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# OPTIMIZING THE POSITIONING OF SERVING UNITS IN THE TOURISM BUSINESS

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**Abstract**. In the present paper an idea for optimizing the positioning of serving units in tourism business is revealed using some applications of mathematics in economics. The elaboration has a methodological character. Methodologically the authors assume that some tourist objects (hotels, restaurants, etc.) are given and they try to determine the best possible position for a serving unit (store, office, etc.) using a set of optimization problems. Finally, an approbation of the results with a real example is considered.

*Keywords:* optimization; tourism; serving unit; positioning

Nowadays, when the globalization and the increasing dynamics of the economical processes are more and more significant for economics, the importance of scientific research in the area of optimization of the general costs in all the business activities is also increasing. One of the areas for analysis and optimization is concerning the so called positioning of different objects (stores, offices, serving units etc.) (Nikolaev & Milkova, 2014).

The significance of the aforementioned is obvious in the area of tourism too. If we assume a set of objects is given (hotels, restaurants, animation centers etc.) and if we need to position a serving unit (store, office, information center etc.) it is clear that the position of this unit is of crucial importance (especially when the number of the objects is big). Usually the choice is made in terms of minimizing the total transportation costs (or the total distance) (Nikolaev, 2014). This problem can be interpreted by the question: how can the serving unit be positioned, so that the sum of all the distances from the unit to the objects served is the least possible one. Different authors (Nikolaev, Milkova &Zhelyazkova, 2017), (Nikolaev & Milkova, 2017), (Grozdev & Nenkov, 2012) have made their research on such topics using various conditions, initial information and interpretations.

The aim of the present paper is to construct an appropriate economical mathematical model for positioning of a serving unit for a set of basic tourist objects, such that the sum of the distances between the unit and the objects to reach its minimum.

Let the positioning of $n$ tourist objects $A_1, A_2, ..., A_n$ is known. Let us assume that both ends of $m$ line segments, such that the serving unit $B$ can be built on any of them, are also known. The question is: which is the best line segment for positioning $B$ in terms of minimizing $\sum_{j=1}^{n} \left| A_j B \right|$.

First of all we will define the problem more strictly from mathematical point of view. Let us have a rectangular coordinate system (if necessary chosen appropriately) and let us know the coordinates $(p_j, q_j)$ of the points $A_j$, $j = \overline{1,n}$. We also know the coordinates $(x_1^i, y_1^i)$ and $(x_2^i, y_2^i)$ of both ends of the line segment $i$ $\left(i = \overline{1,m}\right)$. Let us define with $\left(x^i, y^i\right)$ the unknown coordinates of point $B$ if it lies on the line segment $i$ $\left(i = \overline{1,m}\right)$, hence $(x^i, y^i) = \alpha_i (x_1^i, y_1^i) + (1-\alpha_i)(x_2^i, y_2^i)$, $\alpha_i \in [0,1]$ $\left(i = \overline{1,m}\right)$. Then we have:

$$x^i = \alpha_i (x_1^i - x_2^i) + x_2^i ;$$
$$y^i = \alpha_i (y_1^i - y_2^i) + y_2^i ;$$
$$\alpha_i \in [0,1] \ \left(i = \overline{1,m}\right).$$

So if point $B$ lies on the segment $i$ $\left(i = \overline{1,m}\right)$ the following optimization model can be constructed:

First stage:

$$\min : f_i(x^i, y^i) = \sum_{j=1}^{n} \sqrt{(p_j - x^i)^2 + (q_j - y^i)^2} \ , \ \left(i = \overline{1,m}\right) \qquad (1)$$

subject to:

$$x^i = \alpha_i (x_1^i - x_2^i) + x_2^i , \ \left(i = \overline{1,m}\right) \qquad (2)$$
$$y^i = \alpha_i (y_1^i - y_2^i) + y_2^i , \ \left(i = \overline{1,m}\right) \qquad (3)$$
$$\alpha_i \in [0,1] \ \left(i = \overline{1,m}\right). \qquad (4)$$

Second stage:

For all the optimal points from the first stage $(x^{i*}, y^{i*})$ and for $\min f_i(x^i, y^i) = f_i(x^{i*}, y^{i*}) = f_i^*$, $\left(i = \overline{1,m}\right)$ we find

$$\min_{1\le i\le m}\left\{f_i^*\right\}=f^* \tag{5}$$

and if $f^*=f_{i_0}^*$, $i_0\in\{1,2,...,m\}$ then the optimal position of the serving unit will be $\left(x^{i_0*},y^{i_0*}\right)$.

So at the first stage we have *m* optimization problems (for determining the optimal position on each of the *m* line segments), while at the second stage we choose the optimal segment for *B* so that the sum of the distances is minimum. Constraints (2) and (3) determine the variable $\alpha_i$, $\left(i=\overline{1,m}\right)$ for each of the *m* optimization problems.

When applying the algorithm, for each problem from the first stage we find consecutively:

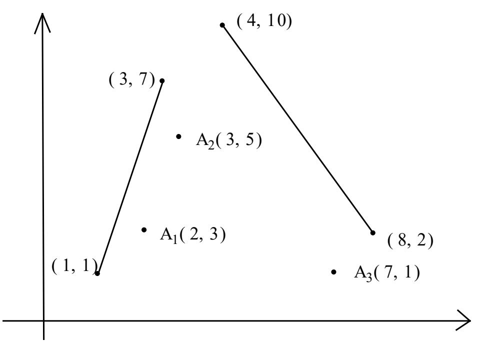
$\alpha_i^*$, $\left(i=\overline{1,m}\right)$;

$(x^{i*},y^{i*})$, $\left(i=\overline{1,m}\right)$;

$f_i^*$, $\left(i=\overline{1,m}\right)$;

while at the second stage from (5) we determine the final solution in the problem $i_0$ from the first stage.

As a demonstration of these theoretical statements we will take an exemplary situation for approbation of the constructed model. Let the positioning of three restaurants in a complex is known and a serving store with products for the restaurants has to be positioned on one of two given streets (line segments). In this interpretation $i=2,\ j=3$. Let us take the coordinates of the restaurants in a coordinate system (chosen appropriately with kilometer measure unit) – $A_1(2,3)$, $A_2(3,5)$ and $A_3(7,1)$. Let both line segments have coordinates $(1,1)$, $(3,7)$ and $(4,10)$, $(8,2)$, respectively (Graph 1).




**Graph 1**

From (2) and (3) we have:

$$x_1 = 3 - 2\alpha_1 ; \qquad x_2 = 8 - 4\alpha_2 ;$$
$$y_1 = 7 - 6\alpha_1 ; \qquad y_2 = 2 + 8\alpha_2 .$$

From the first stage of the model (1) – (4) we formulate the following two optimization problems:

Problem 1 $(i = 1)$:

$$\min : f_1(\alpha_1) = \sqrt{(2\alpha_1 - 1)^2 + (6\alpha_1 - 4)^2} + \sqrt{4\alpha_1^2 + (6\alpha_1 - 2)^2} + \sqrt{(2\alpha_1 + 4)^2 + (6\alpha_1 - 6)^2} ,$$

subjected to $\alpha_1 \in [0,1]$.

Problem 2 $(i = 2)$:

$$\min : f_2(\alpha_2) = \sqrt{(4\alpha_2 - 6)^2 + (1 - 8\alpha_2)^2} + \sqrt{(4\alpha_2 - 5)^2 + (3 - 8\alpha_2)^2}$$
$$+ \sqrt{(4\alpha_2 - 1)^2 + (1 + 8\alpha_2)^2} ,$$

subjected to $\alpha_2 \in [0,1]$.

Both problems can be solved in many different ways (also by a computer using the MS Excel Solver, for example). The results are:

$$\alpha_1^* = 0,5587 ; \qquad f_1^* = 9,0322 ;$$
$$\alpha_2^* = 0,2556 ; \qquad f_2^* = 12,2216 .$$

At the second stage we have:

$$\min\left\{ f_1^*, f_2^* \right\} = \min\{9,0322 ; 12,2216\} = 9,0322 .$$

Then $\alpha^* = \alpha_1^* = 0,5587$ and the point *B* must be positioned on the first line segment and must have coordinates:

$x^* = 3 - 2.0,5587 = 1,884$ ;

$y^* = 7 - 6.0,5587 = 3,6478$ ,

i.e. $B(1,884 ; 3,6478)$ .

Finally, we can make the following analysis, based on the constructed economic mathematical model and its solution – the optimal option for positioning the serving unit (store) is a point at distances 1,884 km and 3,6478 km from both coordinate axis, respectively. Then the sum of the distances from that unit to all the restaurants will be the least possible – 9,032 km, which is obviously the optimal option in terms of distance (if we do not have some other more important considerations).

As a conclusion we can summarize that the problem, treated in the present paper, can be easily extended. One of the directions may be connected with the objects on which the serving unit is positioned. In this material we used line segments (interpreting streets) but curves, regions, etc. could be used too.

Another direction for future research is to increase the number of the serving units. This will surely complicate the model but with all the contemporary technologies the solution could be found without excessive difficulties.

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# ОПТИМИЗАЦИЯ НА ПОЗИЦИОНИРАНЕТО НА ОБСЛУЖВАЩИ ЕДИНИЦИ В ТУРИСТИЧЕСКИЯ БИЗНЕС

**Резюме**. В настоящата статия се разкрива една идея за оптимизация на позиционирането на обслужващи единици в туристическия бизнес с използване на приложения на математиката в икономиката. Разработката има методически характер. Авторите методично предполагат, че са налице някои туристически обекти (хотели, ресторанти и т.н.), които се стремят да определят най-добрата позиция на дадена обслужваща единица (магазин, склад и т.н.), използвайки множество от оптимизационни задачи. В края е разгледан реален пример за апробация на резултатите.

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