

## PROBLEMS ON THE BROCARD CIRCLE

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**Abstract.** We present problems for students about the Brocard circle. The problems are discovered by the computer program “Discoverer”, created by the authors.

**Keywords:** Brocard circle; Euclidean geometry; triangle geometry; computer discovered mathematics; “Discoverer”

The Brocard circle is the circle having the line segment connecting the Circumcenter  $O$  and Symmedian point  $K$  of a triangle as its diameter. See (Weisstein, Brocard circle).

In this paper we present problems for students and teachers on the Brocard circle. The reader may find terminology in (Weisstein). The problems are discovered by the computer program “Discoverer”, created by the authors See e.g. (Grozdev & Dekov, 2015), (Grozdev, Okumura & Dekov, 2017).

A few of the statements of the problems are not valid for all triangles. The readers should investigate the kind of triangle for which the statement of the problem is valid: e.g. obtuse triangle, acute triangle, all triangles.

The readers may find a number of problems in Euclidean geometry in the *International Journal of Computer Discovered Mathematics* (IJCDM): <http://www.journal-1.eu/index.htm>.

### 1 Basic problems

The Brocard circle is the circumcircle of the First and Second Brocard triangles. See (Weisstein, Brocard circle). In addition:

**Problem 1.** Prove that the Brocard Circle is the

- (1) Brocard Circle of the Circum-Symmedian Triangle.
- (2) Brocard Circle of the Circumcevian Triangle of the Schoute Center.
- (3) Brocard Circle of the Circumcevian Triangle of the Center of the Parry Circle.
- (4) Brocard Circle of the Circumcevian Triangle of the First Beltrami Point.
- (5) Brocard Circle of the Circumcevian Triangle of the Second Beltrami

Point.

- 6) Brocard Circle of the Outer Apollonius Triangle of the Lucas Circles.
- (7) Brocard Circle of the Inner Monge Triangle of the Excircles.
- (8) Brocard Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Orthocenter.
- (9) Antimedial Circle of the First Brocard Triangle of the Medial Triangle.
- (10) Nine-Point Circle of the First Brocard Triangle of the Antimedial Triangle.
- (11) Nine-Point Circle of the Excentral Triangle of the First Brocard Triangle.
- (12) Nine-Point Circle of the Hexyl Triangle of the First Brocard Triangle.
- (13) Antimedial Circle of the Medial Triangle of the Second Brocard Triangle.
- (14) Excentral Circle of the Orthic Triangle of the Second Brocard Triangle.
- (15) Nine-Point Circle of the Excentral Triangle of the Second Brocard Triangle.
- (16) Nine-Point Circle of the Antimedial Triangle of the Second Brocard Triangle.
- (17) Nine-Point Circle of the Hexyl Triangle of the Second Brocard Triangle.

Figure 1 illustrates part 1 of Problem 1. In figure 1,

- $O$  is the Circumcenter,
- $K$  is the Symmedian point,
- $PaPbPc$  is the Circum-Symmedian Triangle,
- $c$  is the Brocard circle of triangle  $ABC$ . At the same time,  $c$  is the Brocard circle of triangle  $PaPbPc$ .

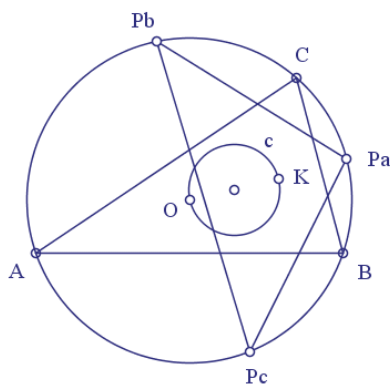


Figure 1

## 2. Concentric Circles

The Brocard circle is concentric with the Lemoine circle. See (Weisstein, Brocard circle). In addition:

**Problem 2.** Prove that the Brocard Circle is concentric with the

- (1) Second Brocard Circle of the First Brocard Triangle.
- (2) Second Brocard Circle of the Second Brocard Triangle.
- (3) Cosine Circle of the Euler Triangle of the Circumcenter.
- (4) Circumcircle of the Euler Triangle of the Symmedian Point.
- (5) Second Brocard Circle of the Euler Triangle of the Symmedian Point.
- (6) Brocard Circle of the Euler Triangle of the Center of the Brocard Circle.
- (7) Lemoine Circle of the Euler Triangle of the Center of the Brocard Circle.
- (8) Antimedial Circle of the Johnson Triangle of the First Brocard Triangle.
- (9) Adams Circle of the Kosnita Triangle of the Second Brocard Triangle.
- (10) Conway Circle of the Kosnita Triangle of the Second Brocard Triangle.

Figure 2 illustrates part 1 of Problem 2. In figure 2,

- $O$  is the Circumcenter,
- $K$  is the Symmedian Point,
- $c$  is the Brocard circle,
- $B_1B_2B_3$  is the First Brocard triangle,
- $c_1$  is the Second Brocard circle of triangle  $B_1B_2B_3$ . The circles  $c$  and  $c_1$  are concentric.

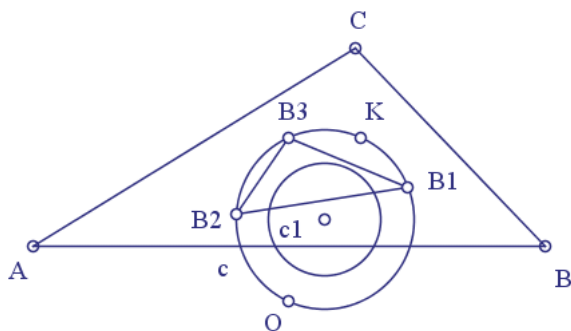


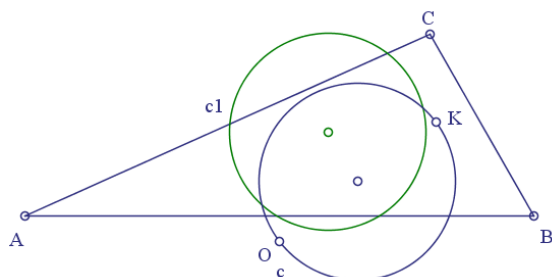
Figure 2

## 3. Congruent Circles.

**Problem 3.** The Brocard Circle is congruent with the

- (1) Brocard Circle of the Johnson Triangle.
- (2) Brocard Circle of the Circumcevian Triangle of the Circumcenter.
- (3) Brocard Circle of the Circumcevian Triangle of the First Brocard Point.
- (4) Brocard Circle of the Circumcevian Triangle of the Second Brocard Point.

- (5) Brocard Circle of the Anticevian Euler Triangle of the Centroid.
- (6) Excentral Circle of the Half-Circumcevian Triangle of the Center of the Brocard Circle.
- (7) Antimedial Circle of the Half-Circumcevian Triangle of the Center of the Brocard Circle.
- (8) Brocard Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Tarry Point.
- (9) Brocard Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Steiner Point.
- (10) Brocard Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Euler Reflection Point.
- (11) Brocard Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Parry Point.
- (12) Brocard Circle of the Triangle of the Orthocenters of the Triangulation Triangles of the Gibert Point.
- (13) Brocard Circle of the Triangle of the Orthocenters of the Anticevian Corner Triangles of the Incenter.
- (14) Brocard Circle of the Triangle of the Nine-Point Centers of the Anticevian Corner Triangles of the Centroid.
- (15) Brocard Circle of the Triangle of the Symmedian Points of the Anticevian Corner Triangles of the Centroid.
- (16) Brocard Circle of the Triangle of the Gergonne Points of the Anticevian Corner Triangles of the Centroid.
- (17) Brocard Circle of the Triangle of the Nagel Points of the Anticevian Corner Triangles of the Centroid.
- (18) Excentral Circle of the First Brocard Triangle of the Medial Triangle.
- (19) Excentral Circle of the Second Brocard Triangle of the Medial Triangle.
- (20) Antimedial Circle of the Second Brocard Triangle of the Medial Triangle.
- (21) Nine-Point Circle of the Second Brocard Triangle of the Antimedial Triangle.
- (22) Excentral Circle of the First Brocard Triangle of the Euler Triangle.
- (23) Excentral Circle of the Second Brocard Triangle of the Euler Triangle.
- (24) Antimedial Circle of the Second Brocard Triangle of the Euler Triangle.
- (25) Excentral Circle of the Orthic Triangle of the First Brocard Triangle.
- (26) Antimedial Circle of the Orthic Triangle of the First Brocard Triangle.
- (27) Excentral Circle of the Euler Triangle of the First Brocard Triangle.
- (28) Excentral Circle of the Medial Triangle of the Second Brocard Triangle.
- (29) Antimedial Circle of the Orthic Triangle of the Second Brocard Triangle.
- (30) Excentral Circle of the Euler Triangle of the Second Brocard Triangle.
- (31) Antimedial Circle of the Euler Triangle of the Second Brocard Triangle.



**Figure 3**

Figure 3 illustrates part 19 of Problem 3. In figure 3,

- $O$  is the Circumcenter,
- $K$  is the Symmedian point,
- $c$  is the Brocard circle,
- $c_1$  is the Excentral Circle of the First Brocard Triangle of the Medial Triangle.

Circles  $c$  and  $c_1$  are congruent.

#### 4. Orthogonal Circles.

The Brocard circle is orthogonal to Parry circle. See (Weisstein, Brocard circle). The “Discoverer” has discovered approximately 500 notable circles orthogonal to the Brocard circle. A few of them are listed below.

**Problem 4.** The Brocard Circle is orthogonal to the

- (1) Stevanovic Circle of the First Brocard Triangle.
- (2) Parry Circle of the First Brocard Triangle.
- (3) Stevanovic Circle of the Second Brocard Triangle.
- (4) Parry Circle of the Second Brocard Triangle.
- (5) Lester Circle of the Outer Kiepert-Gallatly Triangle.
- (6) Second Brocard Circle of the Antipedal Triangle of the First Brocard Point.
- (7) Second Brocard Circle of the Antipedal Triangle of the Second Brocard Point.
- (8) Moses Circle of the Antipedal Triangle of the First Beltrami Point.
- (9) Parry Circle of the Circum-Symmedian Triangle.
- (10) Parry Circle of the Circumcevian Triangle of the Schoute Center.
- (11) Parry Circle of the Circumcevian Triangle of the Center of the Parry Circle.
- (12) Parry Circle of the Circumcevian Triangle of the First Beltrami Point.
- (13) Parry Circle of the Half-Circumcevian Triangle of the First Isodynamic Point.
- (14) Second Brocard Circle of the Triangle of the Circumcenters of the Triangulation Triangles of the First Brocard Point.

- (15) Second Brocard Circle of the Triangle of the Circumcenters of the Triangulation Triangles of the Second Brocard Point.
- (16) Stevanovic Circle of the First Brocard Triangle of the Antimedial Triangle.
- (17) Stevanovic Circle of the Intouch Triangle of the First Brocard Triangle.
- (18) Stevanovic Circle of the Excentral Triangle of the First Brocard Triangle.
- (19) Stevanovic Circle of the Hexyl Triangle of the First Brocard Triangle.
- (20) Parry Circle of the Second Brocard Triangle of the First Brocard Triangle.
- (21) Stevanovic Circle of the Orthic Triangle of the Second Brocard Triangle.
- (22) Stevanovic Circle of the Intouch Triangle of the Second Brocard Triangle.
- (23) Stevanovic Circle of the Excentral Triangle of the Second Brocard Triangle.
- (24) Stevanovic Circle of the Antimedial Triangle of the Second Brocard Triangle.
- (25) Stevanovic Circle of the Hexyl Triangle of the Second Brocard Triangle.
- (26) Parry Circle of the Second Brocard Triangle of the Second Brocard Triangle.
- (27) Parry Circle of the Fourth Brocard Triangle of the Outer Kiepert-Gallatly Triangle.
- (28) Inverse Circle of the Moses Circle in the Second Brocard Circle.
- (29) Inverse Circle of the Half-Moses Circle in the Second Brocard Circle.
- (30) Inverse Circle of the Gallatly Circle in the Second Brocard Circle.
- (31) Inverse Circle of the Stevanovic Circle of the Lucas Central Triangle in the Circumcircle.
- (32) Inverse Circle of the Stevanovic Circle of the First Brocard Triangle in the Parry Circle.
- (33) Inverse Circle of the Parry Circle of the First Brocard Triangle in the Parry Circle.
- (34) Inverse Circle of the Stevanovic Circle of the Second Brocard Triangle in the Parry Circle.
- (35) Inverse Circle of the Lester Circle of the Outer Kiepert-Gallatly Triangle in the Parry Circle.
- (36) Circle having center at the Third Power Point and passing through the First Kenmotu Point
- (37) Circle having center at the Schoute Center and passing through the First Isodynamic Point.
- (38) Circle having center at the First Beltrami Point and passing through the First Isodynamic Point.
- (39) Circle having center at the Second Beltrami Point and passing through the First Isodynamic Point.

- (40) Circle having as its diameter the line segment connecting the Centroid and Steiner Point.  
 (41) Circle having as its diameter the line segment connecting the Centroid and Euler Reflection Point.  
 (42) Circle having as its diameter the line segment connecting the Brocard Midpoint and Third Power Point.  
 (43) Circle having as its diameter the line segment connecting the Kosnita Point and Orthocenter.  
 (44) Circle having as its diameter the line segment connecting the First Brocard Point and Third Power Point.  
 (45) Circle having as its diameter the line segment connecting the First Brocard Point and Second Beltrami Point.  
 (46) Circle having as its diameter the line segment connecting the Second Brocard Point and Third Power Point.  
 (47) Circle having as its diameter the line segment connecting the First Beltrami Point and Second Brocard Point.  
 (48) Circle passing through the Incenter, Inverse of the Incenter in the Circumcircle and Second Isodynamic Point.  
 (49) Circle passing through the Centroid, Circumcenter and Euler Reflection Point.  
 (50) Circle passing through the Centroid, Clawson Point and Euler Reflection Point.  
 Circle passing through the Centroid, Euler Reflection Point and Incenter.

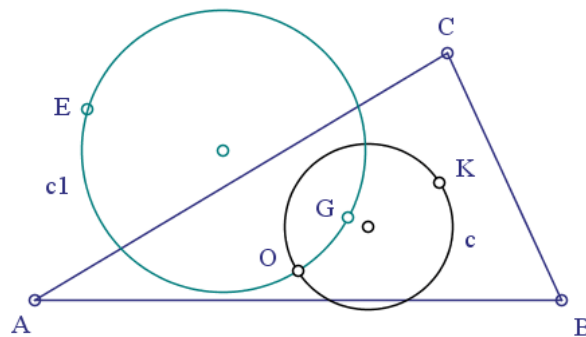


Figure 4

Figure 4 illustrates part 49 of Problem 4. In figure 4,

- $O$  is the Circumcircle,
- $K$  is the Symmedian point,
- $c$  is the Brocard circle,
- $G$  is the Centroid,
- is the Euler reflection point,

–  $c_1$  is the circle passing through points  $O, G$  and  $E$ . Circles  $c$  and  $c_1$  are orthogonal.

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### 5. Points on Brocard Circle.

The First and Second Brocard points lie on the Brocard circle See (Weisstein, Brocard circle). In addition:

**Problem 5.** Prove that the following points lie on the Brocard circle:

- (1) Gibert Point of the First Brocard Triangle.
- (2) Tarry Point of the Second Brocard Triangle.
- (3) Steiner Point of the Second Brocard Triangle.
- (4) Parry Point of the Second Brocard Triangle.
- (5) Gibert Point of the Second Brocard Triangle.
- (6) Symmedian Point of the Antipedal Triangle of the First Brocard Point.
- (7) Symmedian Point of the Antipedal Triangle of the Second Brocard Point.
- (8) Incenter of the Orthic Triangle of the First Brocard Triangle.
- (9) Feuerbach Point of the Excentral Triangle of the First Brocard Triangle.
- (10) Feuerbach Point of the Hexyl Triangle of the First Brocard Triangle.
- (11) Feuerbach Point of the Excentral Triangle of the Second Brocard Triangle.
- (12) Feuerbach Point of the Antimedial Triangle of the Second Brocard Triangle.
- (13) Feuerbach Point of the Hexyl Triangle of the Second Brocard Triangle.
- (14) Kiepert Center of the Excentral Triangle of the First Brocard Triangle.
- (15) Kiepert Center of the Hexyl Triangle of the First Brocard Triangle.
- (16) Kiepert Center of the Excentral Triangle of the Second Brocard Triangle.
- (17) Kiepert Center of the Hexyl Triangle of the Second Brocard Triangle.
- (18) Anticomplement of the Isotomic Conjugate of the Complement of the Cyclocevian Conjugate of the Symmedian Point.
- (19) Inverse of the First Brocard Point in the Parry Circle.
- (20) Inverse of the Second Brocard Point in the Parry Circle.

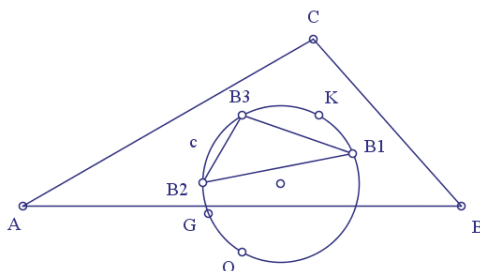


Figure 5



Figure 5 illustrates part 1 of Problem 5. In figure 5,

- $O$  is the Circumcenter,
- $K$  is the Symmedian point,
- $c$  is the Brocard circle,
- $B_1B_2B_3$  is the First Brocard triangle,
- $G$  is the Gibert point. Point  $G$  lies on circle  $c$ .

## NOTES

1. Weisstein, E.W., MathWorld – A Wolfram Web Resource, <http://mathworld.wolfram.com/>.

## REFERENCES

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