

STUDY ON THE PERCEPTIONS OF TEACHERS IN MATHEMATICS RELATED TO THEIR PEDAGOGICAL KNOWLEDGE

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Abstract. The article presents a survey among teachers in mathematics with the idea to evaluate their professional and methodological skills as a factor in the results of national external assessment of 15-year-old students in Bulgarian school. The sample is representative - the opinion of teachers in mathematics from all over the country is taken. They are also part of the team trained in the methodology for assessing the exam. The results show that teachers have a high level of professional training, but the methodological expertise is less. The authors conclude that not only what is taught is important, but also how it is presented.

Keywords: mathematics training; exam; methodology

Introduction

National External Assessment is a final form of testing that allows the assessment of general mathematical preparation of students for completion of primary education. This exam is designed to test that students have met the existential minimum of knowledge, skills and mathematical competence, and there is a grading feature for the upper secondary education level. The success of solving the tasks of the exam is conditioned not only by the good knowledge of the subject, but also by the proper preparation for this exam by the students. National external assessment after grade 7 in this format has existed since 2009/2010 academic year, but has been implemented in the country long before in 1949. Regardless of the stated purpose, the serious analytical and methodological activity of the experts, preparing the measurable materials of knowledge in mathematics, experimental and experimental verification, the results of NGOs in mathematics after the 7th grade signals that there is more to be desired in mastering learning material in student achievement. The results of the NGOs after the 7th grade in 2019 indicate very low average results: the average score in the math exam is 33.62 with a maximum of 100 points.

Many value the idea of NGOs with a certain amount of skepticism. At the internet, online forums, in conversations with educators, students and parents, many claims to

the form and content of the exam can be seen and heard, the procedure for conducting it, including the expert evaluation of the two modules of the exam: the first module, consisting of consists of 17 choice-answer and 3 short-answer tasks (which should be solved in no more than 60 minutes), and the second module – 2 short-answer and 3 description tasks whose solution requires no more than 90 minutes.

All of these discussions require examiners to have clear and accurate knowledge of both seventh-grade math content and specific knowledge of criteria and evaluation. The main problem with valuation is that, despite clear standards or criteria for how it can be carried out so that there is no significant difference in valuation, valuation is a subjective factor. The introduction of a new model of mathematics exam for students taking external assessment after grade 7 is necessary by the need to improve the form of ultimate control in today's and ever-evolving world. The mathematics exam given to students after seventh grade for external assessment requires them to analyze information provided in graphs and tables, to evaluate geometric structures, to be able to construct and study mathematical models, to use acquired and validated knowledge and skills in mathematics in practical work.

Exposure

However, for best assessment of exam results, it is a must that teachers should have sufficient knowledge of the content of mathematics for the respective grade, with all possible misconceptions and possible errors that students of this grade can do. Recently, a number of scholars have been trying to determine the necessary elements and predictors of the basic knowledge of teachers needed to teach mathematics. Some authors have found that teachers acquire the necessary knowledge for teaching mathematics more by entering the education system and 'adjusting' to the requirements imposed by the MES than at the university (Baal 2005). For the first time, the pedagogical knowledge that teachers need to have is classified by Schumann in three categories: subject content knowledge, curriculum knowledge, and pedagogical content knowledge (ways to transform and transfer subject knowledge in this way, to be accessible and understandable to students) (Shulman 1987). The latter is important because two teachers with the same level of knowledge can transfer their knowledge in a completely different way when teaching mathematics (Bukova-Güzel 2013; Ernest 1989).

The three categories of pedagogical knowledge identified by Shulman (Shulman 1986) are universal in all fields of teaching, but many researchers involved in mathematics teaching use them as a basic approach in analyzing the pedagogical knowledge that a teacher should have in mathematics. For example, the notion of pedagogical content knowledge, which is used to distinguish teachers' knowledge that combines pedagogy and content, is used as a basis for defining the specific mathematical knowledge needed by teachers. Shuman (1986) also notes that teachers who have knowledge of pedagogical content should also have knowledge

of optimal and functional presentation of concepts, knowledge of what can facilitate or complicate the learning process, knowledge of delusions either misconceptions of students knowledge on diving sufficient stock of examples or explanations with which to assimilate new concepts, knowledge of perceptions general the culture that students have about the subject at different ages. According to (Bransford, 2000) the content of knowledge implies a deep foundation of factual knowledge, understanding of facts and ideas within a particular conceptual framework and organization of knowledge in a way that facilitates the extraction and application. This is the knowledge that enables teachers to engage in specific learning tasks, such as how to accurately represent mathematical concepts and ideas, provide mathematical explanations for common rules and procedures, and explore unusual methods for solving problems and problems (Hill 2008). Ball and Bass (2000) use the term mathematical knowledge to teaching to outline the complex relationship between knowledge and the teaching of mathematics content, and they also distinguish two key concepts in terms of knowledge: 'conventional' knowledge of mathematics, which assumes that each person must have 'specific' o “mathematical knowledge (knowledge of processes, models, functions, and algebra), which is more characteristic of mathematics teachers and other experts in the field (Baal 2005; Bukova-Güzel 2013).

Not much research is devoted to the study of mathematics teachers' knowledge. An, et al. (An 2004) conducted a comparative study on pedagogical content knowledge and examined differences in the knowledge of mathematics teachers in China and the USA. They find that mathematical content knowledge differs between the two countries: Chinese teachers rely more on developing procedural and conceptual knowledge through traditional teaching practices, while their US counterparts focus on promoting creativity and providing activities designed to develop understanding of mathematical concepts from students. Other studies such as that of (Tchoshanov 2017) analyze the results of Russian and American students in mathematics (both countries are high in world scores: Russia in 6th place and the US in 9th) by section and see significant differences, e.g. Russian students have higher algebra and geometry scores, but American students are better at probability theory and graph analysis. The same authors conclude that the differences between Russian and American teachers are rooted in teacher training themselves: on average, 240 hours are taught in mathematics teacher training programs in Russia, and 120 hours in the US. A detailed analysis of the content of the curriculum reveals that in Russia, more emphasis is placed on algebraic problems and less on probability theory, while in the USA more hours on probability theory and graph analysis are laid. Russian curricula focus on the evidence in mathematics courses, while in the US they are only offered in elective academic courses. Bulgarian mathematics curricula are close to those of the Russians, both in content (mainly disciplines related to algebra and geometry) and in the hours of study at different universities:

230 – 240 hours are intended for studying school mathematics and another 90 – 95 hours – for hospiting and teaching experience.

These differences in curriculum outcomes make it necessary to establish the specificity of teachers' knowledge on topics and sections included in the secondary school curriculum in order to explain these differences in knowledge and among students. There is research on teachers' knowledge in various fields, including algebra (McCrary 2012) and geometry Nason, et al. (Nason 2012) and others. Over the last decade, the number of international and cross-cultural studies on teacher knowledge and training has increased significantly to understand where students' differences in international test scores such as TIMSS and PISA originate (Tchoshanov 2017), (Wang 2005). TIMSS results show a steady decline in student achievement in 8th grade mathematics in Bulgaria (Bankov 2007, 2015). Researchers address these differences through a variety of quantitative, qualitative and mixed methods, focusing on characteristics such as teachers' perceptions of teaching mathematics effectively (Hemmi 2015), as well as teacher knowledge (Tchoshanov 2017). Regarding differences in teaching mathematics and teacher approaches, the Swedish-Finnish study of Hemmi and Ryve (2015) found that in Sweden for effective teaching, teachers emphasize interaction with individual children, while in Finland, educators rely on providing a clear picture of mathematics combined with homework. However, there is a lack of research in Bulgaria that provides a detailed analysis of the various aspects of teacher knowledge.

Kinach (2014), Özdemir & Soylu (2017) distinguish 4 levels of relational understanding: conceptual level (different meanings, identifying patterns and relationships and categorizing phenomena), problem solving level (includes analytical methods such as induction, deduction and modeling), epistemic level (contains information about the knowledge itself, ie explanation of the facts, causes and concept of problem solving) and level of study (advanced problem solving in which new knowledge, problems or theorems are offered).

Authors such as Lucas (2006) compare the teaching experience of 18 educators with respect to pedagogical content knowledge from complex functions. Teachers provided detailed answers to controversial questions. According to the results of this study, teaching experience does not influence the content of teachers' knowledge of teaching composite functions. The hypothesis of researchers that with the increase of work experience in school mathematical knowledge of teaching teachers will increase, is not confirmed. The authors explain this phenomenon by saying that the educational system does not focus sufficiently on mathematical knowledge for teaching, but rather focuses on mathematical content. As a result, these authors recommend focusing more on how to teach rather than what is taught (Akzu 2016). Killpatric (2001) examines five concepts that determine mathematical skill: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive propensity.

Experiment

The purpose of this study is to establish teachers' perceptions of their methodological knowledge of mathematics. On the one hand, Bulgarian students occupy one of the leading places in the world of mathematics (288 medals have been won since 1959, 54 of which are gold), but on the other, the overall presentation of TIMSS and of NGOs after 7 Grades 1 show a steady decline in students' achievement in mathematics, and this necessitates a study to reveal the mechanisms for both the good preparation of students in world competitions and the overall decline in mathematical literacy among the population.

In the present study teachers' perceptions of their pedagogical knowledge of mathematics content will be reviewed. Such a study will identify the components of pedagogical knowledge that teachers consider to be their vulnerable unit and, consequently, enhance the preparation of these components through specialized training and thus lead to even higher mathematical literacy outcomes among students. The survey would also be beneficial for principals, administrators and teachers themselves, as it aims to inform all subjects in the education system how to review courses in the mathematics teacher training program, and optimizing this process would it also allowed for a better response and response to the needs of students in mathematics classes.

The study was conducted during training on the topic "Harmonization of the assessment criteria for mathematics of NGOs after grade 7". The purpose of the training was to introduce clear indicators and criteria for the assessment of NGOs in mathematics, so that the differences in the assessments of NGOs checking teachers are minimized.

Toolkit

Scale for Pre-service Mathematics Teachers' Perceptions Related to their Pedagogical Content Knowledge by Mathematics Teachers of Bukova-Güzel et al. (2013). The original version of this methodology contains 17 versions, divided into 5 scales: knowledge of teaching strategies; knowledge of language and symbols in mathematics; knowledge of misconceptions; knowledge of learners; knowledge of the curriculum. The scale measures responses using a 7-dimensional Likert-type scale.

Feedback scale for mathematics training on the topic "Uniformity of the assessment criteria for mathematics of NGOs after grade 7". Contains eight questions in a 7-dimensional Likert-type scale, related to satisfaction with the training. It contains questions such as "To what extent was your training clear and comprehensible", "To what extent was your training interesting?", "Will you be able to put the knowledge gained into practice?", Etc. The purpose of the feedback is to measure the overall satisfaction with the teacher training.

Researchers

113 mathematics teachers who are experts in the field of doctology and in particular - the external assessment of the mathematics exam after 7th grade. Of these, 94.5% had a Master's Degree and the rest had a Bachelor's or Doctoral degree. About 90% are female and the rest are male. 38% said they teach high school students, 11% teach in high school, and the remaining 51% percent teach high school and high school students. 56% teach in primary school, 12% – in vocational high school, 32% – in specialized high school. 8% have internships in the specialty up to 10 years, 19% – up to 20 years, 47% – up to 30 years, and 26% – over 30 years.

Results

The scale of perceptions of mathematics teachers related to their content knowledge is not standardized and validated for Bulgarian conditions.

Factor analysis was performed to determine the structure and validity of the scale. The Kaiser-Meyer-Olkin coefficient is 0.887, which is a good prerequisite for factorization of the verses. Varimax with Kaiser normalization was used as the rotation method, and the factor extract showed 4 rather than 5 scales as in the original scale. The distribution of variables by scale is shown in *Table 1* below.

Table 1. Analysis of the main components

	Rotated Component Matrix	Component			
		F1	F2	F3	F4
1	I can think of appropriate activities and tasks for presenting mathematical concepts.	0,817	0,161	0,271	0,123
2	I can relate mathematical concepts and understandings to everyday life in task instructions.	0,889	0,151	0,050	0,253
3	I can use analogies for mathematical concepts in instructions.	0.853	0,272	0,129	0,255
4	I can use mathematical language correctly when presenting mathematical concepts.	0,386	0,304	0,425	0,410
5	I can use mathematical symbols correctly.	0,084	0,354	0,753	0,166
6	I know the possible misconceptions and "stumbling blocks" of students on a given topic.	0,321	0,212	0,340	0,599
7	I can think of activities that will not cause students to develop misconceptions about a particular topic.	0185	0.315	0,216	0,792
8	I can choose appropriate examples according to the level of development of the students in my lessons.	0,269	0,265	0,580	0,458
9	I have knowledge of the purposes of the mathematics curriculum.	0,192	0,595	0,474	0,378
10	I can come up with a lesson plan for a topic.	0,160	0,460	0,627	0,328

11	I plan my lessons so that I relate the goals of the math curriculum to the needs of the students.	0,207	0,825	0,187	0,272
12	When designing a curriculum and lessons, I always consider the goals of the topic.	0,208	0,905	0,038	0,032
13	I can use the tools provided to evaluate the mathematics curriculum.	0,439	0,684	0,275	0,211
14	I can evaluate the effectiveness of the activities I carry out in a class for students to understand the material.	0,435	0,393	0,517	0,341

Analysis of the main components:

F1 Knowledge of teaching strategies; F2 Curriculum knowledge; F3 Knowledge of mathematical language, symbols and teaching methods; F4 Knowledge of misconceptions

There are three versions in F1: "I can think of appropriate activities and tasks to represent mathematical concepts", "I can relate mathematical concepts and understandings to everyday life in task instructions", "I can use analogies for mathematical concepts in instruction", "I can use the results of my assessments when designing and adjusting tasks." The semantic analysis of these versions shows that this is a knowledge of mathematics teaching strategies. F2 includes four versions: "I have knowledge of the goals of the math curriculum", "I plan my lessons so that I relate the goals of the math curriculum to the needs of the students", "When designing a curriculum and lessons I always take into account the goals of topic", "I can use the tools provided to evaluate the math curriculum." The meaning around all of the versions is oriented towards the knowledge of the curriculum. F3 includes the following version: "I can use mathematical language correctly when presenting mathematical concepts", "I can use mathematical symbols correctly", "I can select appropriate examples according to the level of students' development in my lessons", "I can devise a plan for a lesson on a topic", "I can evaluate the effectiveness of the activities I apply in a class to understanding student material", "I can anticipate possible difficulties for students on a topic." The semantics of these statements are based on knowledge of mathematical language, symbols and teaching methods. There are only three statements in F4: "I know the possible misconceptions and" stumbling blocks "of students on a topic", "I am aware of the students' prior knowledge of a topic" and "I can think of activities that will not make students develop the wrong perceptions on a particular topic, that is to say semantics is about knowing false and contemptible perceptions".

Analysis of the reliability (Cronbach's alpha) showed that if deleted from the "Knowledge of teaching strategies" factor, the stated "I can use the results of my assessments in designing and adjusting tasks" increases from 0.722 at 0.915. As for the second factor, Curriculum Knowledge, Cronbach's ratio is 0.868. In the third factor, after the item "I can expect possible difficulties for students on a topic" is deleted, the reliability of the scale increases from 0.714 to 0.877. In F4, too,

when deleting the “I am aware of students' prior knowledge of a subject” reliability factor increases significantly, from 0.425 to 0.756. After the deletion of these three versions 14 of them still remain present in teachers' methodology.

Factor analysis and reliability analysis have shown that this methodology is a valid and reliable tool for examining mathematics teachers' perceptions of their knowledge of pedagogical content.

In order to determine the relationship between the different subscales, as well as between the different scales and the feedback for the training, a correlation analysis was done. The results are presented in *Table 2* below:

Table 2. Correlation analysis

		Teaching strategies	Subject – specific vocabulary and methods	Curriculum	Misconceptions	Satisfaction with learning
Teaching strategies	Pearson Correlation Sig. (2-tailed) N	1 113	0,542* 0,000 113	0,403* 0,000 113	0,540* 0,000 113	0,491* 0,000 113
Subject – specific vocabulary and methods	Pearson Correlation Sig. (2-tailed) N	0,542* 0,000 113	1 113	0,725* 0,000 113	0,588* 0,000 113	0,566* 0,000 113
Curriculum	Pearson Correlation Sig. (2-tailed) N	0,403* 0,000 113	0,725* 0,000 113	1 113	0,573* 0,000 113	0,472* 0,000 113
Misconceptions	Pearson Correlation Sig. (2-tailed) N	0,540* 0,000 113	0,588* 0,000 113	0,573* 0,000 113	1 113	0,377* 0,001 113
Satisfaction with learning	Pearson Correlation Sig. (2-tailed) N	0,491* 0,000 113	0,566* 0,000 113	0,472* 0,000 113	0,377* 0,001 113	1 113

*The correlation is significant at the 0.01 level (two-sided)

The correlation analysis shows that the highest correlation is between the “Knowledge of the syllabus” and “Knowledge of mathematical language, symbols and teaching methods” ($r = 0.725$; $p < 0.001$). It can be assumed that mathematics teachers' knowledge of the curriculum is directly related to their knowledge of

mathematical symbols and teaching methods. In fact, the curriculum is a summary document outlining the content and amount of knowledge and skills that each student should have. In order to be able to provide quality teaching material to their students, teachers must be proficient in the curriculum requirements and also have sufficient knowledge of mathematics.

The correlation between the Knowledge of Misconceptions scale and the Curriculum Knowledge scale ($r = 0.573$; $p < 0.001$) is also medium and statistically significant. Knowing the curriculum for each class, the teacher can also orient themselves to the source of potential misconceptions or misconceptions in students. The teacher must be aware of the knowledge and skills of his / her students (including knowledge of the curriculum) in order not to go beyond this framework, and must also be able to judge whether a definition or statement is mathematically correct (Bankov 2015).

The correlation between “Knowledge about teaching strategies” and “Knowledge about students' misconceptions” ($r = 0.540$; $p < 0.001$) is also of medium and statistically significant significance. Mathematics teachers who are aware of possible misconceptions or misconceptions in the teaching material have knowledge of how to present the teaching material so that they can cope with the mathematically incorrect knowledge of the students. According to Bankov (2015), the mathematics teacher must be able to present abstract and complex mathematical concepts in such a way that they are understood by the students, that is, to “unpack” what the mathematicians have compressed to the students (Bankov 2015).

The relationship between the Knowledge of Teaching Strategies and the Knowledge of Mathematical Language, Symbols and Teaching Methods ($r = 0.542$; $p < 0.001$) is of medium and statistically significant significance. This interrelation indicates that good theoretical training of mathematics teachers is directly linked to knowledge of teaching methods and the selection of a particular teaching strategy to optimally convey students' teaching material related to basic mathematical concepts and operations. In order to be able to undertake a teaching strategy (Shelehova, 2015) believes that the teacher should be able to build a connection between the object and its symbol, which implies: 1) perception and comprehension of the content of the task, distinguishing the elements of the internal structure of the task in the process of analysis and 2) presentation of the whole structure of its entirety, as well as its individual parts, established connections between the various components of the structure. All this will show the linguistic nature of mathematical knowledge and will orient students to mastering mathematics as one of the languages aiming at preserve and processing the information about the world, human beings, and expand and enrich communication opportunities.

The correlation analysis between the Satisfaction with the Satisfaction with the conducted training “Uniformity of the assessment criteria for mathematics of NGOs after 7th grade” and the other scales of knowledge shows that all the relationships are statistically significant at the level of 0.01. The highest correlation was with the

“Knowledge of mathematical language, symbols and teaching methods” scale ($r = 0.566; p < 0.001$). Teachers who evaluate their knowledge of mathematical concepts and principles are greatly satisfied with the teaching of mathematics assessment. Gaining new knowledge of the assessment criteria for mathematics after grade 7, they select exactly what they need to know about mathematical symbols and concepts that they need when examining students' writing. Specific criteria for evaluating identical expressions with clearly defined norms or linear inequalities with a single uncertainty $ah + b = 0$ and equations that are reduced linearly by equivalent transformations, etc. facilitate educators in the assessment process. The correlations between the training provided and “Knowledge of teaching strategies” ($r = 0.491; p < 0.001$) and “Curriculum knowledge” ($r = 0.472; p < 0.001$) are slightly lower. The lowest correlation was found between the training conducted to align the assessment criteria in mathematics and the Knowledge of Misconceptions ($r = 0.377; p = 0.001$). Those teachers who evaluate their knowledge of the inaccurate and perceptions and misconceptions of students are less satisfied with the training. It is assumed that these teachers, knowing well the potential misconceptions or misconceptions of the students, are aware of the assessment of these inaccuracies and are less appreciated of their satisfactory training.

Cluster analysis

Figure 1 shows the approximate dependencies between the rocks. There are two clusters that are separated. The clusters closest to each other are the Knowledge of

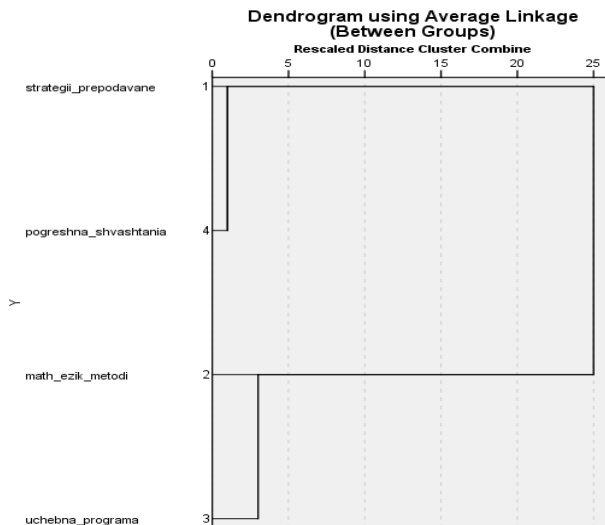


Figure 1. Approximation dependencies between rocks

Teaching Strategies and Knowledge about Student Misconceptions scales. Teachers who are aware of possible mistakes that students can make (ie, poor reading and misunderstanding of the task, technical mistakes) have teaching strategies in place to respond to such situations. Typically, in the case of incorrect, inaccurate or incomplete understanding of mathematics teaching material, teachers apply two types of strategies: constructing logical sequential actions in describing any tasks and their overall solution; building strategies for the dynamics of thinking.

The second cluster includes the scales “Knowledge of mathematical language, symbols and teaching methods” and the scale “Knowledge of the curriculum”. Understanding curriculum content and goals, seventh grade mathematics training (such as linear equations with one unknown $ax + b = 0$ and equations that reduce linearly through equivalent transformations; Elements and properties of ridge bodies (cube, rectangular parallelepiped, straight prism, correct pyramid), etc.) give an idea and guidance to the teachers how to teach the mathematical concepts then learned, such as axioms, theorems, identical expressions, sum of angles in a triangle, etc.

Influence of gender and internship on the scales of perception of mathematics teachers' knowledge.

After conducting a multivariate analysis of variance (MANOVA), it was found that there was a statistically significant relationship between the Knowledge of Misconceptions scale and gender $F(6,762) = 4.237; p = 0.046$. Women report higher values for having more knowledge of students' misconceptions. The difference on this scale is not very large between the two genders, and it should be noted that in the whole sample men are 10%, which is an indicator that a mistake is possible.

Statistically significant differences were observed in the pedagogical factor factor on the “Knowledge of teaching strategies” scales $F(24,89) = 4,560; p = 0.001$, “Knowledge of mathematical language, symbols and teaching methods” $F(16,228) = 2,699; p = 0.027$ and “Knowledge of students' misconceptions” $F(9,414) = 5,898; p < 0.001$. At all three rocks, it is observed that with the increasing number of years of teaching experience at school, teachers are gaining more and more knowledge about teaching mathematics.

After a one-way ANOVA, it was found that on the “Satisfaction with Learning” scale the only statistically significant difference was observed by the “ACS” factor $F(144,35) = 5,560; p = 0.027$. Teachers with a Bachelor of Arts Degree are more satisfied with the training than those with a PhD. It can be assumed that these teachers, experts in the assessment of NGOs in mathematics after seventh grade with the Doctoral degree program have higher criteria for acquiring new knowledge and are more familiar with the tendencies for criterion evaluation of the written works of students who have graduated seventh grade.

The new stage in the development of school education is related to the implementation of a competency-based approach for content formation and

organization of the learning process, with the need for educated students to apply the learned knowledge and teachings to specific learning or life situations. The implementation of a competency approach implies forecasting the results of the content of the training, which also requires changes in the system of assessment of the level of academic achievement. The application of a criterion-based assessment system is determined by the contemporary strategic objectives of education, the need to increase the level of education, taking into account international standards and requirements for quality education, with the need to develop uniform requirements for evaluating students' academic achievement after seventh grade in order to maximize the objectivity of the results

The current study found that, in order for teachers to evaluate students' knowledge after an NGO after the seventh grade, they must have very good knowledge of both assessment criteria and pedagogical content (An 2004) for effective education and training in mathematics, because knowledge of pedagogical content is a major factor in developing students' understanding. The students' understanding of the mathematics material at school is mainly realized in the process of teaching at school. Therefore, teachers need to have a conceptual level knowledge of pedagogical content and a conceptual level of understanding of mathematics material. Therefore, it is possible to improve knowledge of the pedagogical content of teachers through training courses in line with the learning objectives. This study was conducted to assess the level of knowledge of the mathematical content of mathematics teachers. Such studies can also be conducted by teachers in other fields.

In conclusion, the most important for the successful passage of the NGO after the seventh grade in mathematics is the meaningful and sufficient mastering of all the main content and methodology of the mathematics course, knowledge of the cultural and mathematical language and evidential reasoning, the basics logic and general laws for solving equations, inequalities, geometric problems, etc. But all this becomes efficient only if maths teachers are methodically and scientifically prepared, without neglecting a good knowledge of the assessment system, eliminating as much as possible subjectivity in this process.

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