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„TAP, TAP WATER“ QUANTUM TUNNELING DEMONSTRATION

Katarina Borković, Andreana Kovačević Ćurić
General Gymnasium of the Catholic School Centre in Banja Luka
Bosnia and Herzegovina

Abstract. Quantum tunneling is a phenomenon in which an atom or a subatomic particle appears on the other side of a potential barrier that should be impenetrable to the particle, given the difference between the kinetic energy of the particle and the potential energy of the barrier, the former being significantly lesser. However, if the barrier shrinks enough the particle will be able to tunnel right through it. This is among the most well-known quantum physics phenomena. Historically, physicists struggled to grasp the wavelike and particle-like duality of light and subatomic particles. The aforementioned phenomenon occurs due to the wavelike behavior of photons. Quantum phenomena, such as quantum tunneling, might appear quite convoluted, therefore a simple experimental demonstration was performed to highlight the basic mechanisms of the phenomenon in a manner comprehensible to the wide masses. The demonstration, however, is just what we are in fact observing instead of quantum tunneling – "optical tunneling", perhaps more commonly known as Frustrated Total Internal Reflection (FTIR). This mathematically analogous phenomenon serves as a mere tool for explaining quantum tunneling while aiding our understanding of it and facilitating its visualization. The experimental demonstration shall prove that quantum tunneling is a daily occurring phenomenon and that it is possible once the barrier is thin enough. If these arguments are indeed met with a successful visual demonstration a wavelike character of the photons happens to be confirmed as well.

Keywords: quantum tunneling (process); potential barrier; kinetic energy; potential energy; photons; Frustrated Total Internal Reflection (FTIR)

1. Introduction

For a complex concept such as quantum tunneling to seem even comprehensible a brief insight into historical events, emphasizing the nature of quantum logic, as different from the logic of classical physics we know, shall prove to be helpful. Classical physics was entrenched as a seasoned discipline capable of explaining particle motions in unambiguous and deterministic ways, especially in the 17th century. However, it did not encompass all aspects of the physical world that were known at the time. The unresolved matter of the nature of light was one of the lingering questions that undermined the view that all big ideas of physics were now known and all that remained is to sort out the specifics with more precision. It was not until the 19th century that significant progress was made in determining the nature of light. Thomas Young produced highly persuasive evidence in 1801, demonstrating that light has the characteristics of wave motion. Young's primary findings focused on what we now term interference phenomena (e.g., Newton's rings). His views seemed to have put an end to the situation. Light has a wave-like appearance. The same claim has been backed up by the electromagnetism theory of James Clerk Maxwell in 1873. However, much more evident and baffling was another problem, originally identified by Lord Rayleigh in 1900 and dubbed "the ultraviolet disaster". The calamity occurred because classical statistical physics estimates that each degree of freedom in the system will get the same fixed amount of energy, a number that is only dependent on temperature. Max Planck had devised an ingenious solution to the problem proposing that radiation was released or absorbed in discrete packets of energy. He stipulated that the energy level of one of these quanta (or packets) would be proportional to the frequency of the radiation. The proportionality constant, now known as Planck's constant, was assumed to be a universal constant of nature. Concurrently with these developments in physics, chemists provided helpful data regarding atom composition. J. J. Thomson discovered in 1897 that the negative charge in an atom was carried by small particles, which were later dubbed 'electrons.' The balancing positive charge was thought to be simply distributed throughout the atom. This concept was named "the plum pudding model", with electrons playing the part of plums and positive charge playing the role of pudding. Ernest Rutherford began investigating how microscopic, positively charged projectiles known as α -particles behaved when they collided with a thin gold film in 1911. Many α-particles passed through little affected but some were substantially deflected. The positive charge of the gold atoms, which would repel the positive α-particles, could not be spread out as in a 'pudding' but must all be concentrated at the center of the atom. The plum pudding model instantly gave way to the 'solar system' model of the atom. Rutherford had discovered the atomic nucleus. Niels Bohr proposed a breakthrough idea in 1913. He applied to atoms the same ideas that Planck applied to radiation. A classical physicist would have assumed that electrons orbiting a nucleus may do so in orbits of any radius. Bohr advocated replacing this continuous option with a discrete constraint that the radii only take one of a set of unique values that could be enumerated (first, second, third etc.). Additionally, he provided a clear prescription on how to compute these potential radii using Planck's constant, h. Albert Einstein revealed the enigma of the photoelectric effect in 1905, which proved to be the next phase in the unfolding saga of quantum theory. He noticed that below a certain critical frequency, no electrons were emitted, however intense the beam might be; above that frequency, even a weak beam could eject some electrons. This perplexing behavior became instantly perspicuous when the light beam was seen as a stream of persistent quanta. An electron would be expelled because one of these quanta collided with it and gave up all of its energy. According to Planck, the quantity of energy in the quantum was proportional to the frequency. However, these findings indicated that light was particle like, which arose confusion among scientists. Modern quantum theory was entirely enhanced between 1925 and 1926, for which two men were predominantly responsible. Werner Heisenberg had been striving to fathom the subtleties of atomic spectra until 1925, when he achieved a significant breakthrough. The computations appeared to be rather complex, as they involved manipulation of mathematical structures known as matrices. As a result, Heisenberg's finding became known as matrix mechanics. Following Heisenberg's discovery, a radically different-looking version of quantum theory emerged, based on the considerably more approachable mathematics of wave equations. This second interpretation of quantum theory was known as wave mechanics, it was found in its fully evolved form by the Austrian scientist Erwin Schrödinger. Prior to this, in his PhD dissertation Prince Louis de Broglie boldly proposed that if undulating light exhibited particle like features, then particles such as electrons should have wavelike properties. De Broglie was able to quantify this hypothesis by generalizing the Planck formula. It had rendered particle like property – energy to be proportionate to wavelike property – frequency. He then proposed that another particle like property – momentum is inversely proportional to another wavelike property – wavelength, with Planck's universal constant serving as the relevant constant of proportionality once more. These equivalences served as a sort of mini-dictionary for converting particles to waves and vice versa. Early in 1926, Erwin Schrödinger published the famous equation that bears his name. Reversing the kind of reasoning that had led from wave optics to geometrical optics, according to Schrödinger, may lead to the discovery of this wave mechanics. In this manner, he found Schrödinger's equation. This equation is the fundamental dynamical equation of quantum theory. Heisenberg and Schrödinger had clearly made significant strides. However, the manner they presented their ideas seemed to be so dissimilar that it was unclear if they had made the same discovery, but articulated it differently, or whether they were presenting two competing ideas. Immediate clarification work ensued, with Max Born in Göttingen and Paul Dirac at Cambridge playing key roles. Dirac's "Principles of Quantum Mechanics", initially published in 1930, encapsulated the general principles most succinctly. The concept that mater acts as both a wave and a particle is central to quantum mechanics. Now interposing core concept of this experimental demonstration:

Quantum tunneling is a microscopic phenomenon where a particle can penetrate, and in most cases, pass through a potential barrier. The maximum height of the barrier is assumed to be higher than the kinetic energy of the particle, therefore such a motion is not allowed by the laws of classical dynamics. Tunneling occurs in all quantum systems. It is crucial for nucleosynthesis in stars and may have had an essential role in the evolution of the early universe. Since its inception quantum tunneling has remained a hot topic, with myriad applications to this day, especially in modern electronics technology (programmable devices, the tunnel diode, quantum computing, and the scanning tunneling microscope). Among all the triumphs of quantum mechanics, as it developed in the third decade of the twentieth century, none was more astounding than the comprehension of the tunnel effect (Polkinghorne 2002; Razavy 2003; Wichmann 1967).

2. Experimental quantum tunneling demonstration in everyday life

When assessing incredibly small objects, principles of classical mechanics / physics do not operate so well, assuming that we are observing an electron. If that is the case, we can no longer predict where it will be at some future time, but we must provide a probability curve of where it is most likely to be at the time of measuring. As a result, some peculiar things can occur, one of which is quantum tunneling. But even on the quantum scale, if something is fairly thick, there is a near-zero probability that you will ever measure the electron on the other side of such a barrier. However, if that barrier becomes very thin, there is a nonzero probability that you will find the electron on the other side of the barrier without ever breaking the barrier.

2.1. Frustrated Total Internal Reflection as demonstrated analogy of quantum tunneling

For a long time Frustrated Total Internal Reflection (FTIR) at plane interfaces has been regarded as an optical counterpart of quantum mechanical tunneling through a potential barrier *(hence the other name – “optical tunneling”)*, attracting considerable interest as an experimentally accessible system for measuring tunneling times (Chiao, Kwiat & Steinberg 1991; Smith & Blaylock 2017), studying wave packet reshaping (Dyrting, Milburn & Holmes 1993), and providing an optical realization of strange quantum-mechanical effects such as quantum evaporation (Heller 2001). The premise for the quantum-optic analogy is the formal resemblance between the temporal Schrödinger equation for a quantum particle (such as an electron) in potential wells driven by an external electromagnetic field and the scalar and paraxial optical wave equation describing the spatial propagation of a monochromatic light beam through weakly curved (or twisted) guiding dielectric structures. Exact details revealing the mathematical nature and conditions of the analogy between quantum and optical tunneling is discussed by Longhi in “Resonant tunneling in frustrated total internal reflection” and “Dynamical tunneling theory and experiment” (Longhi 2005; Longhi 2011).

“When light is incident on the interface of two media, it is partly transmitted into the second medium and partly reflected back into the first one. If, however, the index of refraction of the first medium (n_1) is greater than the index of refraction of the second medium (n_2) and the angle of incidence exceeds the critical angle [$\phi_C = sin^{-1}(n_2 / n_1)$], total internal reflection occurs. The incident light is reflected completely back into the first medium.” (Fig. 1) (Zhu, Yu, Hawley & Roy 1986).

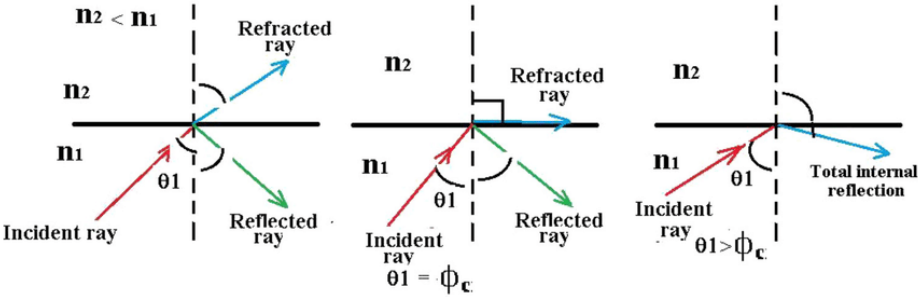

Figure 1. Critical angle behaviour[1)]

In contrast to what the first figure implies, things are not as straightforward as they seem. In fact, the totality of total internal reflection is a rather debatable affair, as it is ensued by another phenomenon – evanescent waves. When light strikes the interface of two mediums past the critical angle it doesn’t completely reflect, part of it (evanescent waves) penetrates the second medium where it gradually fades out (Fig. 2).

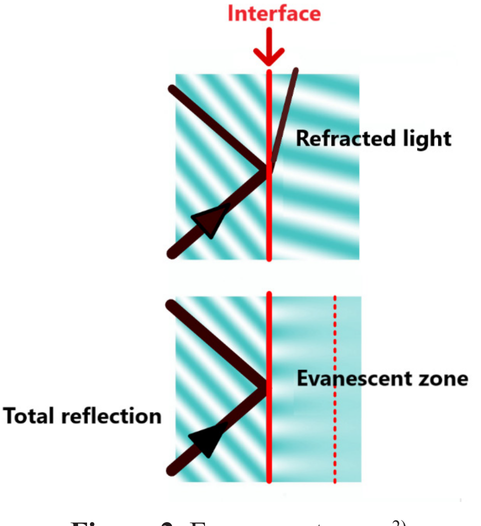

Figure 2. Evanescent wave[2)]

Even if it seems indiscernible to the naked eye during the total internal reflection, the waves are somewhat existent in the second medium as well, which enables phenomena such as FTIR and Attenuated Total Reflection (ATR) to occur. Both of the aforementioned phenomena occur once the third medium with a refractive index similar to that of the first one *($n_3 \sim n_1$)* is introduced to the system at close proximity. Such an array of the three mediums makes it so that the second medium now appears as a gap between the first and the third one. Once the second medium is thin enough (thickness about equal to the incident wave's wavelength) the evanescent wave doesn't die out in the second medium but is rather 'frustrated' so it passes through the second medium into and possibly beyond the third medium as well. Described occurrence is the aforementioned FTIR (Fig. 3). If, however, the second medium happens to possess absorbing qualities which make it a constant 'drain' to the incident wave that is being reflected we are discussing ATR.

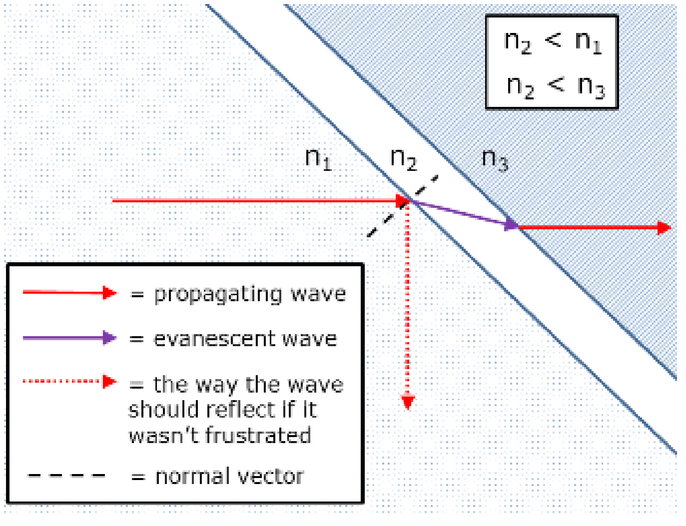

Figure 3. Frustrated total internal reflection diagram[3)]

2.2. Quantum tunneling from the standpoint of physics

A potential barrier is a circumstance in which a free particle encounters an area of space where a repulsive force acts upon it, and the potential energy V_0 of that interaction is greater than the particle's kinetic energy E (Fig. 4). If the barrier has a finite thickness d, as shown in the figure, the exact solutions of the Schrödinger equation reveal that the particle is most likely to be reflected from it. However, beyond the barrier, the wave function is not equal to zero, and there is a finite probability that the particle is on the opposite side of the barrier. The exiting wave function has a reduced amplitude but the same wavelength as the incident one, therefore its energy remains constant as it passes through the barrier. Because each wave function must be continuous and smooth, and therefore must transit from one region to another without fractures nor interruptions, oscillatory wave functions on both sides of the barrier are connected by an exponentially decaying function within the barrier. If it were a classical particle, this would contradict the rule of conservation of energy and could not happen. Comparable phenomena occur with classical electromagnetic waves, and similarly particle's wavelike nature qualitatively interprets this phenomenon (Feynman, Leighton & Sands 1977).

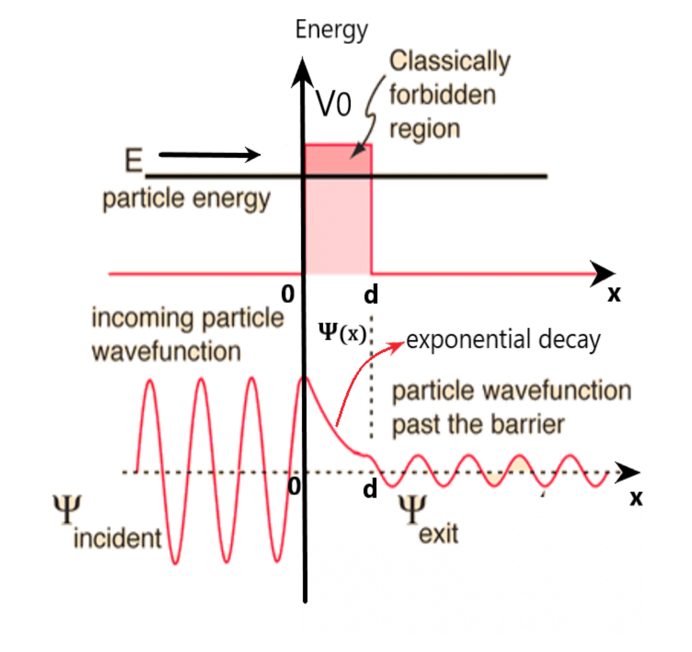

Figure 4. The potential barrier and wave function of a particle before it encounters a barrier, while within a barrier, and after passing through the barrier[4)]

2.3. The experiment and materials required

Materials required for observing an enticing phenomenon, such as quantum tunneling could not be any more accessible. The observation apparatus consisted solely of a glass filled with water. It may appear straightforward, but there are certain details to bear in mind. The observed system could easily be affected by environmental factors (e.g., thermal reactions). To avoid any interference with the demonstration's outcome, we utilized a cold glass (room temperature) and room temperature water (which was 22 degrees at the time in our lab). If by any chance it was not so, some visibility restrictions on the glass and disturbances would trespass such as condensation if the water were cold or the evaporation if water was hot, also the heated glassware once met with cold water is at risk of shattering. The experiment will be discussed now that the observation circumstances have been established.

Material	Index of Refraction (n)
Vacuum	1.000
Air	1.000277
Water	1.33
Ice	1.31
Glass	About 1.5
Diamond	2.417

Figure 5. Refractive index of some common materials[5)]

The particles observed in the experiment are photons, and the impenetrable barrier is in fact a reflective surface – the boundary between water and outer air. Because water and air have different indexes of refraction (Fig. 5), the boundary between the two can behave as a completely reflective mirror. When light is shone out of water past a critical angle, it does not leave the water, but rather reflects off its surface and returns to the water, a phenomenon known as total internal reflection (Fig. 1).

When holding the glass sideways (Fig. 6), the fingers are observable on the other side of the glass, but once when held upright (Fig. 7), the size of the glass changes to that of a mirror, and nothing outside of it is visible. When moving anything down the side of the glass, it is visible, but as soon as it sinks below the surface of the water, it is no longer apparent, thus this water glass boundary is acting as a barrier, not allowing any light to pass through, therefore no light is getting from the fingers to the outside of the glass up through the camera, as is depicted below. This is happening because the air has a lower refractive index than that of the glass and water, so the probability of finding a photon omitted is higher.

Figure 6. Glass held horizontally[6)]

Figure 7. Holding glass vertically[7)]

However, if the boundary is to be made small enough, that is if that layer of air becomes thin enough, the light is observable, so the photons from my finger can quantumly tunnel through the boundary layer of air and get to the camera. If we press extremely hard and attempt to reduce the air barrier between the finger and the glass as much as possible, the fingertips become visible. (Fig. 8) As per the figure, one may argue that what we observe is not FTIR given the absence of a chromatic effect (such as coloration due to the frustration of the evanescent wave). However according to Snell's Law and Electrodynamics in general the wave (evanescent or regular) when trespassing into another medium only changes in regard to its speed and wavelength, while the color-dependent component - frequency remains a constant. Although transmitted evanescent waves do die out eventually which is apparent by the fingertips being engulfed in a white-misty color that contrasts the reddish skin tone.

The only way these photons are getting through this barrier is because we`ve reduced the barrier to a small enough thickness that the photons actually tunnel right through it. Another way to do this instead of pressing really hard with the fingers is reducing that boundary of air in between just by getting the finger wet. Now that the fingers are wet instead of air in between photons and reflective surface there's water. The water has roughly the same index of refraction as the glass and the water inside of the glass, so you can easily see the fingers. Having that water on the finger quickly reduces the layer of air so that the photons from the finger can get to the camera (Fig. 9).

Figure 8. Pressing the glass[8)]

Figure 9. Touching the glass with wet finger[9)]

3. Results and discussion

The experiment itself was made up of four assessment points, each providing useful information. The first assessment point (Fig. 6) was mainly referential (glass held sideways). The second assessment point however demonstrated that an invisible barrier in fact does exist (Fig. 7), even if one is to claim otherwise due to glass and water being transparent materials. The third assessment point (Fig. 8) confirms the hypothesis of tunneling occurring once the barrier happens to be tightened enough. Also, this point demonstrates that doing so mechanically will provide wanted results. However, the last assessment point (Fig. 9) emphasizes the role that index refraction values hold in this scenario. The experiment as a whole confirms the wavelike behavior of photons. The main purpose of the demonstration lies in fact in its simplicity. It is easy to replicate and further develop, but still effectively manages to visually demonstrate complex phenomena such as quantum tunneling, even if it is represented by a mathematically analogous phenomenon (FTIR).

4. Conclusion

The main purpose of the experiment was to visually demonstrate quantum tunneling and to confirm the principles by which it operates using its optical counterpart (FTIR). The materials utilized in an experiment themself indicate that a phenomenon can occur even in the simplest systems (water-glass-air), hence proving the daily presence of the tunneling effect in our lives. Since the fingers were visible once the glass had been mechanically pressed, the hypothesis regarding the barrier reduction has been proved as well. Further analyses of the experiment using different mediums substituting water could prove to be useful and could possibly emphasize the importance of refraction indexes even more. Also, the system once spectroscopically assessed could provide interesting information regarding energy during the tunneling effect occurring, and could perhaps give insight into more crucial processes.

NOTES

1. Figure 1: Critical angle behaviour. Available at: https://www.researchgate.net/figure/Shows-a-critical-angle-behavior-12_fig4_355432907 [Accessed 10 Aug, 2022].
2. Figure 2: Evanescent wave. Available at: https://commons.wikimedia.org/wiki/File:Evanescent_wave.png. [Accessed 15 August 2022].
3. Figure 3: Frustrated total internal reflection diagram. Available at: https://study.com/cimages/multimages/16/frustrated_tir_diagram_fixed.png. [Accessed 15 August 2022].
4. Figure 4: The potential barrier and wave function of a particle before it encounters a barrier, while within a barrier, and after passing through the barrier. Available at: http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/barr.html. [Accessed 25 June 2022].
5. Figure 5: Refractive index of some common materials. Available at: https://www.scienceabc.com/pure-sciences/what-index-of-refraction-defintion-examples-water-air-glass.html. [Accessed 25 June 2022].
6. Figure 6: Glass held horizontally. Borković, K., 2022. Glass held horizontally. [image]. [Accessed 25 June 2022].
7. Figure 7: Holding glass vertically. Borković, K., 2022. Holding glass vertically. [image]. [Accessed 25 June 2022].
8. Figure 8: Pressing the glass. Borković, K., 2022. Pressing the glass. [image]. [Accessed 25 June 2022].
9. Figure 9: Touching the glass with wet finger. Borković, K., 2022. Touching the glass with wet finger. [image]. [Accessed 25 June 2022].

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✉ **Katarina Borković, student**
General Gymnasium of the Catholic School Centre
Srpska 30, Banja Luka 78000
Bosnia and Herzegovina
E-mail: kacaborkovic@gmail.com

✉ **Andreana Kovačević Ćurić, mentor**
ORCID ID: 0000-0002-1773-6765
General Gymnasium of the Catholic School Centre
Srpska 30, Banja Luka 78000
Bosnia and Herzegovina
E-mail: andreanakc88@gmail.com