek philosophers or mathematicians, such as Aristotle, there is a fear of infinity and that, essentially, what Aristotle constructs is a finite world (Garcia 2023). Here, we will show that Aristotle's attitude towards infinity is more nuanced and that he allows infinity to exist in time but not in space (just as Plato does implicitly), while he distinguishes between potential and actual infinity in numbers. Numbers, as a model of the world for him and for other philosophers like Plato, do not permit the description of the undefined and thus cannot themselves be infinite. Here, we attempt to explain this dual attitude towards infinity as a consequence of aesthetic criteria that have influenced decisions and theories in mathematics throughout the centuries up to the present day (Vargolomova & Tomov 2022), including the selection of problems to solve, as well as the means, methods, and notation (Ivanova 2017, Tomov 2016).

Infinity as indeterminate and boundless

The concept of "apeiron" (ἄπειρον) by Anaximander is used to denote the boundless, the indefinite – both notions are interconnected. Anaximander belongs to the Ionian tradition of materialism, which allows for the existence of countless worlds, and apeiron represents the self-moving and self-generating substance of infinite nature (Andrei 2022, p. 692). The author here is deeply mistaken in his claim that the Ionian philosophers and materialists (Democritus, Anaxagoras, Anaximander) acknowledge natural laws and thus reject metaphysics. The propositions of countless worlds and infinite nature are actually given to avoid describing nature through natural laws, as these would require reason as a source, or a Creator (Tomov 2020). Countless worlds can exist because there are countless principles (something Aristotle strongly opposed, and which contradicts the essence of developed mathematical science, where we aim to reduce everything to a minimal number of principles, as can be seen from the way Euclid's "Elements" are constructed). This is an approach that serves as a model for constructing any theory in mathematics and physics after him (Andrade-Molina & Ravn 2016).

In fact, the propositions for an unlimited, boundless nature directly contradict the notions of rigor at that time, where the indefinite could not be handled, and for this reason, actual infinity was excluded from the reasoning of mathematicians from the times of Theaetetus to Archimedes. Before Aristotle, there was no other definition for the infinite except that it was indefinite and not subject to measurement – infinity, when added to or subtracted from itself, still yields infinity, and therefore it could not be defined, as it was understood back then. The proposal for a universe that emerged from a substance indefinite in its dimensions is in direct conflict with the philosophical tradition from which mathematics arose.

Archimedes wrote the treatise "The Sand Reckoner" (Archimedes 1979 pp. 131 – 142) to demonstrate that this concept is not applicable to the material world, in a finite universe. Everything, even the unimaginable number of grains of sand, can be determined if sufficiently large numbers are constructed— and with this, he creates hyper-exponentiation and reaches a number so large that it is still used today: 1080. The number of all protons and neutrons in the visible universe is of the same order. For Archimedes, just as for other mathematicians of ancient Greece, a number cannot be infinitely large; it must be determinable.

Potential and actual infinity

The understandings of mathematicians were transferred into philosophy as we understand it today by Aristotle. He explicitly introduced the concepts of "potential infinity" and "actual infinity," distinguishing between infinity in terms of multiplication and division, with division being allowed (Hardie & Gaye 1984 p. 47, 207a32-207b15):

"It is reasonable that there should not be held to be an infinite in respect of addition such as to surpass every magnitude, but that there should be thought to be such an infinite in the direction of division. For the matter and the infinite

are contained inside what contains them, while it is the form which contains. It is reasonable too to suppose that in number there is a limit in the direction of the minimum, and that in the other direction every amount is always surpassed. In magnitude, on the contrary, every magnitude is surpassed in the direction of smallness, while in the other direction there is no infinite magnitude. The reason is that what is one is indivisible whatever it may be, e.g. a man is one man, not many. Number on the other hand is a plurality of 'ones' and a certain quantity of them. Hence number must stop at the indivisible; for 'two' and 'three' are derivative terms, and so with each of the other numbers. But in the direction of largeness it is always possible to think of a large number; for the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, never actual: the number of parts that can be taken always surpasses any definite amount. But this number is not separable, and its infinity does not persist but consists in a process of coming to be, like time and the number of time"

In this paragraph, Aristotle separates number from magnitude in geometry (lengths and other measures) due to the fact that not every two magnitudes can be commensurable through a common measure in the form of a number, which is why numbers are separated from geometry. Here we see a curious asymmetry in thinking – there is a limit in the direction of the smallest in terms of number, but not in the direction of the largest, which in itself is not an assumption accepted in modern mathematics, where these are viewed symmetrically. In magnitude, there exists something smaller than any given size, but not something larger than every size. The same asymmetry exists, but magnitude is allowed to be smaller than any other size, whereas a limit is defined for numbers. This is a secondary manifestation of asymmetry in thinking. Both manifestations of asymmetry here, in our view, are aesthetically motivated: a) the separation of numbers from magnitude due to the impossibility of commensuration of some magnitudes through numbers in a finite manner; b) allowing magnitude to be arbitrarily small, but not number, as a result of a). The latter shows the special role of number for Aristotle, where it serves to count sets. Here, counting and measuring are not equated—something that develops in modern mathematics of the 19th-20th century with measure theory, where the measure of a set is not related to the measure of its elements (Grünbaum 197 pp.158 – 76). A third aesthetic consideration is the principled acceptance of the infinite, as long as it is contained within some finite space, which makes it "thinkable," possible to grasp in the mind, seen as a single image, i.e., its true nature can be safely ignored. This is an expression of a sense of beauty through the ignoring of the unthinkable, the indefinite, the indescribable.

In regard to magnitude, it cannot be thought that there is a minimal magnitude, as this would create serious problems in mathematics concerning the method of exhaustion. Therefore, Aristotle allows the smallest not to exist (Aristotle, Guthrie. W.K.C 1939 Fifth Chapter 271b).

„ This, then, is now clear, and we must turn to consider the rest of our problems, of which the first is whether there exists any infinite body, as most of the early philosophers believed, or whether that is an impossibility. This is a point whose settlement one way or the other makes no small difference, in fact all the difference, to our investigation of the truth. It is this, one might say, which has been, and may be expected to be, the origin of all the contradictions between those who make pronouncements in natural science, since a small initial deviation from the truth multiplies itself ten-thousandfold as the argument proceeds. Suppose for instance someone maintained that there is a minimum magnitude; that man with his minimum would shake the foundations of mathematics.The reason for this is that the beginning is more significant for its potential for development than for its size, because what is small at the start eventually becomes vast."

Therefore, there cannot be a smallest magnitude (geometrical, this is not referring to a number). But can there be an infinitely small magnitude? The manifestation of potentiality here lies in the fact that magnitude is allowed to surpass any limit, but it cannot be infinite at any specific moment (Hardie & Gaye 1984, p. 49 3:8, 208a22 – 208a23):

"Magnitude is not infinite either in the way of reduction or of magnification in thought "

According to Aristotle, we cannot have a smallest magnitude, but we also cannot have an infinitely small one in the actual sense – there can always be something smaller (potential infinity). A key sentence in this regard can be read at the end of Physics 3.6 (Hardie & Gaye 1984, p.46 207a15 – 207a31).

"But it is absurd and impossible to suppose that the unknowable and indeterminate should contain and determine"

Why does Aristotle not admit actual infinity? For him, considering an infinite set as complete creates a paradox—since something can always be added, how could it be complete?

However, is this distinction valid from a logical and mathematical point of view? Can we distinguish potential from actual infinity? What is the stance of modern mathematics after Cantor and Boltzmann? In modern mathematics, actual infinity is foundational to axiomatic systems such as ZFC (Zermelo-Fraenkel with the Axiom of Choice) with the so-called Axiom of Infinity out of necessity. This necessity is well illustrated by examples in mathematics, where all objects are finite and infinity is absent from definitions and statements, yet is necessary for their proofs (Simpson, 1987).

Axiom of Infinity (ZFC): There exist sets with infinitely many elements.

The Zermelo-Fraenkel theory + Axiom of Choice is consistent, as far as we know, with no contradictions.

So, where does the paradox arise for our mind (and Aristotle's)?

Actual infinity cannot be grasped by the mind; it is neither knowable nor definable, and therefore it cannot be understood, nor can one derive pleasure from it through reasoning (Aristotle 1986, p.4):

"Another reason for this is the fact that knowledge is supremely pleasurable not only to philosophers but also to other people, with the difference that the latter engage with it only briefly."

Knowledge is supremely pleasurable, while the unknowable is unpleasant. It is what our mind can grasp, imagine, and see. Actual infinity cannot be grasped; it is not a Whole. The Whole is finite (Hardie and Gaye 1984 p.46 207a32–207b15). Aristotle formalizes concepts that were developed by mathematicians among the Greek philosophers (Euclid 1972, pp.13 – 14).

Postulate 2: Any finite straight line can be extended indefinitely.

Axiom 8 The whole is greater than the part.

In this translation Axiom 8 is the same as Common notion 5 in the standard English translations. Postulate 2 allows the potential infinite, while Axiom 8 forbids the actual infinite.

The modern definition of an infinite set (according to Dedekind) is a set that is equivalent to one of its strict subsets, meaning that we can establish a one-to-one correspondence. In other words, an infinite set is one where the whole is (in some sense) equal to a part of it. An example would be even numbers and natural numbers: 1, 2, 3, 4, 5 | 2, 4, 6, 8, 10 – for every odd number such as 1, 3, 5, 7..., there corresponds a number – 1, 2, 3, 4..., meaning we know the position of a given odd number using the formula 2n-1. For example, from 2n-1 = 99, we see that n = 50, meaning 99 is the 50th odd number. Likewise, each position corresponds to a number, i.e., the 100th odd number is 2*100-1 = 199. In this way, half of the natural numbers are "numbered" with all the natural numbers. This is precisely what Axiom 8 aims to prevent, likely introduced because of Zeno's paradoxes, which demonstrate the correspondence between a line segment and a part of it in the exact same way but through geometric means.

However, the mathematical works of Archimedes, Pappus of Alexandria, and even Euclid are not entirely isolated from the use of actual infinity (Mendell 2019). In Book 1, Proposition 12, Euclid creates an infinite line to avoid having to find where a circle intersects a finite line segment (Euclid 1972, p.23).

"To draw a perpendicular straight line to a given infinite straight line from a point not lying on it."

The term ἄπειρον was used in the original Ancient Greek – meaning infinite, undefined, limitless. Despite the efforts of ancient mathematicians, actual infinity sneaks into their work, both explicitly and implicitly. Therefore, it is worth asking whether there is a difference between actual and potential infinity from a logical standpoint. For a long time, it seemed that there was, as Cauchy's approach (the epsilon-delta definition of limits) in analysis was considered logically rigorous, while Newton's infinitesimals sparked serious debates with Bishop Berkeley (Grabiner 1997). However, in the twentieth century, Abraham Robinson's non-standard analysis emerged, in which infinitesimals are rigorously defined and used directly. This rigor, however, involves an extended model of the real numbers, where infinite numbers are included on the number line, and it represents rigor in the modern sense, where mathematics is viewed purely formalistically as a game with rules, without concern for the "truth" of its statements. We are only concerned with whether they are provable according to the axioms we accept and whether the axiomatic system is complete and consistent, i.e., it cannot prove arbitrary statements (ex falso quodlibet). This non-standard analysis not only uses infinite sets and considers infinity as completed, but it also includes infinitely large and small numbers as a core part of the theory. The overall equivalence between this approach and the Cauchy-Weierstrass approach, which uses potential infinity (Tomov 2021), suggests that the distinction between the two types of infinity has purely aesthetic roots. It transforms what is incomprehensible to the mind into something comprehensible and enjoyable to study.

The history of mathematics shows us that the movement towards actual infinity stems from the break with the ancient understanding of logical rigor and the shift towards a new approach to this concept—one associated with the abandonment of the search for meaning and truth in favor of the logical discipline of following rules. The beginning of this theory was marked by infinite sets and further developed with the inclusion of infinite numbers on the number line. The stricter modern mathematics may seem in our eyes, the less rigorous it would appear to the ancient Greek mathematician, the founders of the science. This raises the interesting question of what "logical rigor" actually means when we speak of mathematics. However, the answer to this is a different investigation.

The permitted and prohibited infinities

In "Om the heavens," the universe is infinite in time but not in space (the modern understanding in physics is that they are largely equivalent) (Aristotle 2006, p. 88).

" In this case, we speak of infinite time because the infinite can be defined in some way, namely as that which has nothing greater than it. On the other hand, what is infinite in some direction of extension cannot be said to be either infinite or finite in the absolute sense."

Time and space are largely connected. Aristotle himself defines time as a measure of change (Coope 2006) and, by extension, of motion. What causes this distinction? Why can't we say that what is presented in a certain direction of extension is infinite or finite?

Aristotle links the impossibility of infinity in a certain direction of extension with the definition of the infinite as that which has no greater, using number as a model for space (Aristotle 2006, p. 88).

"I call 'distance' that which exists between lines, beyond which there is no magnitude that touches the lines. It is necessary for this to be infinite because when it comes to finite lines, the distance between them will always be finite. Furthermore, compared to something already given, there can always be something greater, just as with the number we call infinite, since in reality, there is no greatest final number."

What lies behind Aristotle's arguments is the following: space is something we can see with our eyes, something we can represent in our minds. Our mind cannot grasp the infinite, and what is unknowable does not exist. Therefore, infinite space cannot exist. Time, however, is not representable in our mind; we cannot see it, we cannot even attempt to hold it in our mind – it is always just a moment in it. Therefore, it is knowable; it is always potentially infinite for us, for our subjective sensations. We can let time be actually infinite because we cannot even attempt to perceive this actuality. For us, there is always just one moment, which is why we perceive it as potentially infinite. This is a purely aesthetic distinction, because time can also be measured and counted, and then it would be easy to see, by the same arguments, that it cannot be infinite, because there is no largest number to measure it. The choice not to represent time with numbers is entirely aesthetic, stemming from the internal feeling of the eternal "now" – from the senses and sensations comes the representation of extension in space in a different way, through numbers. Geometry is what we see.

Symmetry and infinity

Now for that Living Creature which is designed to embrace within itself all living creatures the fitting shape will be that which comprises within itself all the shapes there are; wherefore He wrought it into a round, in the shape of a sphere, equidistant in all directions from the center to the extremities, which of all shapes is the most perfect and the most self-similar, since He deemed that the similar is infinitely fairer than the dissimilar..

(Plato trans. Lamb 1925, 33b.)

The symmetry of the sphere O(3) represents "actual infinity" – all rotations and reflections that leave the sphere coincident with itself. In the cited passage, the preservation of the sphere upon rotation is referenced, or at least this is how it would be interpreted by a modern mathematician. The context of the dialogue, related to the regular Platonic solids and other geometric forms used, suggests that this interpretation is not arbitrary (Lloyd 2010). At the same time, Plato rejects actual infinity even in this very dialogue (Plato 1990, p. 511, 55d).

"If someone reflects on all of this, they would quite logically struggle with whether to believe that an infinite number of worlds exist, or that their number is finite. The first assumption—that there are an unlimited number of worlds—would actually be considered the opinion of someone with boundless ignorance in a field where they should be knowledgeable."

Here, in the Greek original, "unlimited ignorance" is a repetition of two completely different meanings of the same word ἄπειρος—unskilled, ignorant, and unlimited, infinite. And yet, Plato creates boundlessness, infinity in his cosmogony by giving the world a spherical shape.

On the other hand, Plato is against the inclusion of time in mathematics, the expression of actions within time (Plato 1981, p.338).

"– Therefore," I continued, "no one, even if they have little experience in geometry, would doubt that this science does everything contrary to the words used in it by those who practice it."

"– But how?" he asked. "

– Indeed, they speak rather amusingly and, compelled by necessity, as if they are doing some sort of work, and for the sake of that work, they use all their specialized terms, saying 'let us construct a quadrilateral,' 'let us extend the line,' 'let us add,' and they pronounce all the terms in this manner. Yet, this entire science is designated for knowledge."

"– That is indeed true," he said.

"– Should we not also agree on this?"

"– On what?"

"– That it is for the knowledge of what always exists, and not for what comes into being or perishes."

"– That is very acceptable," he said, "because geometrical knowledge is about what always exists."

The introduction of time is necessary precisely from the perspective of potential infinity – any finite line can be extended indefinitely. Each subsequent number is generated at the moment we need it. The requirement that the infinite set should not be considered complete inevitably introduces time, and Plato is against this, be. Is he in contradiction?

The key lies in symmetry.

In the dialogue "Parmenides," as well as in "Philebus," Plato defends the thesis of an organic combination of the finite and the infinite and the possibility of considering the many as one (Tomov 2023, p. 314). Later, Georg Cantor would give this as a definition for a set. As can be seen from the dialogue "Philebus," Plato accepts the infinite insofar as it can be represented in a finite way, and the sphere is precisely an expression of this representation. The sphere, being a form, something ideal and timeless, a form that is infinitely symmetrical and represents the infinite in a finite way, thus giving determinacy to the indeterminable, is beautiful. Therefore, Plato is opposed to infinity that cannot be given an "acceptable," knowable form – an infinity expressed in numbers. Hence, we argue that the root of his ambivalent attitude towards the apeiron is purely aesthetic.

The same applies to "all right angles," which are equal to each other (Postulate 4) (Euclid 1972, pp.13 – 14). In Euclid, there is no formal approach with a universal quantifier, which is why, purely intuitively, "all right angles" is a description of a completed infinite set. Why do we accept this description? Because when we imagine equality, we always compare only two things – we do not see the entire set at once. This is something we cannot avoid when enumerating or imagining infinitely long straight lines.

In both specific cases, we do not enumerate angles, and we do not have numbers that cannot take infinite values; the completed infinity does not appear in our minds. The knowable infinite is beautiful because it is acceptable. We see actual infinity, but it does not trouble us because it can be comprehended visually and, from there, mentally. It is because we see, we know, and because we know, we love. The beauty of symmetry lies in the ability to grasp the many as one. Infinite symmetry leads to infinite beauty. Moreover, it leads to infinite simplicity. Symmetry is a central theme in mathematics and physics because it greatly simplifies reasoning and conclusions. It is also a central theme in the classical approach to physical beauty, manifested in art (sculptures, buildings). It is naturally the most knowable thing to us and, therefore, the most pleasurable – this is a manifestation of actual infinity that we accept because it does not impose itself on our vision. The very concept of Cosmos is linked to the idea of beauty, which symmetry provides in astronomy – the circular orbits of planets, the musicality, i.e., the rationality in the relationships between their orbital periods, the repetition of their movements. The Cosmos is beautiful because it is orderly, repetitive, and predictable, and this makes it knowable, or at least that is how we perceive it once we manage to ignore the actual infinity that manifests itself in every aspect of it.

Logical rigor as an aesthetic criterion and 'non-rigorous' formalism

"... After one of Kemer's lectures, Dyson, Harish-Chandra, and Kemer were walking to lunch. Harish-Chandra, who until that moment had been a student of physics at Cavendish, made the following comment: 'Theoretical physics is in such a mess that I decided to switch to pure mathematics,' to which Dyson replied: 'That's strange. I decided to switch to theoretical physics for exactly the same reason!" (Schweber 1994, p.491)

The concept of logical rigor originated in ancient Greece, along with the concept of proof based on logic. Logic in ancient Greece was not formal, and the concepts in mathematics, though abstract, were connected to the real world and human intuition. In Greek mathematics, statements had to "make sense," meaning they had to correspond to the mathematician's internal sense, expressed through logic. This internal sense is the same that tells a modern person that it seems strange we can choose in only one way from zero objects, i.e., 0!=1, or that 0.9999…=1. It is the same sense that told medieval mathematicians that numbers make sense when raised to powers of 1, 2, or 3, corresponding to length, area, or volume (the area of a square, the volume of a cube), and therefore any number raised to any power should be expressible in terms of these degrees, which led to cumbersome notation and strange terms like *zenzizenzizenzic* (Vargolomova & Tomov 2022). It is the same sense that tells us we cannot view the set of natural numbers as complete since there is no largest natural number. From an intuitive standpoint, the existence of transfinite numbers larger than the largest natural number seems absurd, since no such number exists; and yet, the definition of ordinal numbers by von Neumann is considered "rigorous" by modern mathematics – the set of all natural numbers is larger than any natural number.

What is essential in the modern approach to rigor is the acceptance of limiting cases – for example, a triangle with an angle of 180 degrees, which degenerates into a straight line, is still accepted as a triangle in order to preserve the generality of conclusions. Parallel lines, which do not intersect at any finite point, are accepted as intersecting at an infinitely distant point, and this simplifies many of our theorems.

In this way, however, there is essentially no difference between a straight line and a circle – a straight line is a circle with an infinite radius. Every acceptance of limiting cases, including 0.999999=1, implicitly involves accepting actual infinity, even in cases where we can define it with a delta-epsilon limit. Including the unattainable limit on equal footing with other cases is precisely the acceptance of actual infinity, and it is this which violates the intuitive concept of rigor in ancient Greek mathematics – the original mathematics. Other violations include generalizations of concepts that "do not make sense," such as choosing between zero objects and 0!=1, or the existence of the number 0 at all, as well as it being even. From a purely logical standpoint, the existence of zero presents a contradiction: it exists both as a number and as a representation of non-existence – the tool we use for counting exists, yet what it counts does not.

Modern mathematics frees itself from this uncomfortable feeling of over-generalization and conflict with intuition through its formalist approach – there is no problem in positing the existence of infinite sets as an axiom or including infinity on the number line, as is done in measure theory – it's just a symbol we operate with. Mathematical concepts are devoid of any "meaning"; mathematics is a game according to the rules of logic, and anything that follows those rules is allowed.

The shift in attitude towards infinity and the generality of concepts in mathematics, the replacement of the intuitive approach with the formalist one, parallels the change in the perception of beauty within mathematics The focus on notation and the elegant manipulation of symbols, and the pursuit of maximum generality with a minimal number of symbols such as Einstein's equations in the general theory of relativity is the new focus (Vargolomova & Tomov 2022). This is related to the revolution brought about by Descartes, which transformed geometric problems into algebraic ones, though the question of causal connection is not entirely clear—whether the change in the attitude towards infinity allowed the revolution, or whether the revolution facilitated this change. It remains a subject for future work to explore the mutual relationship between these two changes and to attempt to answer the question to which this shift leads, namely: "What is logical rigor in mathematics?".

Conslusion

Mathematics is a creation of Greek philosophy, and its foundation is logical rigor. However, in relation to some of the most important aspects of mathematics, such as infinity, aesthetic preferences are hidden behind logic, especially in the case of philosophers who think about mathematics, as opposed to the practice of mathematics itself. The concepts of potential and actual infinity in Aristotle are examples of distinctions driven by aesthetic considerations; the same applies to his dual attitude towards actual infinity in the context of time and space; and similarly with Euclid's postulates in relation to the practice of mathematicians and Euclid himself; as well as the acceptance of actual infinity, which is hidden behind the symmetry of the sphere. What can be grasped by the mind can be accepted because infinity remains in the background. Where it cannot be grasped (extension, infinite numbers), ancient criteria for rigor are applied. The modern understanding of infinity is based on a shift in the criteria of rigor – towards formalism in mathematics – which are again aesthetically grounded due to a shift in the source of enjoyment from the work, from the intuitive connection of mathematical objects and concepts in a meaningful and logical way, to the pursuit of maximum generality and conciseness in notation. The aesthetic roots of logical rigor in mathematics, and the changes in these criteria over time, raise the question of defining the concept of "logical rigor" in relation to mathematics.

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✉ **Dr. Latchezar P. Tomov, Chief Assist. Prof.**
ORCID ID: 0000-0003-1902-6473
WoS Researcher ID: AAR-2987-2020
New Bulgarian University
21, Montevideo Blvd,
1618, Sofia, Bulgaria
Sofia University "St Kliment Ohdridski"
E-mail:lptomov@nbu.bg