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PRACTICE-ORIENTED TASKS FOR SCHOOLS: METHODOLOGY FOR FUTURE MATHEMATICS TEACHERS

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Abstract. This article presents the results of the research into teacher training (mathematics) at the pedagogical university. The author addresses training undergraduate students in designing practice-oriented tasks. It is discussed that such training is part of the general system of teacher training for the practical application of mathematics at schools, and the task itself is a comprehensive educational product. As part of such training, the proposed approach allows to highlight the levels of complexity of practice-oriented tasks, methodological requirements for such tasks, requirements for the educational and scientific context of the task. It has been confirmed that tasks of this type can be organised into systems with different internal organisation. Examples of practice-oriented tasks, designed by undergraduate students, are given.

Keywords: teaching mathematics at schools; teacher training; practice-oriented task

Introduction

Nowadays the subject of teaching schoolchildren the practical application of studied material is becoming especially relevant. This trend stems from the increasing importance of mathematics in the general system of knowledge in recent decades (Rozov, 2013). Mathematical methods overlap with various fields of activity and are the underlying principle in the world-changing information technology. Such tendency should be reflected when selecting the training content. Thus, the Concept for the Development of Mathematical Education in the Russian Federation emphasises the need for schoolchildren to acquire "knowledge and skills used in everyday life and professional activity."[1)]

At present, practical applications of mathematics do not play a utilitarian role in teaching at schools, i.e. students are not provided with the algorithm of action in real-life conditions. In the process of teaching mathematics, the teacher forms students' ideas about the place of mathematical knowledge in the general system of knowledge about the world. In this sense, practical application allows solving an important didactic problem of developing students' mathematical literacy. This is what the Programme for International Student Assessment (PISA) describes as "the ability of a person to define and understand the role of mathematics in the world in which he lives, to express well-founded mathematical judgments and to use mathematics in such a way as to satisfy the present and future needs inherent in a creative, interested and a thinking citizen."[2)] The development of this ability in schoolchildren is impossible without studying the practical applications of mathematics nor the method of mathematical modelling at an accessible level. This means that such work requires special methodological teacher.

In this article, we discuss the peculiarities of the methodological preparation of a bachelor, a future teacher, for teacher practical applications of mathematics at schools. In particular, we will consider one of the aspects of such training - drawing up practice-oriented tasks with various educational functions.

The aim of the study is to answer the following question: how to prepare students of a pedagogical university for the independent selection of practical applications of mathematics and the compilation of practice-oriented problems for schoolchildren on the basis of those?

In their research, scholars from different countries pay special attention to the training of a school teacher (Borisenkov, 2018) and the study of the methods of teaching mathematics in various aspects (Grozdev et al., 2014). In connection with teacher training, the issues of applying mathematics in the educational process and teaching mathematical modelling are also considered.

A study by Wubbels, T., Korthagen, F. & Broekman, H. found that the changes in the content of school mathematics education in the Netherlands (those concerning the so-called "realistic approach") require adjustments to the teacher's work. (Wubbels et al., 1997) The authors presented an analysis of an educational programme focusing on preparing future mathematics teachers to use real-life situations as examples when teaching students to identify mathematical facts and patterns in the world around them. It is noted that students experienced significant difficulties in mastering the proposed programme due to the unconventional approach used in teaching mathematics at school. According to the authors, it is necessary to search for new methods and strategies in teacher training in the chosen direction to overcome these difficulties.

Sevinc, S., Lesh, R. believe that it is crucial to introduce special training courses that will allow future teachers to acquire professional skills for teaching practical applications of mathematics as a means of substantiating abstract mathematical ideas, and for assessing students' performance and acquisition (Sevinc et al., 2018). Consequently, the authors have implemented the binary use of practical applications in the courses designed for students: as a means of teaching mathematics and as a learning goal. It is important to note that when constructing the plot of the task, teachers were asked to take into account not only the education content, but the age-related interests of students. The researchers point out that teachers have difficulty both in selecting examples of practical applications, composing tasks based on them, and in using such tasks in teaching mathematics.

Blum, W., Leiß, D. identify a number of criteria that determine the quality of teaching mathematics at school, as well as some of the shortcomings of the existing teaching process. (Blum, W., et al. 2007) In particular, it is noted that teachers do not know how to teach students mathematical modelling, they do not use practice-oriented tasks in class. At the same time, the authors state that when practical applications do get included in the lesson content, students' motivation increases, and the quality of subject acquisition improves. It is noteworthy that Blum, W. participated in the development of PISA tasks for testing mathematical literacy.

Most of the research focuses on the context of practice-oriented tasks (Sevinc et al., 2018; Nestorova, 2018), the implementation of the stages of the mathematical modelling method (Blum & Leiß, 2007), the attitude of students and teachers to this type of task (Ang, 2013), and identifying the goals of teaching schoolchildren mathematical modelling (Ikeda, 2013). The questions of the methodological requirements for practice-oriented tasks for school environment and their correlation with proficiency levels are still to be studied thoroughly. The solution of these issues will allow to design and select practice-oriented tasks, to establish their didactic function in teaching mathematics at school, both in the methodological training of bachelors and in the practice of teachers.

Methods

In the context of this study, teaching practical applications of mathematics at school (or practice-oriented learning) is defined as a specially organised cognitive process aimed at forming students' ideas about mathematics as a method of grasping reality, allowing them to describe and study real objects, as well as to develop the ability to apply the studied mathematical concepts, results, methods for studying the simplest objects of reality, solving practice-oriented tasks. (Egupova, 2014).

The idea behind teaching mathematics in such manner was obtained via the analysis of approaches to understanding the relationship and interaction of two equally important components of mathematics - theoretical and applied, about the role of mathematics in the cognition of the objective world, which are described in the works of A. Alexandrova (1986), N. Vilenkina (1980), A. Kolmogorov (1988), and other well-known scholars – mathematicians and teachers.

A distinctive feature of the interaction between mathematics and the real world is that the practical tasks either come to the fore and lead the development of mathematics, or they serve as a means of testing newly created theories. (Kolmogorov A., 1988) All particular mathematical methods used to solve certain classes of applied problems constitute a single mathematical method for processing reality. (Kline, 1985) These ideas point to the inseparable link between mathematics and its applications, and also serve as a rationale for including practice-oriented problems in teaching mathematics at schools.

A practice-oriented task is a task *based on a meaningful model* of a real object, a mathematical model of which can be built using the means of school curricula mathematics contents. (Egupova, 2014)

According to A. Myshkis, the meaningful model of a real object is a physical, biological, social, etc. model of an object or a combination thereof. (Myshkis, 2007) It is clear that the meaningful model determines the context in which the student's ability to formulate, apply and interpret mathematics will be formed, i.e. his/her mathematical literacy.

It is important to understand that not every plot-based task is practice-oriented. The more the meaningful model in the task will be "cleared" of reality, the higher the degree of its abstractness and conditionality, the less practice-oriented-ness in such a task.

It is known that mathematical modelling is the leading method for studying the surrounding reality and that it plays a fundamental role in numerous applications of mathematics, acting as a generator of the most progressive areas in the development of science and technology (I. Blekhman, 2007).

As a result of a comparative analysis of a number of studies in the field of applied mathematics (Myshkis, 2007; Tikhonov et al., 2013), four stages of the mathematical modelling method that can be implemented in school practice when solving practice-oriented problems are distinguished: mathematisation (condition analysis), formalisation (constructing the mathematical model of the condition), intra-model solution, interpretation of the result.

In contrast to other studies of teaching students mathematical modelling (Fridman, 1984), here it was decided to single out the stage of mathematisation, during which in applied mathematics the problem that can be solved by means of mathematical theories is isolated from the real situation.

Also, this stage is key for establishing the level of complexity of a practice-oriented task. In the context of teaching mathematics at schools, at the stage of mathematisation, real objects and relationships between them are singled out, their mathematical equivalents are selected to build a model. According to the degree of complexity of this stage, four levels of complexity of practice-oriented tasks have been identified:

I. In the text of the task, a direct reference to the mathematical model is present.
II. There is no direct reference to the model, but the objects and relations of the problem are unambiguously correlated with the corresponding mathematical objects and relations.
III. Objects and relations of the task are related to mathematical objects and relations, but not uniquely. Considering the actual conditions is necessary.
IV. Objects and relations of the task are not clearly distinguished or their mathematical equivalents are unknown to schoolchildren.

In other studies, the levels of complexity of practice-oriented tasks are not indicated by any features.

Results

For students, the compilation of practice-oriented tasks is included as a main component in the methodological system of preparing a teacher for practice-oriented teaching of mathematics at schools.

In terms of theory, preparing bachelors for constructing such tasks implies studying the following issues:

– the method of mathematical modelling in solving applied problems in science and school practice;

– an understanding of the social, cultural and historical factors of the formation of mathematics as science and mathematics within school curricula;

– applied mathematical skills of schoolchildren and methods of their formation;

– methodological and didactic requirements for practice-oriented tasks, their classification by various criteria;

– functions and significance of practice-oriented tasks in class.

Hands-on training includes creating various educational products to teach students the applications of mathematics. A practice-oriented task is one of such products, which is considered as a goal and a result of student learning (Perevoshchikova, 2020).

The process of composing a task includes the selection of material and text editing. The methodological requirements that a practice-oriented task must satisfy are the following:

I. Requirements for the plot.

I. 1. Reflecting a real object and its properties in the text.

I. 2. Demonstration of the connection of mathematics with other sciences, practical areas of activity.

I. 3. The presence of an issue or properties of an object, in the study of which mathematics plays an essential role.

I. 4. Compliance of the plot with the age characteristics (cognitive interests, the leading type of activity) of the student.

I. 5. Clarity of the plot: the non-mathematical terms used are known to students as a result of studying other disciplines, they are easily defined or intuitively clear. II. Requirements for the mathematics.

II. 1.The mathematical content of the solution.

II. 2. Correspondence of the numerical data of the problem to real value.

II. 3. Correspondence of the actual data, assumptions made and simplifications to the real situation described in the task.

II. 4. Correspondence of the mathematical model of the problem, the complexity of the intra-model solution to the content of teaching mathematics at schools.

Guided by these requirements, undergraduate students select the material to compose tasks. These can be well-known problems with an outdated storyline or not fully satisfying the methodological requirements formulated above. The necessary material can also be found in popular science. With a certain experience and observation, periodicals (interesting scientific facts are often printed), instructions for household appliances, schemes for calculating building materials, etc. can also be the sources for the plot.

To compose a task, students are encouraged to complete the following learning steps:

– analyze the presented material as the possibility of conveying some real issue;

– determine the mathematical means by which it can be solved and correlate these means with the school mathematics course to establish the possibility of solving the proposed problem by students of a particular class;

– construct the plot of the task;

– draw up guidelines for using the task in teaching mathematics at school.

Composing the text. With the text of a practice-oriented task being perceived as educational and scientific, the main requirements for such a text are presented further. They are stated according to the analysis of the difficulties that students experience in compiling such problems.

1.Maintaining the educational and scientific style of the text: operating primarily terminological, general scientific and common vocabulary. The use of terminology known or intuitively clear to schoolchildren, its informativeness.

2. Compliance with stylistic features: accuracy, clarity, consistency, generalisation, impersonality, abstraction, expressiveness of presentation. Use of turns of speech without speech errors.

3. The brevity and reliability of the meaningful model: on the one hand, avoiding an overly detailed description of a real object, and on the other, its inappropriate simplification and primitivisation.

4. Stating the question: specificity and complexity, lack of direct indications of a possible mathematical model, making the construction of such a model trivial.

Here are some examples. Figure 1 shows an abstract from the "Komsomolskaya Pravda"[3)] newspaper page, translated into English. The author composes the task and suggests ways to use it in teaching geometry at school.

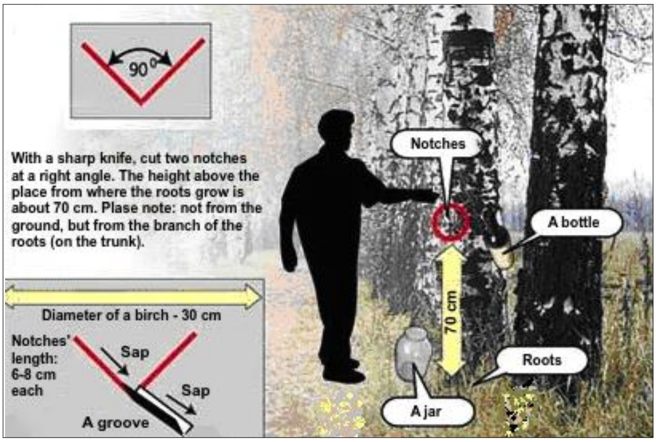

Figure 1. How to extract birch sap

The task. *Birch sap is a healthy and tasty drink. It is usually extracted 4-7 days before the swelling of the buds. For this, two notches are cut with a sharp knife at an angle of 90° on the birch trunk. Then, you insert a groove, along which the juice will flow. The cuts (notches) should be made at a height of 70 cm from where the roots begin (Fig. 1). However, it is only possible to collect the gifts of nature from those trees, the trunk's diameter of which at the notches' level is at least 30 cm. Otherwise, the tree may die after extracting the sap! How do you determine the diameter of a birch trunk?*

Methodical commentary*:* The task is supposed to be used in class for the topic "Circle" when introducing the formula for the circumference to motivate the study of theoretical material.

A methodological commentary is drawn up by a student according to the following plan:

– the topic, for studying which the task may be used; the information necessary to solve it;
– the place of the task in teaching mathematics: a lesson or after school hours;
– functions of the task in teaching: assimilation of mathematical concepts, teaching evidence; creating a problem situation; teaching elements of research, etc.;
– difficulty level of the task;
– source of information.

Practice-oriented tasks can be organised into systems with different internal organisation. By system of tasks, according to G. Kovaleva, one implies a set of ordered and selected tasks in accordance with the set goal, acting as a whole, the relationship and interaction of which lead to the intended result (Kovaleva, 2012). As practical experience has shown, chains (with linear connections between tasks) are the simplest for students to independently compile task systems.

Students learn to compose *the following three types of chains:* 1) tasks that ensure the formation of one mathematical concept and have plots on one topic; 2) tasks with different plots and providing the acquisition of one mathematical concept; 3) tasks that provide the formation of several mathematical concepts and have plots on one topic.

The students' assignment to draw up task chains is formulated as a list of learning activities. To sum up:

– Conduct a methodological analysis of the theoretical material of the mathematics textbook on the selected topic. Identify possible practical applications of the studied theory and select the tasks associated with these applications. Organise the selected tasks in chains, indicating the purpose of each task when studying this material. Provide a short solution to the tasks.

Here is an example of the chain of practice-oriented tasks of the second type, united by the topic "Circumference". The plots of the tasks are taken from everyday life and from the field of geography. The tasks are designed for a geometry lesson in the 9th grade, the geographical information used by Russian schoolchildren has already been studied by this time. Various didactic functions can be assigned to the tasks.

Figure 2. Method of measuring the diameter of a tree trunk

1. Figure 2 shows a method for measuring the diameter of a tree trunk. What geometric statement is it based on?
2. The shape of the Moon, satellite of the planet Earth, is not spherical, but more like an egg. The average radius of the moon is 1738 km. Determine the length of the Moon's equator corresponding to this radius.

3. The diameter of the well gate, on which the rope is wound, is 0.24 m. To pull the bucket from the bottom of the well, you have to do 10 turns. How deep is the well?
4. Calculate the length of the parallel at the latitude of Franz Josef Land (latitude 80°).
5. Determine the geographic latitude of the settlement where you live on the map. Calculate the length of the corresponding parallel.
6. At what latitude of the Earth is the parallel length half as long as the equator?

An important factor that determines the success of teaching students to compose tasks for applications is the analysis of successful tasks, a discussion of how the meaningful model was found, the joint revision of the text, and the compilation of similar tasks. To organise such educational work, the teacher, of course, must have his/her own experience in drawing up tasks, which he/she could share with the students. Collective discussion allows the teacher to evaluate the tasks using elements of the expert assessment method. Students, acting as experts, together with the teacher, check the task for compliance with methodological requirements, assess the educational and scientific nature of the text. This is the final stage of learning how to compose practice-oriented tasks.

Discussion

Taking everything into consideration, teaching students to compile practice-oriented tasks for schoolchildren independently is a complex process, which is one of the components of the methodological system of teacher. This process consists of several stages: theoretical preparation; selection of practical applications of mathematics and composing the text; expert assessment of the task based on group discussion.

The result of the student's methodological preparation for compiling practice-oriented tasks is the ability to create their own educational product that meets the methodological requirements and the requirements for an educational and scientific text. The composition of such an educational product includes a practice-oriented task presented in text, graphic or combined form (or a system of such tasks), accompanied by its solution by the method of metamathematical modelling, a methodical commentary on the task that determines the place of the task in teaching schoolchildren.

NOTES

1. The official website of: Ministry of Science and Higher Education (Russia); the Concept for the Development of Mathematical Education in the Russian Federation; an e-source, access via URL (last visited on 5th September 2020): http://минобрнауки.рф/документы/3894.

2. Education Quality Assessment Centre; web site about PISA; an e-source, access via URL (last visited on 5th September 2020): web site about PISA: http://www.centeroko.ru/pisa06/pisa06.html.
3. "Komsomolskaya Pravda" newspaper (5th May 2006) *Kak sobrat' sok?*

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