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# MATHEMATICAL SKILLS ARE REQUIRED IN THE CHANGING WORLD

**Desislava Georgieva**
*University of Ruse "Angel Kanchev"*

*"In mathematics, know-how is more important than mere possession of information."*
*George Pólya*

**Abstract.** Different treatments of the notions "skill" and "mathematical skill" are presented in the paper with regard to their relation to the notion of competence. Old and new educational standards of different countries are collected and analysed, thus outlining the framework of the mathematical skills which are necessary to face the needs of the 21st century education.

*Keywords:* mathematical skill, competence, iceberg model, KSAVE model.

## INTRODUCTION

The globalisation of the intensively changing world and the goal to realize knowledge-based economies set new challenges for the government policies in Europe. In response, the Lisbon European Council (March 2000) decided a new European framework as a key measure that should define basic skills through lifelong learning. The framework is restated regularly during meetings of the Brussels European Council. In 2006 the European Parliament and the Council determined eight key competences for lifelong learning with long-standing importance. One of them is the mathematical competence, which is connected with basic competences in science and technology. The European Key Competence Network for school education between 2012 and 2014 focuses on identifying and analysing initiatives for the implementation of key competences in primary and secondary school education (Kearney, 2013)

It is noted in a report on mathematics education in Europe, that the PISA research accounts for a considerably lower average achievement of the students in Bulgaria, Romania and Turkey in comparison with the other members of the Eurydice network (Education, Audiovisual and Culture Executive Agency, 2011, p. 15). Thus, a natural task for the Bulgarian educators is to find ways for the improvement of the Bulgarian educational system in regard with the world's standards of student knowledge and

skills. It is very important for the Bulgarian school authorities to be acquainted with the needed school skills, which should be acquired in accordance with the curriculum topics and the corresponding particular classes, determined in respecting the class working time. The compulsory part of the curriculum should be approved by the school management, while the regional inspectorates should approve the optional one.

**ON THE CONCEPT OF SKILL**

In the scientific literature there are diverse interpretations of the concept "skill". Although very similar, they have some important differences.

According to the English dictionaries, a skill is "*an ability to do an activity or job well, because you have practiced it*"[1), 2)]. Russian encyclopaedias add: "*to perform fast, exactly and conscientiously*" and "*in accordance with the received knowledge*"[3), 4)]. Anglo-saxon authors accept the notion under consideration to be natural and do not try to explain it. On the contrary, many helpful definitions appear in Russian scientific materials:

1) gained means for action executions [5)];

2) stage of learning new methods [6)];

3) mental ability of individuals, connected with the internal ability to realize activities most successfully (Леонтьев, 1980);

4) exteriorization of knowledge and habits in real operations [4)];

5) extremely complicated structural unification of personal sensual, intellectual and emotional qualities that are formed and exhibited consciously through appropriate and successful actions in the attainment of the activity objective in changing conditions" (Милерян, 1973, pp. 51-52).

One of the Bulgarian Psychology Dictionaries describes the skill notion "*as an opportunity for success in task performance, exercised as a result of repetition, experiments or tries, as a way of self-usage of knowledge and concepts, as a mental process and physical action to solve theoretical and practical tasks"* (Десев, 1999).

The content of the skill concept can be summarized as a physical ability or mental property of the individual to determine successfully, exactly and conscientiously performances of physical or mental activities in changing conditions in accordance with the received knowledge. Operationally skills are formed in exercising repeating actions resulted in learning. They are related to knowledge and habit applications in new situations.

**SKILLS AS PART OF COMPETENCY**

N. Ogata measures and compares social science competences of graduates in Japan and the Netherlands, describing them by the so called "iceberg model" (Ogata, 2007, pp. 55-56). He outlines two kinds of skills – soft skills and hard skills:

1) Personality and motives, lying at or below "sea level" and therefore hidden, are referred to as soft skills – in many cases innately determined and difficult to be developed.

2) Conversely, knowledge and skills, lying above "sea level" and therefore more visible, are called hard skills, which are possible to be acquired post natally, and are easy to be developed. So, education may develop this "hard skills", which educators and governments have framed in national and cross-national standards.


Source: Based on Spencer and Spencer (1993)

**Fig. 1.** Iceberg model of competencies (Ogata, 2007, p. 52)

Concerning the broad spread terms in the framework, the soft and the hard skills are not so correct, because skills, lying above "sea level" can be shaped and developed, thus allowing to label them as "soft skills". More than fifteen years politics, researchers and educators in Europe, the United States and many other countries have realized projects, in which educational frameworks of relatively new term competencies [8)] have been used. But what is the relation between this term and the well-known skill notion?

The Department of Education in Washington has proposed a pyramid assessment model - Fig. 2.

Every person has individual characteristics which are prerequisites for acquiring different skills and knowledge through learning activities. Various combinations of the acquired skills and knowledge determine correspondent competencies and their individual variety is crucial for an effective realization of tasks (Jones, 2002, p. 8).

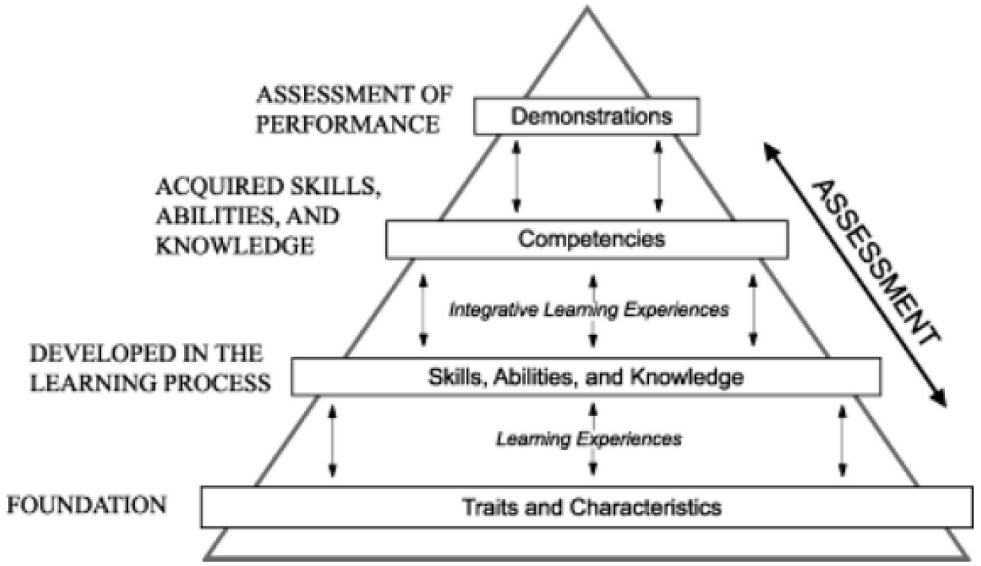


**Fig. 2.** Position of the skills in the hierarchy of acuiring and assessing individual's performance (Jones, 2002, p. 7)

The conclusion is that skills can be developed through learning processes resulting in different stages of the acquired competence.

**HISTORICAL RETROSPECTION OF MATHEMATICAL EDUCATION STANDARDS**

The basic mathematical skills are divided into ten areas by NCSM [9] since the 70-ies of the past century. All of them are important for the development of pupil ability for effective reasoning in various situations: Problem Solving; Mathematics Applications to Everyday Situations; Alertness to Result Reasonableness; Estimation and Approximation; Appropriate Computational Skills; Geometry; Measurement; Reading, Interpreting, and Constructing Tables, Charts, and Graphs Using Mathematics to Predict; and Computer Literacy (NCSM, 1977, pp. 4-6).

In 1995 the National Centre for Education Statistics outlined, that "*student skills are required for a success in mathematics*": Remember formulae and procedures; Think in sequential manner; Understand concepts; Think creatively; Understand math use in real world; and Support solutions (National Center for Education Statistics, 2000, p. 212).

The NCTM "process standards" are the first standards with longstanding importance in mathematics education in the USA and Canada. They include problem solving, reasoning and proof, communication, representation, and connections (National Council of Teachers of Mathematics).

Next are the strands specified in the National Research Council's report "Adding It Up":

1) conceptual understanding – comprehension of mathematical concepts, operations, and relations;

2) procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

3) strategic competence – ability to formulate, represent, and solve mathematical problems;

4) adaptive reasoning – capacity for logical thought, reflection, explanation, and justification;

5) productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick, Swafford, & Findell, 2001, p. 116).

**NECESSARY SKILLS FOR EVERYBODY NOWADAYS**

During the last decade many initiatives were carried out to improve the economies of the states and to base them on knowledge. The main goal is to get more and more people being enough educated, competent, enterprising and innovative. Among the documents with remarkable importance are the Bologna decree for Life Long Learning, the European Qualification Framework 2006…, the National Qualification Frameworks, the Key Competence Network, the research initiatives of large dimensions like TIMS and PISA, etc. Quite precise but complicated at the same time is the "KSAVE1) [10)]. Model of the Twenty-First Century Skill framework", which is described in the second chapter of the book "Assessment and Teaching of the 21st Century Skills". Researchers have compared twelve relevant frameworks drawn from a number of states and have organised the nowadays necessary skills. The detailed model includes:

1. Ways of Thinking (Creativity and innovation; Critical thinking, Problem solving, Decision making; Metacognition);

2. Ways of Working (Communication; Collaboration);

3. Tools for Working (Information literacy, ICT literacy);

4. Living in the World (Citizenship; Life and career; Personal and social responsibility) (Binkley, et al., 2012, pp. 36-58).

The Problem solving skills demand the Cognitive Process Skills to be divided into the following categories (2012, p. 47):

1. Task Regulation Skills

1.1. Organisation (problem analysis);
1.2. Set goals;
1.3. Resource management;
1.4. Flexibility and ambiguity;
1.5. Collections of information elements;
1.6. Systematization;
2. Learning and Knowledge Building Skills
2.1. Relationships (representations and formulations);
2.2. Rules: "If ... then";
2.3. Hypothesis "what if..." (reflections and monitoring)).

No doubt, that the thinking skills need to be explicitly modelled, drawn out and reapplied in different contexts, one of them being a technology-rich environment (Hesse, 2015, p. vii).

The effective teaching of transferable thinking skills with ICT applications contains the following elements or some of them:

1) teaching 'thinking' vocabulary and giving explicit explanation of the thinking skills that learners should acquire;

2) observing expert task performance (modelling);

3) in-time performance feedback (formative assessment);

4) direct support in the task-learning early stages (scaffolding) and a gradual move towards self-regulation and autonomy (teacher fade-out);

5) the opportunity to articulate thinking strategies and discuss them with other learners (thinking together);

6) explicit demonstrations of how acquired thinking strategies in one subject area can be used to solve problems in another area (bridging) (Wegerif, 2002, p. 34).

The cross national research PISA measures these skills, but note, that in Bulgaria it has been carried out in few specialized mathematical schools only and this is one of the reasons for the Bulgarian low average score.

In 2015 some details of the KSAVE Model of the Twenty-First Century Skills are shown schematically in the book *Assessment and Teaching of 21st Century Skills. Methods and Approach* - Fig 3.

Becoming more and more important the creative and innovation skills are comprehensively and operationally determined in (Piirto, 2011). The conclusion is that the mathematical education is closely related to the first group of skills, i.e. to the *Ways of Thinking*, but the others may and must be improved directly and indirectly in the learning process.

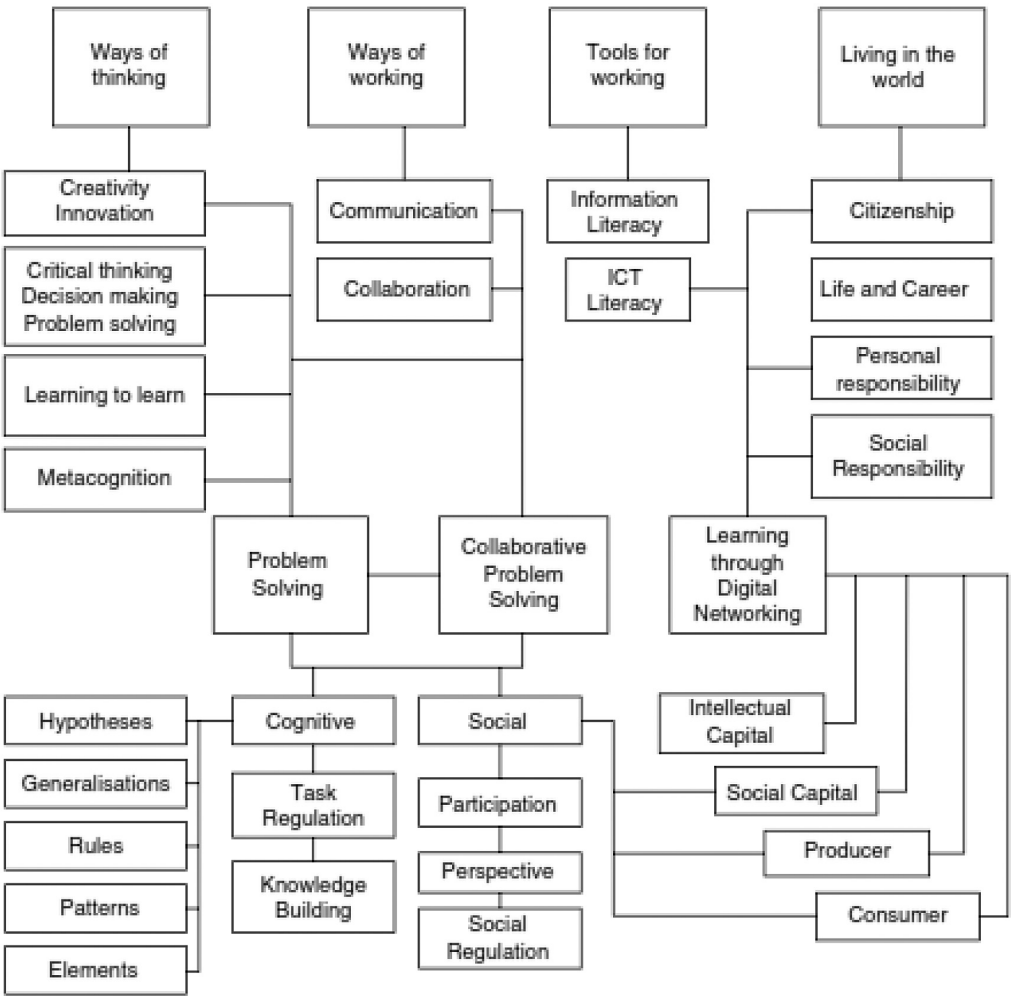


**Fig. 3.** The KSAVE Model of the Twenty-First Century Skills framework

**SUMMARY**

The result of the standard and notion analysis concerning mathematical competency, mathematical literacy, mathematical proficiency, etc., is given in the proposed framework below (Table 1), which outlines the necessary mathematical skills for facing the needs of the twenty-first-century education.

**Table 1.** Classes of the needed skills for today's mathematical education

| Problem solving skills | Thinking skills | Additional skills in mathematics performance |
|---|---|---|
| Goal settings and persistence in meeting them | Abstract and quantitative reasoning | Metacognitive skills |
| Observing, classifying, using space/time relationships | Finding regularity in repeated reasoning | Looking for and using structures |
| Interpreting data, selecting and controlling variables | Convergent thinking | Communicating mathematically |
| Specifying relationships | Divergent thinking | Collaborating |
| Mathematising (Niss, 2015, p. 51) | Reasoning inductively and deductively | Applying mathematics to everyday situations |
| Choosing appropriate computation | Critical reasoning | Procedural fluency |
| Constructing viable arguments | Reflective thinking | Using appropriate tools strategically |
| Inferring, predicting | Being alert to result reasonableness | Attending of precision |

The Problem solving skills are concrete skills involved in searching and finding solutions of tasks or problems. The skill, which is important not only in mathematics but in the real life too, is the ability to set goals and to persist in achieving them. Nowadays, the attention should be concentrated on data interpretation concerning different sources like tables, charts, diagrams and tools. With essential importance for business and engineering is the skill of inferring and predicting changes in various events, using parameters and functions that are involved by means of mathematical modelling and statistics.

The thinking skills are developed strongly through the process of mathematical problem solving. They are useful for other learning subjects and for the everyday activities. Since Pólya's prominent book "How to solve it" we know that the reflective thinking after the finish of the solving process is significantly important for outlining methods and approaches that we may be used in other problems. To think convergently in reaching the goal has been substantial for many decades but today it is more sensible

to think divergently (creatively and innovatively) because one problem may have many solutions, which are applicable to a given situation. In the rapidly changing labour market people must learn continuously, so they must be in possession of metacognitive skills (skills for learning). The skill topic is large and the frames of the present paper do not allow exhausting it.

## CONCLUSION

New standards for what students should be able to do must replace the basic skills and knowledge expectations of the past. To meet this challenge, schools must be transformed in ways that will enable students to acquire the sophisticated thinking, flexible problem solving skills, collaboration and communication skills they will need to be successful in work and life (Binkley, et al., 2012, p. 18).

## NOTES

1. Cambridge dictionary: http://dictionary.cambridge.org/dictionary/british/skill
2. Oxford dictionary: http://www.oxforddictionaries.com/definition/learner/skill
3. Официальная терминология, Словари и энциклопедии на академике: http://official.academic.ru/27502/%D0%A3%D0%BC%D0%B5%D0%BD%D0%B8%D1%8F , (2015, Aug, 18).
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8. At the end of twentieth century
9. Network Communicate Support Motivate
10. Knowledge, Skills, Attitudes, Values and Ethics

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# В ПРОМЕНЯЩИЯ СЕ СВЯТ СА НЕОБХОДИМИ МАТЕМАТИЧЕСКИ УМЕНИЯ

**Резюме.** В статията са представени различни схващания за понятията „умение“ и „математическо умение“ от гледна точка на отношението им към понятието за компетентност. Събрани са и са анализирани стари и нови образователни стандарти от различни държави и по този начин е очертана рамката на математическите умения, които са необходими във връзка с изискванията на образованието през XXI век.

✉ **Mrs. Desislava Georgieva, PhD student**
Department of Mathematics
University of Ruse “Angel Kanchev”
8, Studentska Str.
7017 Ruse, Bulgaria
E-mail: dmgeorgieva@.uni-ruse.bg.