From the Research Laboratories

MEASURING THE COLLISION TIME OF A BALL WITH HIGH COEFFICIENT OF RESTITUTION

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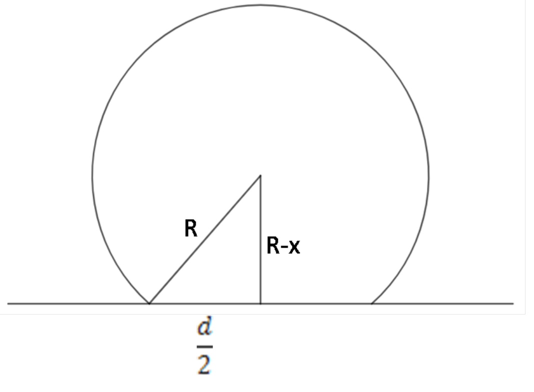
Abstract. The purpose of this paper is to measure the collision time of a ball with a high coefficient of restitution (SuperBall) with a hard surface. The manuscript discusses the theoretical model of the collision and approximations used; the direct methods for measuring the collision time - shooting with a high speed camera from the side; shooting with a high speed camera from below; verification of the theoretical model; measuring the voltage of a photodiode, and the non-direct methods used - component frequency analysis of the collision sound; measuring the spot left from the falling ball. Results and conclusions are highlighted.

Keywords: collision; coefficient of restitution; time; theoretical model; surface

Theoretical model

During the collision there are 2 forces acting on the ball – gravity and the normal force, which changes throughout the collision. We shall assume the total force is proportional to the deformation of the ball. Also due to the high coefficient of restitution (0.9) one can assume no energy is lost. We assume the ball remains spherical except for the deformed part, which is flat with the ground. This assumptions hold for drop heights less than 3m, after which the deformation of the whole ball significantly influences the spot measuring method.

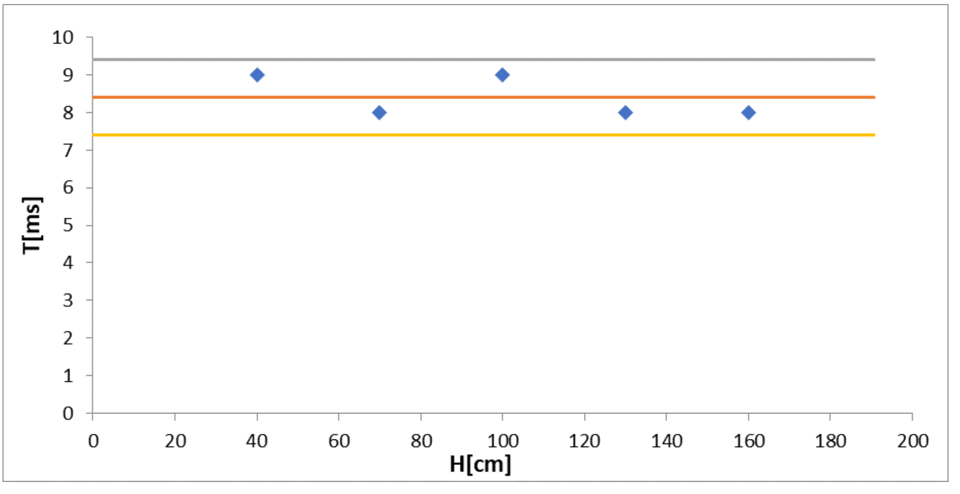


Shooting the collision from the side

A high speed camera of 1000 fps was utilised. The ball is dropped on a hard surface, with the camera shooting at the collision site. From the number of frames in which the ball is touching the surface one can infer the collision time.

Measurements were made for drop heights from 40 to 160 cm every 30 cm, with the collision time averaging at 8.4ms regardless of the drop height. This result leads us to hypothesize the ball acts as a harmonic oscillator.

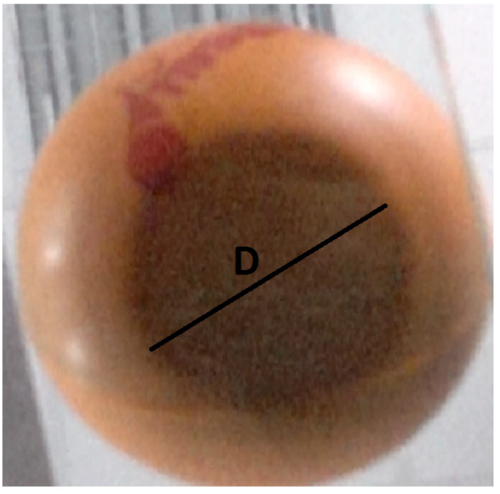
The error in the measurement is ±1ms, or the time between consecutive frames. It can be improved with increasing the frame rate. The graph below shows the collision time vs drop height, which is flat and averaging at 8.4ms.



Shooting the ball from below

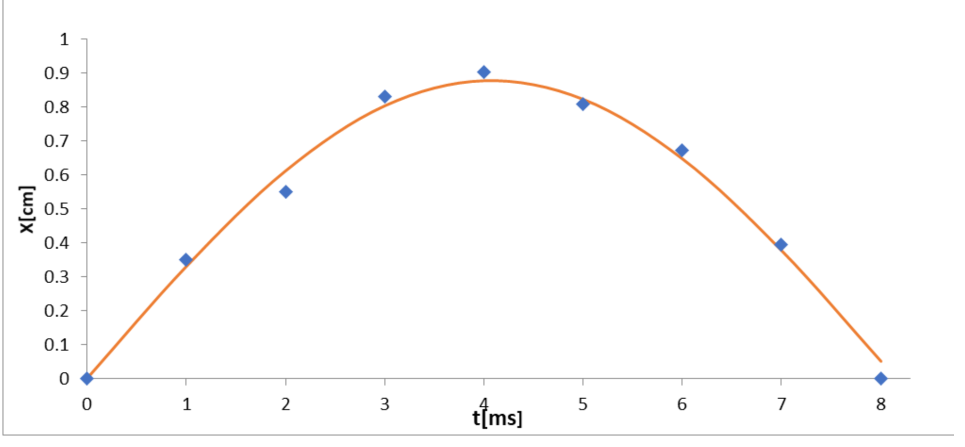
Purpose of the experiment: To obtain the relationship between contact area and time since the ball touches the surface. Using this, one can obtain the deformation of the ball vs time.

Setup: A glass surface is suspended from two sides, and the camera shoots from below. The ball is dropped and for each frame the diameter of the deformation is measured. Using the Pythagorean Theorem the deformation can be computed for each frame.



$$(R - x)^2 + \frac{d^2}{4} = R^2$$

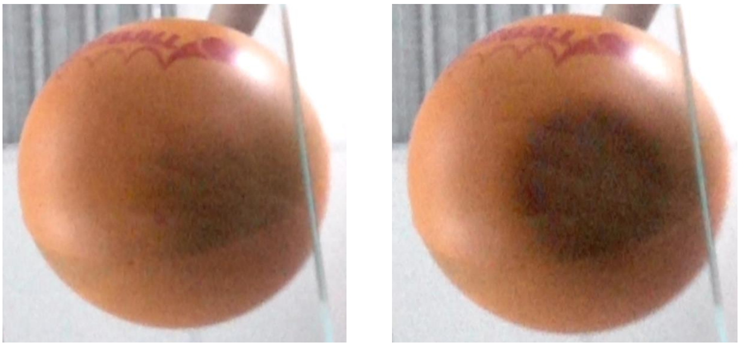
$$x = R - (R^2 - \frac{d^2}{4})^{1/2}$$



The graph shows the deformation of the ball vs time. If the ball acts as a harmonic oscillator this should be a sine wave. The red curve is the best fitting sine wave (using the least squares method), which adequately describes the experimental data. This proves the model of a harmonic oscillation and describes the collision well enough.

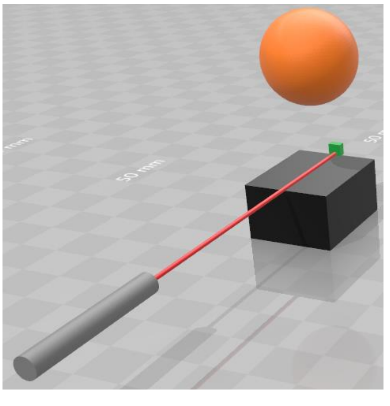
Measurements were made for drop heights in range from 30 to 130 cm, with data points for each measurements fitting well on a sine wave. Averaging the half periods of the fitted sine waves gives a collision time of 8.7ms, not far from the 8.4ms obtained from the previous method.

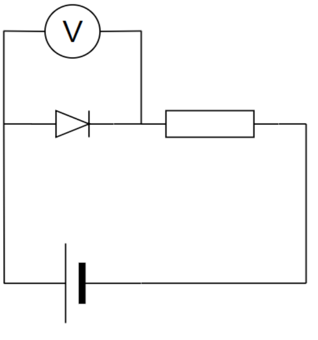
The error in this experiment is larger than the error in the previous one since it is hard to decide the first frame in which the ball touches the glass.



Measuring the voltage of a photodiode

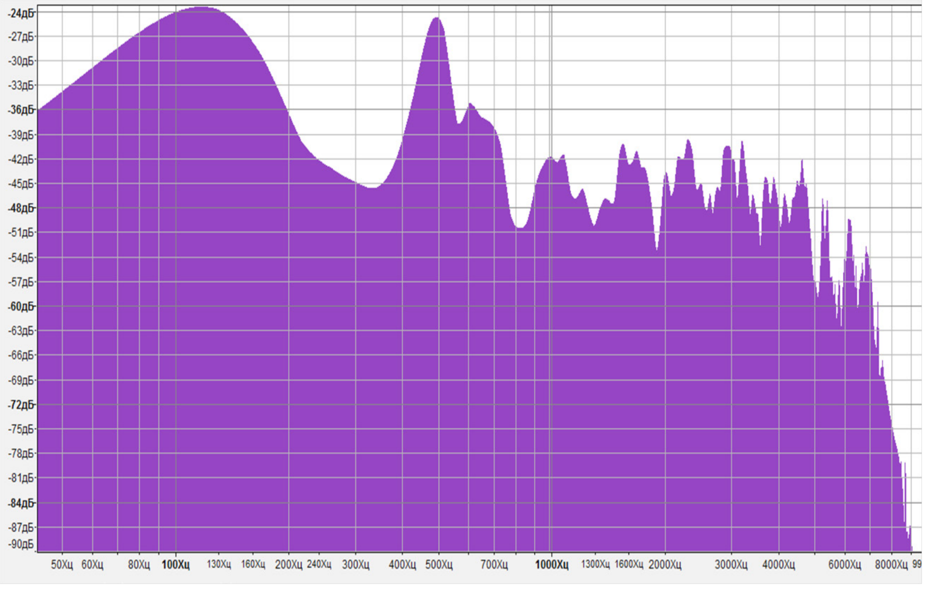
Setup: On the end of the surface a photodiode is attached, and a laser beam is focused on it, just grazing the surface. The ball is dropped on the laser beam, and during the collision the ball obstructs the laser beam. A high speed voltmeter (1000 measurements per second) can measure the peak in voltage, and thus we can measure the collision time. The experiment is restrained in accuracy by how accurately one can drop the ball onto the laser beam. Due to an imperfect drop there will be intervals of time in the beginning and end of the collision where the laser will shine on the photodiode even though the ball is still in contact with the surface. That is why instead of taking the average over many measurements, one should take the maximum time for each drop height. Using this method of data collection an average of 8.4ms is obtained, again in good agreement with the previous two methods.





Sound analysis

The ball is dropped on a hard surface and the sound is recorded. Sound spectrum analysis is performed. There should be a peak in frequency corresponding to the collision time.



A peak in the sound wave is obtained when the deformation of the ball is 0 and parts of the surface are moving the fastest. For one period of oscillation, or twice the collision time, there are 2 peaks in the sound wave. Therefore there is one peak per collision time

Thus the collision time $t = \frac{1}{f}$, where f is a frequency where the sound spectrum has a maximum.

From the previous experiments we know the collision time is between 8 and 9 ms, corresponding to frequencies between 111 and 125Hz. Between these two frequencies there should be a peak in intensity. The sound was recorded in echo-proof room. For all the drop heights the frequency peak in the desired range was approximately equal, and averaging at 117.86Hz (from approximately 100 measurements). Form this a period of 8.48ms is derived, which is the most accurate period so far. The increased accuracy comes from the accuracy of modern microphones, and the longer duration of the sound, which allows for a longer time interval on which Fourier analysis is performed.

Measuring the maximal deformation of the ball

A transparent file folder is filled with a thin layer of flour on which the ball can leave an impression. The diameter of this impression is measured, allowing us to compute the maximum deformation. Since we assume energy is not lost, then all the gravitational potential energy is transformed into elastic potential energy. Let the ball have mass m, coefficient of elasticity k and radius R.

From the law of conservation of energy:

$$mgh = k\frac{x^2}{2}; h = \frac{k}{2mg} * x^2$$

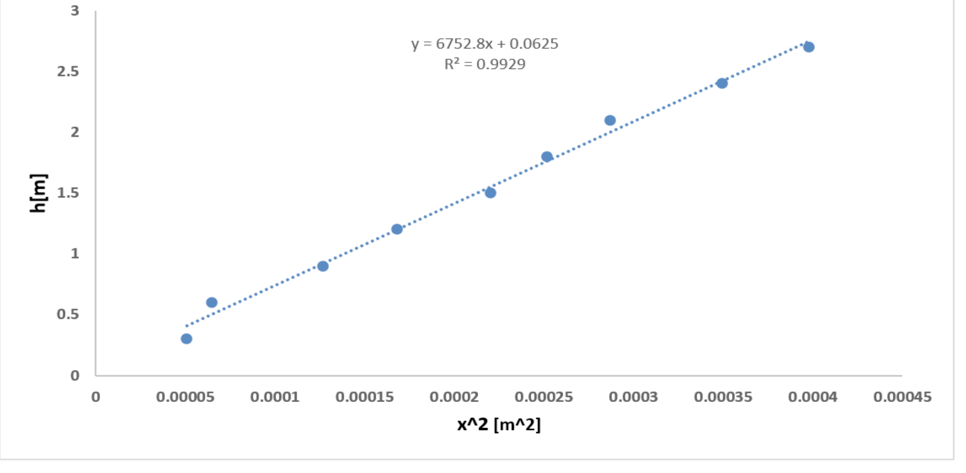
As in the previous experiment, we used the Pythagorean theorem to calculate the deformation *x*.

We can drop the ball from different heights and measure the diameter of the spot *d*, from which we can compute x^2. Plotting h vs x^2 should be a straight line with equation

$$h = \frac{k}{2mg} * x^2 = a * x^2 .$$

From here we can compute the coefficient of elasticity

$$k = 2mg * a$$



From the graph, we can see that the data are fitted by a line. Using the obtained value for the gradient of this line, the collision time can be computed knowing the mass of the ball and g, by the formula for a period of a spring pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{2ga}}$$

The collision time is half of the oscillation period T, or

$$t = \pi\sqrt{\frac{1}{2ga}} \approx 8{,}6\ ms$$

The inaccuracy of this method comes from the inaccuracy due to measuring the diameter of the spot and the drop height. It relies on the assumption that energy is not lost during the collision. At higher collision speeds this would be inaccurate, however since the data points lie almost perfectly on the line of heights of up to 3m, then the model is accurate for drop heights of at least that.

Conclusion

This paper has proven that the collision of Super Balls with a hard surface can be modeled by a harmonic oscillator. By 5 different methods it has been shown that the collision time is between 8.4 and 8.7ms, with the most accurate one of all leading to 8.48ms. Each of the methods confirm that the collision time is independent of the drop height. These experiments are valuable since most of them can be easily reproduced by students of all backgrounds, and they teach the basics of data analysis and experimental methods.

Method	Collision time [ms]
Shooting from the side	8,4
Shooting from below	8,7
Photodiode	8,4
Sound spectrum analysis	8,48
Flour	8,6

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