https://doi.org/10.53656/voc22-401reco

Methodology & Experience
Методика и опит

RECONSTRUCTING THE MATHEMATICS CURRICULUM FOR POST-16 STUDENTS: A COMPARISON OF THE APPROACHES IN ENGLAND AND BULGARIA

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Abstract. A major reform of the Mathematics curriculum was published in England in 2016, implemented in 2017 and examined in 2019. In Bulgaria, the analogous reform was published in 2018, will be implemented in 2020 and examined in 2022.

This paper takes the framework developed by Budgell and Kunchev (2019) and seeks to interpret the new curricula in terms of the **Students**, the **State** and the **Curriculum** with an introduction to **Assessment**.

In terms of the Curriculum, the paper examines, at the highest level, **General and Specific Objectives**; followed by **Overarching Themes**; then the **Topics** covered and finally the **Detailed Content Statements** for each topic.

The paper concludes that the real differences between the teaching of Mathematics in England and Bulgaria lie not in the Mathematics itself but in the overall curriculum and assessment frameworks within which Mathematics is taught.

Keywords: reform; Mathematics curriculum; England; Bulgaria; new curricula; students; assessment; general and specific objectives; overarching themes; topics; detailed content statements

The Context

If, in 2016, you were a mythical visitor from outer space, who happened to be a bilingual mathematician, and you found yourself in either an 'Algebra and functions' lesson or an 'Exponentials and logarithms' lesson in Year 13 (En) or Klas 12 (Bg), you would probably have concluded that the Mathematics curricula in England and Bulgaria are essentially similar.

A few simple enquiries would have informed you that in both countries two levels of Mathematics were taught: Mathematics and Higher Mathematics in England and General Mathematics and Profile Mathematics in Bulgaria. This structure, illustrated in Table 1, would have reinforced the conclusion that the Mathematics curricula were similar.

Table 1

	ENGLAND	BULGARIA
LEVEL I	MATHEMATICS	GENERAL PROGRAMME
LEVEL II	FURTHER MATHEMATICS	PROFILE PROGRAMME

Since then, however, both countries have engaged in a major reform of the Mathematics curriculum. In England, a new Mathematics curriculum was introduced in 2017 and examined for the first time in 2019; whilst in Bulgaria, the new Mathematics curriculum will be introduced in 2020 and examined for the first time in 2022.

The purpose of this paper is to examine in greater depth the similarities and differences in the Mathematic curricula in England and Bulgaria.

The Framework

Budgell and Kunchev (2019) proposed that:

a. a well-understood set of sentences (axioms), basic concepts/terms; and

b. the rules of logic

must be assumed in any discussion of the education system. They propose

1. The Fundamental Axioms – **Students** and **Education**;
2. The Structural Axioms – The **State** and **Schools**;
3. The Functional Axioms – **Subsidiarity**, **Leadership and Management**, the **Curriculum**, **Teachers**, **Assessment** and **Inspection** and **Inspection**.

This paper (**Part 1**) will focus on **The Students**, **The State** and **The Curriculum** with an introduction to **Assessment**; **Part 2** will address **Teaching** and **Assessment** after the new curriculum and examinations have been introduced in Bulgaria

The Students

In England, the majority of students enter a comprehensive secondary school (there are also a small number of state grammar schools and independent private schools) at the age of 11. All students study Mathematics until the age of 16; at the age of 16 students have a free choice of the 3 or 4 subjects that they wish to study. There are neither compulsory subjects nor compulsory examinations – except that you can only study Higher Mathematics if you have chosen to study Mathematics.

In Bulgaria, there is a common curriculum until the age of 14; at the age of 14 they can enter a Profile-oriented school, a comprehensive school or a vocational/ technical school. Students wishing to enter a Profile-oriented school must perform very well in entrance examinations taken at the age of 14. There is a unified curriculum, but in the last two years of study, students are required to take advanced

courses in two or three subjects. Once they have entered a profile (pathway) students have little choice of what they study for 5 years.

In 2019, there were almost 600,000 18-year-olds in England of whom over 91,895 (15.6%) took A Level Mathematics and over 14,527 (2.5%) took Higher Mathematics. In Bulgaria, there were over 60,000 19-year-olds but only 2,200 (3.5%) took the Mathematics Matura as their second subject.

Nevertheless, the students studying Profile Mathematics in Bulgaria or Mathematics/Higher Mathematics in England are amongst the most academically gifted students in their respective countries. In England, those students wishing to study Mathematics after the age of 16 need to have been very successful in the Mathematics examinations (General Certificate of Secondary Education) taken at the age of 16. In Bulgaria, those students wishing to enter a Mathematics Profile-oriented school must similarly have done very well in the 14+ examinations

The State

There are two levels at which the state is involved in planning and organising the curriculum in England and Bulgaria:

– determining the structure and organisation of the curriculum framework that will provide students with the opportunity to **choose** the subjects that they which to study as they mature;

and because the curriculum for older students is driven by external qualifications,

– specifying, in detail, the content and assessment requirements of all subjects taught in schools.

In England, the curriculum is specified by the Qualifications and Curriculum Authority, a non-ministerial department that reports directly to Parliament – not to The Department for Education. Examinations are the responsibility of Examinations Groups who bid in a controlled market for the right to organise school examinations.

In Bulgaria, the Curriculum is specified by and examinations are organised by the Ministry of Education and Science directly.

The Curriculum Framework

Budgell and Kunchev (2019) asserted that all students are entitled to a curriculum which:

– is balanced and broadly based;

– promotes the spiritual, emotional, moral, cultural, intellectual and physical development of pupils at the school and of society;

– prepares pupils for the opportunities, responsibilities and experiences of life by equipping them with appropriate knowledge, understanding and skills; and

– empowers young people to achieve their potential and to make informed and responsible decisions throughout their lives.

As was stated above, the state determines the structure and organisation of the curriculum framework. It is in the curriculum framework for senior high school students that England and Bulgaria diverge.

Up until the age of 16, the curriculum in England meets the requirements for breadth and balance proposed by Budgell and Kunchev. After the age of 16, however, students have a free choice of the 3 or 4 subjects that they wish to study; there are no compulsory subjects. Students in England do not have to study English Language and Literature or Mathematics – there is no equivalent to the General Mathematics courses followed by students in Bulgaria. Schools in England are bigger than schools in Bulgaria in order that they can offer a range of up to 20 subjects.

There can be a wide variation in the students' choice of subjects, for example:

(a) Mathematics,	Higher Mathematics,	Physics,	and Chemistry
(b) Chemistry,	Biology,	English,	and Spanish
(c) English Language,	English Literature,	German,	and French
(d) English Language,	Psychology,	Sociology,	and Mathematics
or			
(e) Geography,	Business Studies,	History,	and Accounting

It is important to stress that it is the students' choice – not the school's, not the municipality's and not the Department for Education's. The one obvious exception has already been referred to above: you can only study Higher Mathematics if you have chosen to study Mathematics. On the other hand, you can study English Language and/or English Literature – or you can take a combined course in English Language and Literature.

The curriculum in Bulgaria does more to meet the requirements of breadth and balance specified by Budgell and Kunchev. It is unified for all schools and across year groups. It includes:

Bulgarian language and literature,	Mathematics,	Modern foreign languages,	History	
Geography,	Physics,	Biology,	Chemistry,	Informatics,
Psychology,	Ethics and law,	Philosophy,	Music,	Arts,
and				
Sports and P.E.				

Table 2 illustrates the balance of subjects studied in Klac 12 in two different profile-orientated schools with two profiles in each school. The number of lessons clearly illustrates the subjects in which students are taking advanced courses. In contrast to England, the students have very little choice after they have entered a profile-oriented course at the age of 14. Table 3 summarises the difference between the curriculum frameworks in England and Bulgaria.

Table 2

Mathematics High School											
Mathematics Profile											
Ma	En	Bul	IT	Sport/PE	Hi	Civ	Ge/ Ru	Inf	**Gg**	**Ph**	
9	5	4	4	4	2	2	2	1	**1**	**1**	
Biology Profile											
Ch	Bi	En	Bul	Sport/PE	**Ma**	Hi	Civ	Ge/ Ru	**IT**	**Gg**	**Ph**
6	5	5	4	3	2	2	2	2	**1**	**1**	**1**
English Language High School											
English with German Profile											
En	Bul	Ge	**Ma**	Sport/PE	Hi	Civ	Ph	Gg	**Ch**		
9	6	5	4	3	3	2	1	1	**1**		
English with French Profile											
En	Bul	Fr	Hi	Sport/PE	Ge	Ch	**Ma**	Civ			
9	6	4	3	3	3	3	2	2			

Table 3

	ENGLAND	**BULGARIA**
Breadth and balance	↓	↑
Choice	↑	↓

In England the high degrees of freedom of choice come at a cost; a lack of breadth and balance. In Bulgaria, on the other hand, the cost of the breadth and balance is the lack of real choice available to the students.

The Mathematics Curriculum

The education of older students is constrained by external qualifications and that requires a detailed specification of the content and assessment requirements of all subjects examined in schools. At the highest level of generality, the Department for Education in England and the Ministry of Education and Science in Bulgaria take similar approaches to the Mathematics Curriculum. They specify, at the highest level, **General and Specific Objectives**; followed by **Overarching Themes**; then the **Topics** covered and **Detailed Content Statements** for each topic.

General and Specific Objectives

The highest-level statements about the Mathematics curriculum are entitled "General and specific objectives" in England, but "Aims of Mathematics Education" in Bulgaria.

Table 4

England	BULGARIA
SPECIFICATION FOR MATHEMATICS	**MATHEMATICS CURRICULUM FOR KLAS XI AND XII**
AIMS AND OBJECTIVES	**OBJECTIVES OF MATHEMATICS EDUCATION**
Understanding Mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study. Extending the range of mathematical skills and techniques. Understanding coherence and progression in Mathematics and how different areas of Mathematics are connected. Applying Mathematics in other fields of study and be aware of the relevance of Mathematics to the world of work and to situations in society in general. Using mathematical knowledge to make logical and reasoned decisions in solving problems both within pure Mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly. Reasoning logically and recognise incorrect reasoning. Generalising mathematically. Constructing mathematical proofs. Using mathematical skills and techniques to solve challenging problems which require students to decide on the solution strategy. Recognising when Mathematics can be used to analyse and solve a problem in context. Representing situations mathematically and understanding the relationship between problems in context and mathematical models that may be applied to solve them.	Deepening of logical knowledge and skills, formation of logical culture and mastering of mathematical language. Mastering the major applications by extending intragranular and intersubject links. Mastering scientific methods and ideas. Establishing such relationships, between the teacher and students, between the students themselves and between the students and the community, enabling the expression of the personal qualities of each student and the formation of civic position: Bringing Mathematics education up to European standards while maintaining national traditions. Building habits for protecting the environment and your own health.
	XI KLAS (GENERAL MATHEMATICS)
	Knowledge of some specific numerical series and their applications Expanding knowledge of trigonometric functions and their applications. Laying the foundations of statistical knowledge through familiarity with the mechanisms for presenting data.
	XI KLAS (PROFILE MATHEMATICS)
	Mastering the properties of logarithms, exponential and logarithmic functions and their applications. Mastering the interrelationships of points, straight lines and planes in space, relations of “parallel” and “perpendicularity” and building skills to prove the claims related to them. Knowledge of the concept of polynomials, some types of polynomials and their elements, knowledge of faces on their surfaces and volumes, and of skills in their application. Expanding knowledge of vectors in space and their applications.
	XII KLAS (GENERAL MATHEMATICS)
	Acquiring knowledge related to the mutual positions of

Reading and comprehending articles concerning applications of Mathematics and communicating their understanding. Using technology such as calculators and computers effectively and recognising when such use may be inappropriate. Taking increasing responsibility for their own learning and the evaluation of their own mathematical development.	points, straight lines and planes in space, relations "parallel", "perpendicularity" and the image "orthogonal design". Acquiring knowledge about the concept of polynomials, about some types of polynomials and their elements, knowledge about the faces of surfaces and volumes of studied polynomials and forming skills for their application. Summarizing the knowledge of the studied numerical sets. Summarizing knowledge of the studied features, their properties and applications. Expanding knowledge of equations and inequalities.
	XII KLAS (PROFILE MATHEMATICS)
	Acquiring knowledge about certain types of rotary bodies, their elements, faces on surfaces and volumes and forming skills for their application. Mastering the elements of mathematical analysis and their applications. Assimilation of elements of analytical geometry in the plane and their applications. Upgrading knowledge by probability and statistics. Introducing the set of complex numbers and forming the skills to perform operations with complex numbers.

Table 4, however, indicates that only General Objectives are published by the Department of Education in England, for example:

– Using mathematical skills and techniques to solve challenging problems which require students to decide on the solution strategy; and

– Representing situations mathematically and understanding the relationship between problems in context and mathematical models that may be applied to solve them.

The list of objectives published by the Ministry of Education and Science in Bulgaria, however, include General Objectives, for example:

– Deepening logical knowledge and skills, forming a logical culture and learning mathematical language; and

Specific Objectives, for example:

– Acquiring knowledge of the concept of polynomials, of some types of polynomials and their elements, knowledge about the faces, surfaces and volumes of studied polynomials and developing skills for their application.

Overarching Themes

Table 5

ENGLAND	BULGARIA
OVERARCHING THEMES	
Argument, language and proof	Knowledge, skills and attitudes
Problem solving	Areas of competence
Modelling	Establishing cross-domain links
The use of technology	Practical activities
The use of data in statistics	Use of information technology

Table 5 indicates that, although the words are not identical, both the DfE and the MOH have adopted similar approaches when giving a high-level overview of the Mathematics curriculum.

Topics

The next level of analysis, at the level of **Topics**, is shown in Table 6. It is at this level that some of the differences in approach become more apparent.

Table 6

ENGLAND	BULGARIA
Topics	**Themes**
MATHEMATICS	**GENERAL MATHEMATICS**
Mathematics	**11 KLAS**
Proof	Exponentials and logarithm
Algebra and Functions	2-Dimensional Geometry
Co-ordinate Geometry in the (x,y) plane	Trigonometry
Sequences and Series	Probability
Trigonometry	**12 KLAS**
Exponential and Logarithms	Statistics
Differentiation	Equations
Integration	Inequalities
Numerical Methods	Extension Activities
Vectors	
Statistics	**PROFILE MATHEMATICS**
Statistical Sampling	**11 KLAS**
Data Presentation and Interpretation	**Geometry**
Probability	Vectors and coordinates
Statistical Distributions	Co-ordinate Geometry in the (x,y) plane
Statistical Hypothesis Testing	3-Dimensional Geometry

Mechanics	**Mathematical Analysis**
Quantities and Units in Mechanics	Polynomials of a variable
Kinematics	Numbers
Forces and Newton's Laws	Functions Series and Differentiation
	12 KLAS
FURTHER MATHEMATICS	**Practical Mathematics**
Proof	Applications of Mathematical Analysis
Complex numbers	Geometric models
Matrices	Empirical distributions
Further Algebra and Functions	**Probabilities and data analysis**
Further Calculus	Probability
Further Vectors	Random value
Polar Coordinates	Binomial distribution
Hyperbolic Functions	Normal distribution
Differential Equations	

First of all, Table 6 suggests that Table 1 is misleading; the levels do not map onto each other. General Mathematics in Bulgaria and Mathematics in England do not map onto each other. In England, there is no equivalent to General Mathematics in Bulgaria. The only 16+ students who study Mathematics in England are those who have chosen to study Mathematics to university entrance level. So, is Mathematics in England equivalent to Profile Mathematics in Bulgaria? Figures 1, 2 and 3 have been constructed to further this analysis.

Figure 1 illustrates very clearly the lack of equivalence between General Mathematics in Bulgaria and Mathematics in England. With the exception that students in Bulgaria study Геометрия and Неравенства, the Mathematics curriculum in England is much wider. So, is Mathematics equivalent to Profile Mathematics? This is addressed in Figure 2.

Figure 2 indicates that that Mathematics in England and Profile Mathematics in Bulgaria map more closely onto one another. Although some topics are included in Profile Mathematics in Bulgaria that are not included in Mathematics in England, for example:

– 3-Dimensional Geometry and Geometric Modelling;

others are included in Mathematics in England but not in Profile Mathematics in Bulgaria, for example:

– Integration, Quantities and Units in Mechanics, Kinematics and Forces and Newton's Laws.

At the level of Topics therefore, as defined by the DfE and the MOH, there is a major overlap between Mathematics in England and Profile Mathematics in Bulgaria.

So, where now does Higher Mathematics fit? This is addressed in Figure 3.

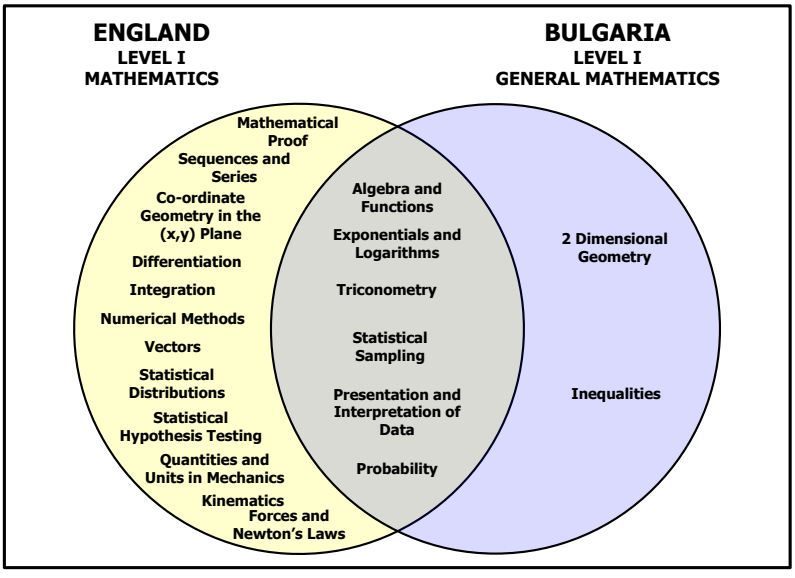

Figure 1

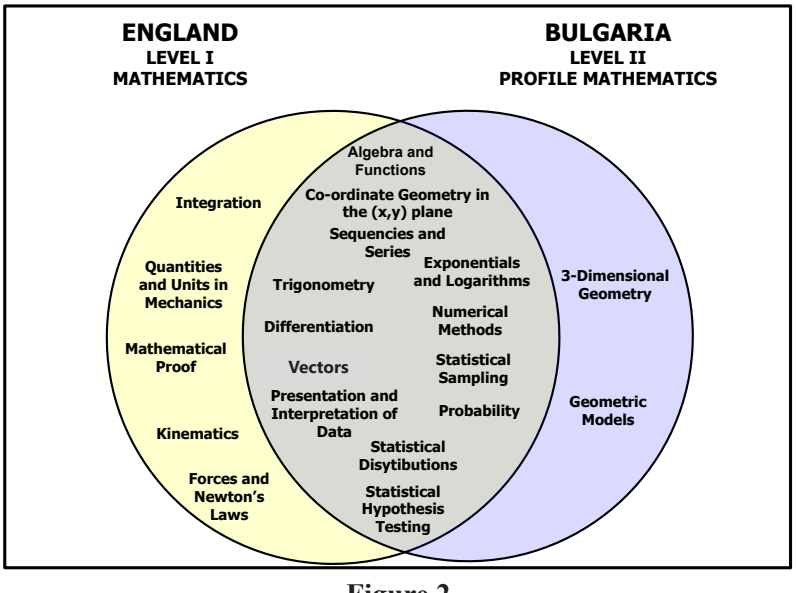

Figure 2

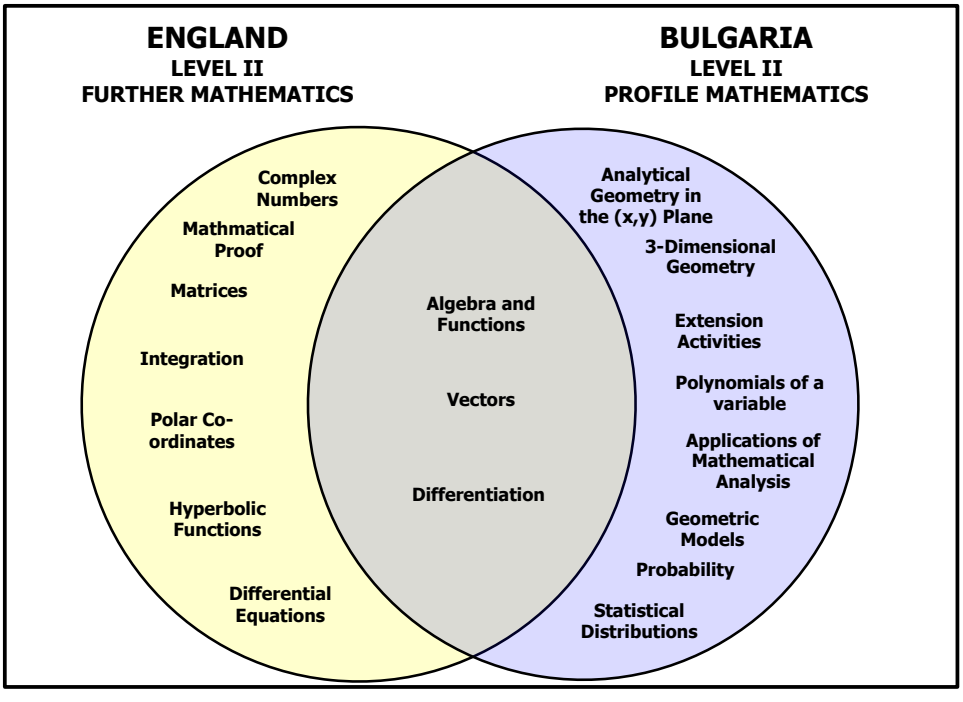

Figure 3

Because the majority of topics covered in Higher Mathematics are clearly distinct from those covered in Mathematics, Figure 3 indicates that are also distinct from the topics in Profile Mathematics. Therefore, with the exception of:

– Differentiation, Algebra and Functions, and Vectors;

the topics covered in Higher Mathematics are clearly distinct from those covered in Profile Mathematics, for example:

– Integration, Complex Numbers, Polar Co-ordinates, Hyperbolic Functions and Differential Equations.

This leaves the question of where Higher Mathematics fits. This will be addressed more fully in the Section on Assessment; suffice it to say at this stage that because of the differences between the Curriculum Frameworks and the Assessment Frameworks in England and Bulgaria, it doesn't.

Detailed Content Statements

Tables 7a and 7b present the detailed content statements for "Exponentials and Logarithms".

Table 7a

England
Detailed Content Statements
Exponentials and Logarithms
Know and use the function a^x and its graph, where a is positive Know and use the function e^x and its graph
Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications
Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 1$ Know and use the function and its graph Know and use in x as the inverse function of e^x
Understand and use the laws of logarithms: $\log_a x + \log_a y = \log_a (xy)$; $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$; $k \log_a x = \log_a x^k$ (Including, for example, $k = -1$ and $k = -\frac{1}{2}$)
Solve equations of the form $a^x = b$
Use logarithmic graphs to estimate parameters in relationships of the form $y=ax^n$ and $y=kb^x$, given data for x and y
Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models

Table 7b

BULGARIA		
Numbers and Algebra		
Knowledge, skills and attitudes **As a result of his training, the student:** • knows the concept of the nth root and its properties; • knows the concepts of exponential and logarithm and their basic properties; • performs identical transformations of irrational expressions containing square and cubic roots and 4th root.		
Topics	**Expected Competencies at the end of the topic**	**New concepts**
1. Exponentials and Logarithms 1.1. Properties of the 3rd root. 1.2. Properties of the nth root.	– knows the concept of root nth and its properties; – be able to translate irrational expressions, containing square and cubic	Third root (cubic root), nth root, logarithm,

1.3. Transformation of irrational expressions. 1.4. Function graphs $y = \sqrt{x}$, $y = x^2$ и $y = \sqrt[3]{x}$ 1.5. Exponential with a rational, exponent properties. 1.6. Convert expressions containing an exponential with a rational exponent. 1.7. Indicative function. Graphics. 1.8. Logarithm. Basic properties. Comparison of logarithms. Graph of logarithmic function. 1.9. Logarithm of a work, privately, exponential and root.	roots, and root 4-you; – knows the concept of degree with a rational indicator and its properties; – able to translate expressions containing degrees with rational indicator; – knows the concept of logarithm and its properties; – be able to apply logarithm properties to expression transformation; – be able to find the elements of a logarithm value, basis or argument in the presence of the other two dimensions; – be able to recognize the power graphs, indicative and logarithmic function; – read and interpret information provided by graphics; – can solve practical problems with a calculator.	basis, logarithm, anti-logarithm.

Tables 8a and 8b present the detailed content statements for "Vectors".

Table 8a

ENGLAND
Detailed Content Statement
Vectors
MATHEMATICS
Use vectors in two and in three dimensions
Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form
Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations
Understand and use position vectors; calculate the distance between two points represented by position vectors
Use vectors to solve problems in pure Mathematics and in context, including forces and kinematics
FURTHER MATHEMATICS
Understand and use the vector and Cartesian forms of an equation of a straight line in 3D
Understand and use the vector and Cartesian forms of the equation of a plane
Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane
Check whether vectors are perpendicular by using the scalar product
Find the intersection of a line and a plane Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane

Table 8b

BULGARIA		
Geometry		
Knowledge, skills and attitudes Vectors are the basis of different parts of physics, especially in mechanics when studying the movement of bodies and the forces that act on them in different situations.		
Topics	**Expected Competencies at the end of the topic**	**New concepts**
1. Vectors		
1.1. Linear dependence and independence of vectors in the plane and in space.	**The student:** – knows the concepts of linear dependence and independence of vectors; – is able to determine linear dependence and independence of vectors.	Linear dependence of vectors, linear independence of vectors.
1.2. Vector base in the plane and in the space.	**The student:** – knows that if it is in the plane (or space) selected vector base, each vector in the plane (or space) can be represented as linear combination of base vectors; – is able to decompose a vector as linear combination of vectors from base to specific situations in the plane and in pace.	Vector base.
1.3. A scalar product of two vector.	**The student:** – knows the scalar product in two vectors; – is able to find a scalar product of two vector; – knows the properties of a scalar product.	Scalar product.
1.4. Scalar application product - vector length; angle between two vectors.	**The student:** – is able to find the length of a vector; – is able to find an angle between two vectors.	Angle between two vectors.
1.5. Vector coordinates in the plane rectangular coordinate system.	**The student:** – knows the concept of vector coordinates in plane rectangular coordinate system; – is able to find the coordinates of a point; is able to find the distance between two points.	Vector coordinates.
1.6. Vector coordinates in the plane rectangular coordinate system.	**The student:** – is able to find the sum of vectors set in coordinate form; – is able to find the product of a number with a vector, set in coordinate form; – is able to determine the coordinates of a vector, set by a linear combination of others vectors; – is able to find the scalar product of two the vector in the plane by their coordinates; – is able to find an angle between two vectors, set by their coordinates.	Sum of vectors given by coordinates, linear combination of vectors, a scalar product in two vectors, given coordinates, angle between two vectors given by coordinates.

Despite the differences in the style of presentation, what is obvious is that the detailed content statements above are almost identical. However, this is not peculiar to "Exponentials and Logarithms" and "Vectors". These were just chosen as examples; it is true of all the topics that are taught in both England and Bulgaria. So, what does this say?

Primarily that the differences in the teaching of Mathematics in England and Bulgaria are a function of the differences in the overall curriculum framework; not differences in the Mathematics.

Of course, this is true, Mathematics is Mathematics; what else would one expect? However, maybe if Ellenberg (2014) (see **Postscript and Paradox**, below) had had more influence and school Mathematics was „simple and profound" rather than being „complex but shallow" students in senior high schools would have to deal with:

– Benacerref's Dilemma (1973) „what is necessary for mathematical truth makes mathematical knowledge impossible";

– Pluralism and the foundation of Mathematics, Hellman and Bell (2006); or

– Pluralism: Beyond the One and Only Truth, Horgan (2019).

Then the content statements might well be very different – and legitimately so.

Assessment

There is no Diploma of Secondary Education in England. The students only have their examination results recorded on certificates from the examination boards.

The students can **chose** at the age of 16 to study Mathematics and two years later they must sit their Advanced Level examinations. This similarly applies to those students who **chose** to study Higher Mathematics. These students can get the equivalent to two Matura: one in Mathematics and one in Higher Mathematics.

The new Mathematics curriculum was examined for the first time in 2019 and consequently the "Assessment Objectives" have already been published, a student must:

– Use and apply standard techniques;

– Reason, interpret and communicate mathematically; and

– Solve problems within Mathematics and in other contexts.

The examination framework has already been published. There are 3 two-hour examinations:

– Paper 1: Pure Mathematics;

– Paper 2: Pure Mathematics and Statistics; and

– Paper 3: Pure Mathematics and Mechanics.

Similarly, the new Higher Mathematics curriculum was examined for the first time in 2019 with the same overall "Assessment Objectives" and the examination framework has also been published. There are 4 1½ hour examinations:

– Paper 1: Pure Core 1;
– Paper 2: Pure Core 2;
and then 2 from:
– Statistics;
– Mechanics;
– Discrete Mathematics; and
– Additional Pure Mathematics.

Teachers play no part in the summative assessment of the students' standards of achievement, although they are (of course) involved in the formative assessment of the students during the 2-year course.

In Bulgaria, there is a Diploma in Secondary Education issued by the school. It is based on the results in the Matura examinations and the teachers' summative assessment of the standards of achievement across the curriculum. The new curriculum will not be examined until 2020, but **The Curriculum for Twelfth Grade General Education Mathematics** contains clear guidance on the assessment of students' achievements by their teachers:

Specific Methods and Forms for Assessing Students' Achievements

Forms of assessment:

Oral examination – assessment of the student's opinion and arguments when solving a particular mathematical problem.

Written examination – assessment of the standards achieved through brief written individual or group tests.

Supervision and classroom work – assessment of the standards achieved at the end of modules and terms

Practical work – homework, project development, etc.

Table 9

Ratio in the formation of term and annual assessment	
Assessments from oral tests	15%
Assessments from written tests	10%
Assessments from control and classroom work	50%
Grades from other participations (hourly work, homework, project work, etc.)	25%

Similarly, **The Curriculum for Profile Mathematics** contains clear guidance on teacher assessment:

Specific Methods and Forms for Assessing Students' Achievements

Assessment of students' knowledge and skills is in line with the expected results and activities foreseen in the program.

The student needs to be informed in advance of the criteria and the system for assessing his/her achievement.

Table 10

Ratio in the formation of term and annual ssessment	
Ongoing classroom work, group discussions, and discussions	20%
Ongoing homework assessments	20%
Current grades from practical assignments in class	25%
Estimates from project work	25%
Assessments from written tests	10%

In Bulgaria, all students have to sit a Matura examination in Bulgarian Language and Literature plus one other subject (a **very** small number take two other subjects). The majority of students in Mathematics profile oriented-classes (be they in Mathematics profile-oriented schools or Mathematics profile-oriented classes in other schools) take the Mathematics Matura as their second subject.

However, the Mathematics Matura must also be available to those students who do not attend a Mathematics profile-oriented school or attend a Mathematics profile-oriented school but are not in a Mathematics profile-oriented class. Consequently, much of Profile Mathematics curriculum studied in Klac XII is not examined in the Mathematics Matura.

In Bulgaria there is no Higher Mathematics Matura, as a consequence, there is no external examination of the full range of the Mathematics curriculum for students in Mathematics profile-oriented classes; i.e. there is no external examination of the 405 minutes (9 lessons) per week of Mathematics studied by students in Klac XII in the Mathematics profile-oriented classes.

In England there is no overall Diploma in Secondary Education, but students take external examinations in the full range of the curriculum in all the subjects they have studied:

– 9 written examinations, if the take 3 subjects; and

– up to 13 written examinations, if they take 4 subjects – and one of them is Higher Mathematics.

Teacher assessment plays no part in the summative assessment of the students' standards of achievement.

Table 11

	ENGLAND	BULGARIA
Teacher Assessment	↓	↑
External Examinations	↑	↓

In Bulgaria, teacher assessment plays a central role in the assessment of students' standards of achievement across the curriculum. The Mature examinations of 4 hours (but usually, only in two subjects) provide additional information for the Diploma in Secondary Education but do not necessarily cover the whole curriculum in Klas 12.

Table 12

ENGLAND			BULGARIA		
Mathematics	91,895	15.6%	Bulgarian Language and Literature	52,650	84.9%
Biology	69,196	11.8%	English	14,612	23.6%
Psychology	64,598	11.0%	Biology	11,076	17.9%
English Language and/or Literature	63,135	10.7%	Geography	<6,000	<9.7%
Chemistry	59,090	10.1%	Philosophy	4,287	6.9%
History	51,438	8.8%	Mathematics	>2,200	>3.5%
Physics	38,958	6.6%	History	>1,000	>1.6%
Sociology	38,015	6.5%	Chemistry	>1,000	>1.6%
Further Mathematics	**14,527**	**2.5%**			

In Bulgaria, all students have to take the Matura examination in Bulgarian Language and Literature **plus** one other subject. Table 12 illustrates the number of students who took the ten most popular subjects in 2019. As has been indicated, within the current curriculum and assessment frameworks in Bulgaria, there is no space for a Matura in Higher Mathematics. In addition, the very low percentage of students taking the Matura in Mathematics (3.5%) suggests there would be no market for a Matura in Higher Mathematics – unlike England, where 14,527 (2.5%) of the students took A Level Higher Mathematics.

Summary

At the level of individual **Topics**, where these are covered in both Mathematics (Level II in England) and Profile Mathematics (Level II in Bulgaria) there is, to

all intents and purposes, complete congruence between the **Detailed Content Statements**.

Step up a level, to the range of **Topics** covered and the pattern is again similar. Of course, some **Topics** are included in Profile Mathematics in Bulgaria that are not included in Mathematics in England, for example:

– 3-Dimensional Geometry and Geometric Modelling;

others are included in Mathematics in England but not in Profile Mathematics in Bulgaria, for example:

– Integration, Complex Numbers, Polar Co-ordinates, Hyperbolic Functions and Differential Equations.

The overall range of **Topics** is, however, similar.

At the highest level of generality, there is a similar approach to the Mathematics curriculum. It is specified in terms of **General and Specific Objectives** and **Overarching Themes** and these are remarkably similar.

The real differences between the teaching of Mathematics in England and Bulgaria lie not in the Mathematics itself but in the overall curriculum and assessment frameworks within which Mathematics is taught. Students in England have a free choice in the 3 or 4 subjects that they study after the age of 16 - there is no General Mathematics programme. Students in Bulgaria elect or are selected to follow profiles (pathways) at the age of 14 within which there is little subsequent freedom of choice.

In England, there is no Diploma in Secondary Education. At the age of 18, Students take external examinations in the 3 or 4 subjects they chose to study two years earlier. Teachers play no part in the summative assessment of the standards of achievement reached by the students.

The Diploma in Secondary Education is determined by summative assessment undertaken by their teachers across the full range of the curriculum, augmented by the results achieved in the two Matura examinations.

Paradox and postscript

Despite the reform in both countries, little has been done to address the challenge issued by Jordan Ellenberg (2014) in "How not to be wrong. The hidden maths of everyday life".

Table 13

	John Ellenberg's Quadrants	
Profound	**This should be the focus for school/college Mathematics** ← ↑	The Riemann Hypothesis, Fermat's Last Theorem, The Poincaré Conjecture, P vs. NP, Gödel's Theorem

Shallow	$1 + 2 = 3$ $\sin(2x) = 2 \sin x \times \cos x$	**Much of school/college Mathematics** $\int x^{-1}\sqrt{x^3-1}.dx =$ $\frac{2}{3}\sqrt{x^3-1}.\left(1 - \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{\sqrt{1-x^3}}\right)$
	Simple	**Complicated**

Simple but shallow

Basic arithmetic facts, like 1 + 2 = 3, are simple and shallow. So are basic identities like

sin(2x) = 2 sin x cos x

or

the quadratic formula: they might be slightly harder to convince yourself of than 1 + 2 = 3, but in the end they don't have much conceptual weight.

Complicated but shallow – Much of school/college Mathematics

You have the problem of multiplying two ten-digit numbers, or the computation of an intricate definite integral. It's conceivable you might, for some reason, need to know the answer to such a problem, and it's undeniable that it would be somewhere between annoying and impossible to work it out by hand; or, it might take some serious schooling even to understand what's being asked for. But knowing those answers doesn't really enrich your knowledge about the world.

Simple and profound - This should be the focus for school/college Mathematics

Mathematical ideas that can be engaged with directly and profitably, whether your mathematical training stops at pre-algebra or extends much further. And they are not "mere facts," like a simple statement of arithmetic— they are principles, whose application extends far beyond the things you're used to thinking of as mathematical. They are the go-to tools on the utility belt, and used properly they will help you not be wrong.

Complicated and profound

This is where professional mathematicians try to spend most of their time. It's where the celebrity theorems and conjectures live:

– The Riemann Hypothesis,
– Fermat's Last Theorem,
– The Poincaré Conjecture,

– P vs. NP,
– Gödel's Theorem …

Each one of these theorems involves ideas of deep meaning, fundamental importance, mind-blowing beauty, and brutal technicality.

NOTES

1. Mathematics for profile preparation. Ministry of Education and Science.
2. Syllabus for the XI class in force from the academic year 2020-2021. Ministry of Education and Science.
3. Syllabus for the XII class in force from the academic year 2021-2022. Ministry of Education and Science.

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