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MATHEMATICAL MODELLING IN THE COMPUTATION OF VOLUMES OF SOLIDS OF REVOLUTION

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Abstract. In this paper, the influence of the mathematical modelling approach in computer-supported collaborative learning (CSCL final) on students' knowledge of calculus contents, particularly the application of definite integral, is examined. The research was conducted with the final-grade students at the grammar school. Two groups of the students, the experimental and the control one, were observed. Both the groups learned in CSCL environment, while in the experimental group, mathematical modelling was applied. The work of the experimental group students during the whole mathematical modelling process was monitored and analyzed. Some examples of the students' solutions are described. After the learning process of the integral contents, the students' learning achievements were tested and compared. The results indicate that the use of mathematical modelling increases the students' interest in solving the definite integral application problems. It is proved that the students who applied modelling process had better results than the students who did not.

Keywords: Calculus; Constructivism; Definite integral; Mathematical modelling; volume

1. Introduction

The process of teaching and learning calculus at the university level is analyzed in numerous papers (Tall, 1992; Tall, 2003; Takači et al., 2015). The integral concept is one of the basic topics in calculus and, therefore, its understanding is crucial in learning calculus and mathematics. Also, this topic often causes difficulties for students (Mahir, 2009; Orton, 1983; Soylu & Tatar, 2007). Both graphical and algebraic representations of the applications of definite integrals and their connections are very difficult for the students to understand (Milovanović et al., 2011; Tall, 2003). The use of dynamic software, such as *GeoGebra*, enables simultaneous multiple representations of functions and related topics. This contributes to overcoming the students' difficulties in learning this calculus content (Arzarello et al., 2012; Božić et al., 2019).

Mathematical modelling in computer-supported collaborative learning (CSCL) is a modern approach to teaching in mathematics education (Komis et al., 2003). Calculus, in particular, integral calculus, is one of the most difficult contents for high school and university students. Therefore, it is challenging to examine the process of its teaching and learning, especially in a mathematical modelling CSCL environment (Tall, 1992).

In this paper, the process of mathematical modelling, in CSCL environment in the last grade of grammar school is analyzed.

2. Literature review

2.1. Constructivism

Constructivism is a learning theory in which the student actively and independently constructs his knowledge, based on its previous experiences. The learners use their previous knowledge as a foundation for building new material. The students explore their learning environment, conduct and monitor their own learning and are responsible for it (Naylor & Keogh, 1999; Taber, 2011; Sjoberg, 2010; Iran-Nejad, 1995).

In the late twentieth and early twenty-first centuries the authors as (Bodner, 1986), (Tobin & Tippins, 1993) and (von Glasersfeld, 1995) contributed to the constructivism. According to von Glasersfeld *the constructivist model implies that knowledge has quality if it is applicable, that is, if it is useful in achieving some goals* (von Glasersfeld, 1995).

The teachers are very important in constructivist learning as in the traditional one. The teacher carefully guides and follows the students' learning process. He is supposed to introduce students to the new contents and then to help them in understanding and adapting existing knowledge to a new one. The teacher is responsible for creating such a learning environment that will help students acquire new skills and knowledge (Tobin & Tippins, 1993).

2.2. Collaborative learning

According to Laal and Ghodsi (2012, p.486), "Collaborative learning is an educational approach to teaching and learning that involves groups of learners working together to solve a problem, complete a task, or create a product." In small collaborative groups, the students work together in order to maximize the effects of learning. Collaborative learning implies that students at various performance levels work together in small groups toward a common goal (Gokhale, 1995).

Collaborative learning is based on constructivism. Constructivism encourages students' discussions among them (Naylor & Keogh, 1999; Taber, 2011; Sjoberg, 2010; Iran-Nejad, 1995), which are very important for collaborative learning. Collaborative work provides excellent conditions for effective constructivist learning for students (Takači et al., 2015).

The groups in collaborative learning can be homogeneous and heterogeneous. The homogeneous small groups consist of students with similar characteristics and the heterogeneous small groups consist of students with diverse characteristics. Heterogeneous groups are more efficient, because the members of the groups help one another in their learning, since students have different views of the given problem and different understandings of the obtained answers. The groups of four members are the most effective (Takači et al., 2015; Johnson & Johnson, 1999; Kagan, 1994; Slavin, 1990).

The development of technology enabled the improvement of collaborative learning using different software. A method of collaborative learning, supported using computers, is called computer-supported collaborative learning (CSCL). This method proved to be very useful in mathematics teaching and learning (Takači, Stankov, & Milanović, 2015; Tall, 1992).

2.3. Mathematical modelling and the multiple representations

Mathematical modelling is considered the transformation of any real problem to mathematical model (Arseven, 2015). It can be considered as an approach for "obtaining a mathematically productive outcome for a problem with genuine real-world motivation" (Galbraith & Stillman, 2006, p. 143).

The mathematical modelling process has several phases (Maaß, 2006; Stillman, 2012) and the best-known is the mathematical modelling cycle (Galbraith & Stillman, 2006). It consists of seven phases, and the most important are the cognitive activities within the transitions between modelling phases (Ferri, 2006; Sekulić et al., 2020). The application of mathematical modelling can improve the teaching and learning of mathematics and the implementation of mathematical modelling in the curricula was the topic of the previous research (Kaiser, 2020; Niss & Blum, 2020).

The realization of the mathematical modelling process can be improved by using modern technologies. The dynamic software, such as *GeoGebra*, enables multiple representations of mathematical objects and manipulations with the algebraic, graphical and numerical representations, contributing to better understanding of the relationships between mathematical model and the real-world problem (Cekmez, 2020; Sekulić, et al., 2020).

Mathematical representations are very important for students' learning. They help them understand and connect the mathematical contents. In mathematics education, in particular, in learning calculus, algebraic, graphical, tabular and verbal representations are used. According to many authors (Božić et al., 2019; Hwang & Hu, 2013; Milenković et al., 2024; Milovanović et al., 2011; Ozgun-Koca, 1998) various representations are necessary for better understanding calculus contents.

The use of the multiple representations contributes to improving the mathematical modelling process (Arseven, 2015; Blum, 2002). The use of dynamic software, such as *GeoGebra*, enables simultaneously dynamic manipulation of algebraic, graphical, tabular, and verbal representations. Due to this, *GeoGebra* software is suitable for mathematical modelling in learning.

3. Research question

The research was conducted with students of the final grade of grammar school, in order to analyze the impact of mathematical modelling in *GeoGebra* collaborative learning environment on students' achievements in learning calculus contents, in particular the application of the definite integral. In that manner, the main questions of our research are:

– (RQ1) Does the application of mathematical modelling in CSCL environment contribute to better students' achievements in learning the application of the definite integral?

– (RQ2) How does the application of mathematical modelling impact students' learning of calculus content, particularly the application of the definite integral in CSCL environment?

4. The methodology of research

4.1. General background

In this research, an experimental approach was applied. According to the aim of the research, the experiment was conducted with two parallel groups – the experimental and the control group.

The students in the experimental group applied mathematical modelling within the learning process and the students of the control group did not. Both the groups learned in a CSCL environment and used *GeoGebra* software. It was chosen because it provides multiple representations in a dynamic environment, and it is easily accessible and simple to manipulate. All the students were well trained in the use of *GeoGebra*. The impact of mathematical modelling in CSCL on the students' achievements in calculus, in particular on the determination of the volume of different solids of revolution, was analyzed.

4.2. Participants

The research was conducted with 126 students during their mathematics course in grammar school in Novi Sad, Serbia, in the 2022 and 2023 school years.

The experimental group in the school year 2023 consisted of 63 students from two classes (32+31 students). They were divided in 8+8 groups. In 15 small groups there were 4 students, and one group had 3 students.

The control group in the school year 2022 consisted of 64 students from two classes, each with 32 students; however, 63 students were considered in the analysis.

4.3. Instruments and procedures

In most high schools in Serbia, elements of calculus contents are taught in the final grade. Students have many difficulties in learning calculus (Tall, 1992; Tall, 2003; Takači et al., 2015). Particularly, integration is often a challenge for students (Dorko, 2012; Milenković et al., 2020; Orton, 1983). In order to overcome these difficulties, the process of mathematical modelling in collaborative *GeoGebra* learning environment was applied for the applications of calculus, in particular for the calculation of the volume of different solids of revolution.

Both the groups did exercises in calculating the volume of solids of revolution in school, over three school lessons (3 × 45 minutes), conducted separately for each class. The experimental group applied the modelling process, and the control group did not.

The students' previous knowledge and skills necessary for learning integral content were checked by the pre-test. The results of the pre-test were analyzed, and there was no statistically significant difference between the experimental and the control groups.

According to the pre-test results, the four-member heterogeneous groups, based on different levels of knowledge, were formed using Kagan's principles (Kagan, 1994) as follows. The list of all students, ranking from the worst to the best, was formed using the results of the pre-test. The first small group was formed by taking the first student, the last one, and two students from the middle of the rank list. When the first group is formed properly, its members were removed from the list, and the process was repeated. In the first class of the experimental group, the first 7 small groups are formed as explained. The remaining three students formed the last group. In second class, there were 32 students, and the teacher formed eight small groups. Similarly, the small groups in control group were formed. All members of the groups were given an excellent mark for successfully completing the task.

In both groups, the experimental and the control, collaborative learning was conducted. All students had smartphones or tablets, and one smartphone or tablet was used as the main device with GeoGebra open. Students performed calculations using GeoGebra and occasionally used it for explanations.

During their work, the students spontaneously shared roles and responsibilities within each small group. In each small group, one student always led the work and directed activities. An example of the students' collaborative work is shown in Figure 1.

In order to help students in their work, the teacher moved among the students and continuously monitored their work, talked to them, and answered their questions. The leading students of the small groups communicated with the teacher.

Figure 1. The students' collaborative work

On the other hand, the control group students solved the usual mathematical tasks, using their textbooks.

5. Analysis of the mathematical modelling process

The students in the experimental group applied the mathematical modelling process. Each small group was asked to bring three objects in the shape of a cylinder, a conical frustum, a vase, or another object in the shape of a solid of revolution.

At the beginning of the mathematical modelling process, the students were given the first real problem: **Determine how much water the cylinder (which you brought) can hold, by pouring water, using GeoGebra, and applying the definite integral.**

Starting from the real problem, the students concluded that they needed to find the volume of the cylinder. In the first and third tasks, they already knew the mathematical model—the formula for determining the volume of a cylinder—but in the second task they had to make several attempts to move from the real-world problem to the mathematical model.

At the beginning, the students filled the cylinder with water and measured its quantity. For example, group G15 measured $0.5\ dl$. All the students knew the mathematical model of the volume of a cylinder, $V = \pi r^2 H$, where r is the radius of the cylinder base and H is the height of the cylinder. Group G15 measured the radius as $r = 1.2cm$ and the height as $H = 10cm$. Then, they calculated the volume as $V = 14.4\pi\ cm^3 \sim 45{,}24\ cm^3 = 0{,}045dm^3 \sim 0.5dl$.

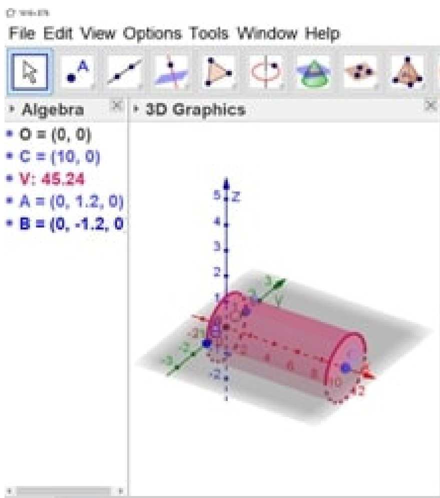

Figure 2. The students' work (group G15)

In order to determine the volume of the cylinder (which they brought) using *GeoGebra*, after brief discussions, all the groups concluded that they needed to draw the cylinder according to the measurements of the cylinder they had. This means that they used only the numerical values of the dimensions. For example, group G15, with radius $r = 1.2cm$ and height $H = 10cm$, determined the points $A(0,1.2,0)$, $B(0,-1.2,0)$, and $C(10,0,0)$. Using the command "cylinder", they obtained a cylinder with volume $V = 45.24cm^3$ (Figure 2).

The students completed the third task using only the formula for calculating the volume of a solid of revolution:

$$V = \pi \int_0^{10} 1.2^2 dx = \pi \cdot 1.44 \cdot 10 \approx 45.24\ cm^3.$$

Several groups, such as small group G4 (which used the same cylinder as group G15), created a cylinder (the green one, Figure 3) by rotating the segment AB around the x −axis. They obtained the same cylinder as the red one in the second example.

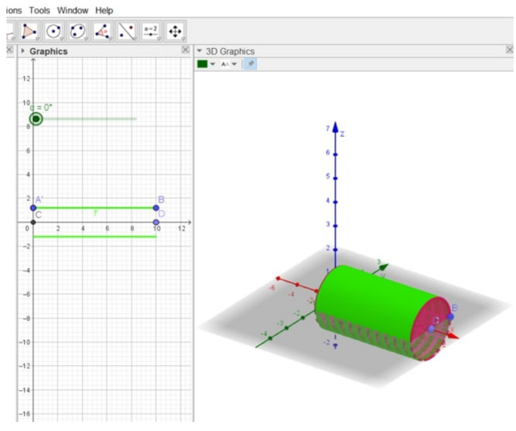

Figure 3. The students' work (group G4)

The other groups did similar tasks. For example, the students from small group G3 used a candle as the cylinder and calculated its volume with *GeoGebra*, using the cylinder command, as shown in Figure 4. They did not take the x −axis as the axis of symmetry for the cylinder (candle), but they obtained its volume accurately.

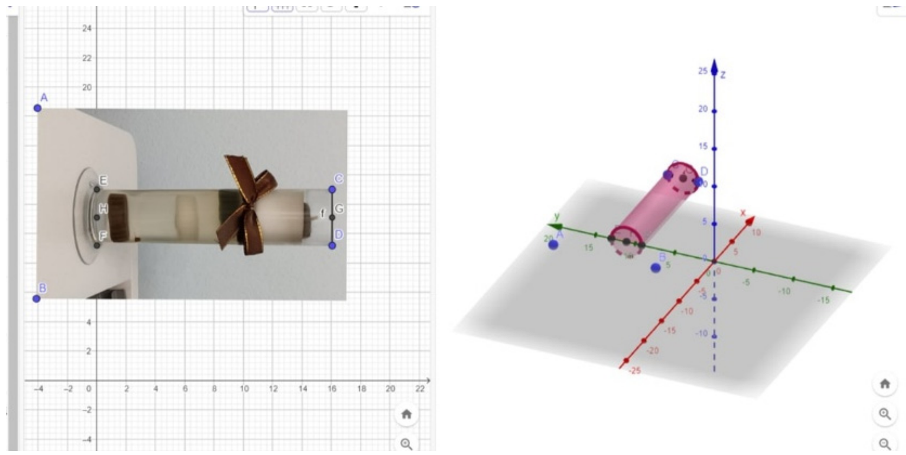

Figure 4. Determining the volume of the candle using *GeoGebra* (group G3)

Then, the students were given the following real problem: **Determine how much water the conical frustum (which you brought) can hold by pouring water, using GeoGebra, and applying the definite integral.**

All the groups brought glasses in the shape of conical frustum. For example, students of group G11 filled the glass with water and measured the volume of water. The volume was approximately $310\,ml$, i.e. $310\,cm^3$. Then they determined the volume of the conical frustum using the direct formula and obtained $V = 312.21\,cm^3$.

After that, they took a picture of the glass and inserted it into *GeoGebra*. They tried to determine the volume of the glass using *GeoGebra*. After discussions with the teacher, most of the groups calculated the volume of the glass using the *GeoGebra* 3D tools. The picture dimensions were adjusted to correspond to the real dimensions of the glass. They observed the 3D view in *GeoGebra* and constructed two cones; thus, the conical frustum corresponding to the glass was obtained by cutting the smaller cone from the larger one. The work of group G11 is shown in Figure 5. They calculated the volume of the truncated cone by subtracting the volume of the smaller cone from that of the larger one and obtained a volume of $312.21\,cm^3$ for the glass.

Some groups used the *GeoGebra* 3D options without taking a picture of the glass.

In order to determine the volume of the glass using a definite integral, the students modelled the glass with an appropriate function. Group G11 set the picture in the *GeoGebra* worksheet so that the *x*-axis became the axis of symmetry of the glass. Then they calculated the proportional ratio between the glass in *GeoGebra* and the real glass. After that, the students drew a line *f* (using GeoGebra) that matched one side of the glass, as shown in the Figure 5. They also marked two points, C at the bottom and D at the top of the usable space of the glass (the line *f* is determined by these points). Then the glass was observed as a solid of revolution generated by the rotation of the line *f* around the *x*-axis and bounded by two parallel planes perpendicular to the *x*-axis. After that, the students calculated the volume of the observed solid figure in *GeoGebra* using a definite integral and converted the obtained volume to cubic centimetres. They obtained that the volume of the usable space of the glass is $311.97\ cm^3$.

At the end, the students were given a similar task, with only one requirement: **Determine how much water can fit in the vase or in the other object you brought that can be considered a solid of revolution.**

In fact, in this case, the students first had to determine the volume of

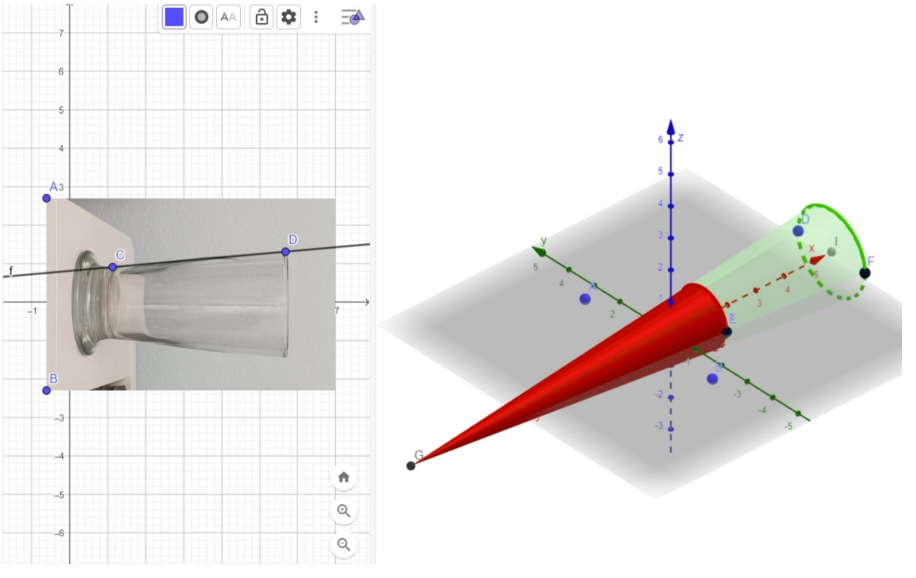

Figure 5. Determining the volume of a glass using *GeoGebra* (group G11)

the vase they brought. They did not have a mathematical model to calculate the volume of the vase, but they knew the general formula for determining the volume of a solid of revolution.

Let us consider the work of group G7. They had a small vase and determined its approximate volume as follows. Firstly, they poured water into the vase and then poured it into a measuring cup. In the students' opinion, it was approximately 130 cm^3. Then they needed to improve this result. Namely, all students had to apply the mathematical modelling process to determine the volume of the vase. They knew the formula for calculating the volume of a solid of revolution obtained by rotating a curve around the x-axis. Therefore, they had to determine such a curve. All the students had already been introduced to curve fitting, and all the groups applied this method.

All students took a picture of the vase and inserted it into *GeoGebra.* They placed it as shown, taking care that the x-axis was the axis of symmetry of the picture (Figure 6). Then they continued by calculating the volume.

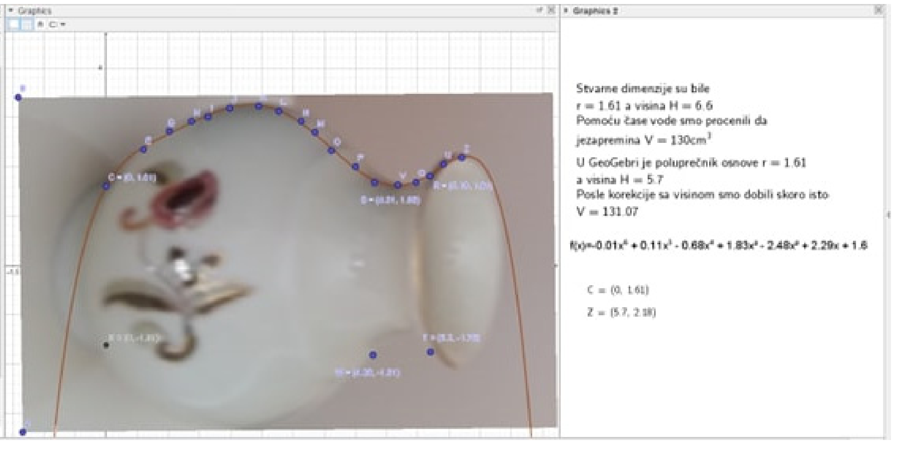

Figure 6. The students' work on calculating the volume of the vase (group G7)

Group G7 inserted the picture of the vase. They determined the curve f (a sixth-degree polynomial, see Figure 6) using the "FitPoly" command. Applying the formula for the volume (using a definite integral) and adjusting the dimensions, they obtained $V = 131.07\text{cm}^3$. Interestingly, this group used the *GeoGebra* environment as a whiteboard for explaining the obtained results.

After that, the students from small group G7 tried to determine different curves. First, they used the command "FitSin". Then, they took more points and combined the function they had used, "FitSin", up to point S, with "FitPoly" between points S and Z. They obtained more or less similar results, but they showed that they are very familiar with *GeoGebra* and curve fitting.

However, during the discussion, some students remarked that the angle of shooting was not appropriate and the photograph of the vase was slanted. These students assumed that there could be deviations in the calculated volume and that it was necessary to take the picture at the correct angle.

For example, the students from small group G12 took a photo of their vase, trying to have the correct shooting angle. They also used more points when fitting and obtained the appropriate polynomial function as the fitting curve (Figure 7). First, they measured (by pouring water into the vase and then into a measuring cup) that the volume of the vase was $460\ ml$, i.e. $460\ cm^3$. After that, they took a picture of the vase, inserted it into *GeoGebra* and followed the previously described process of fitting a curve and calculating the volume of the vase. Unlike the previously observed group, the G12 students did not set the real dimensions of the vase in *GeoGebra*, so they had to recalculate the volume using the proportions. Finally, they obtained that the volume of the vase was $469.21\ cm^3$.

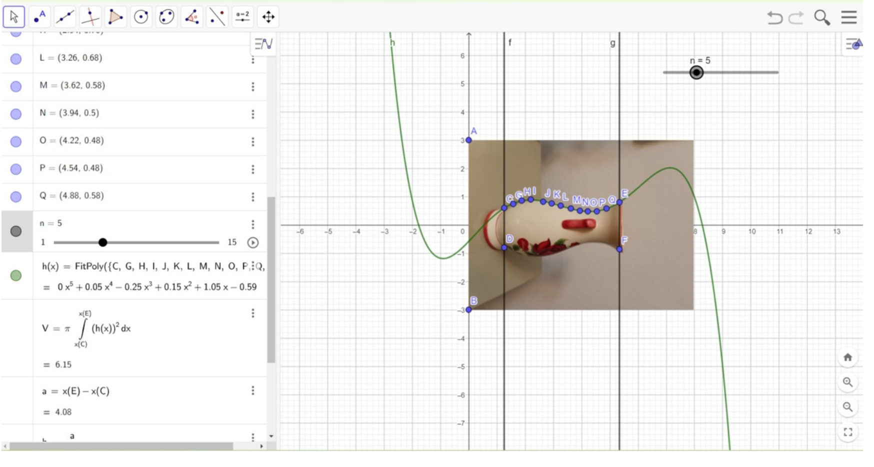

Figure 7. The students' work (group G12)

6. Statistical analysis of test results

Two weeks after the collaborative learning of the application of the definite integral, the students solved a test. The test was administered as a written task with five questions related to integrals. In particular, two questions required the application of the definite integral.

The maximum number of points on the test was 100 (20 points for each task). The average number of points scored by the students in the experimental group was 64.11, and the average number of points scored by the control group was 50.29. In the control group, there were students who scored zero points, while students who achieved the maximum number of points were present in both groups.

Distributions of the points in the experimental and the control groups for the test are presented in graphs in Figure 8. The number of points (from 0 to 100, in intervals of 20) is shown on the x-axis, and the number of students who achieved the corresponding points is shown on the y-axis.

Looking at Figure 8, it can be observed that the red line (the

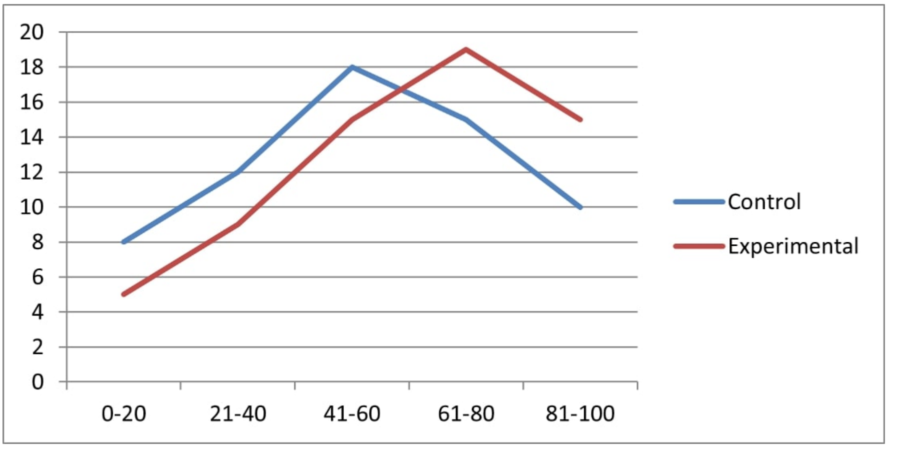

Figure 8. Distributions of the number of students according to the number of points (scored on the test)

experimental group) is below the blue line (the control group) for scores under 60 points, while for scores above 60, the blue line is below the red line. In the experimental group, six students achieved maximum points, whereas this was true for only two students in the control group. The statistical results obtained for the test are shown in Table 1.

It can be concluded that the difference between the test results of the experimental and the control groups is statistically significant at the 0.01 level ($t = 3.0537; p = 0.0028$).

It may also be noted that the effect size of the experimental factors is medium (Cohen's $d = 0.544$), meaning that the obtained difference provides a practical advantage for the experimental group compared to the control group. In fact, we proved that students' learning achievements in integrals are better when mathematical modelling is applied in a dynamic software environment.

Table 1. Statistical results of the test

Group	**Number of students**	**Arithmetic mean**	**Standard deviation**	**Test of difference between arithmetic means**		**Effect size**
	n	*M*	*SD*	*t*	*p* (2-tailed)	Cohen's *d*
Experi-mental	63	50.286	24.904	3.0537	0.0028	0.544
Control	63	64.111	25.906			

Special attention is paid to students' results on the third task, where the students were required to determine the volume of the solid generated by the rotation of a function's graph. The points achieved by the students in this task were important to analyze, because during their learning process, based on mathematical modelling, they solved problems on the volume of solids.

In the third task, as well as in the others, the maximum number of points was 20. The average number of points scored by the students in the experimental group was 12.968 (64.84% of the maximum points for this task), and the average number of points scored by students in the control group was 10.095 (50.48% of the maximum points for this task). The statistical results obtained for the third task are shown in Table 2.

It can be concluded that the difference between the results of the third task for the experimental and control groups is statistically significant at the 0.01 level ($t = 3.1577; p = 0.002$). It may also be noted that the effect size of the experimental factors is medium (Cohen's $d = 0.563$), meaning that the obtained difference provides a practical advantage for the experimental group over the control group in determining the volume of solids.

Table 2. Statistical results of the third task

Group	Number of students	Arithmetic mean	Standard deviation	Test of difference between arithmetic means		Effect size
	n	*M*	*SD*	*t*	*p* (2-tailed)	Cohen's *d*
Experi-mental	63	10.095	5.029	3.1577	0.002	0.563
Control	63	12.968	5.183			

7. Discussion and conclusions

The objective of this research was to analyze the impact of mathematical modelling in the *GeoGebra* collaborative learning environment on the students' achievements in learning the application of the definite integral. In particular, the research questions RQ1 and RQ2 concerned whether and how the application of mathematical modelling in a computer-supported collaborative learning (CSCL) environment contributes to better students' achievements in learning the application of the definite integral. The CSCL process in the experimental and the control groups was observed. In the experimental group, mathematical modelling was applied, while it was not applied in the control group.

By analyzing the test results, it can be concluded that the students from both groups had similar difficulties in solving the tasks. These difficulties, which have also been analyzed in earlier research (Mahir, 2009; Milenković et al., 2020; Orton, 1983; Tall, 2003; Soylu & Tatar, 2007), are reflected in the incorrect determination of the appropriate function and the limits of integration, as well as in errors made during the calculation process. The results of this research showed that the students in the experimental group were more successful in overcoming the noted difficulties, which can be attributed to the application of mathematical modelling during their learning process.

Practice, as well as previous research, showed that most students, when solving the tests, usually avoid tasks where the application of the definite integral in real-world problems is required, because they consider these tasks as too complicated (Orton,1983; Soylu & Tatar, 2007). Also, the students consider determining the volume of solids of revolution using definite integral very challenging (Dorko, 2012; Orton, 1983). There are indications that mathematical modelling, applied in the experimental group learning process, contributed to the increased student interest in solving real problem situations.

Statistical analysis of the test results has shown that the difference between the experimental and the control groups is statistically significant at the 0.01 level ($t(126) = 3.0537$; $p = 0.0028$). The effect size is medium (Cohen's $d = 0{,}544$), which means that the obtained difference provides an advantage for the experimental group over the control group (Table 1).

Based on these results, it can be concluded that the mathematical modelling process, supported by dynamic software within a collaborative learning environment, contributes to better achievement of students in the experimental group on the test, which represents a positive answer to the first research question.

The application of mathematical modelling, supported by dynamic software, in the teaching and learning process enabled the students to connect real-life situations with abstract mathematical concepts, such as the volume of solids of revolution and the definite integral. This result is in accordance with previous research (Kaiser, 2020; Niss & Blum, 2020; Maaß, 2006), which proved that the application of mathematical modelling in teaching contributes to better handling of different problem situations by students.

The results of the conducted research have shown that mathematical modelling increases students' interest in solving real problems. Namely, the students in the experimental group analyzed the problems from different points of view. They tested, changed, and improved their mathematical models. During the process of mathematical modelling, the students discussed their ideas within their collaborative groups, but also with the teacher, which is very important, because in that way they are encouraged to express their views and opinions. All of the above represents an answer to the second research question. These results are in accordance with the conclusions of (Kaiser, 2020), (Niss & Blum, 2020) and (Maaß, 2006).

There are numerous advantages of the approach applied in this research, but it would be important to examine which other factors could improve the teaching and learning of the definite integral and its applications. Certain limitations of the conducted research must also be considered. One of these limitations is a small sample – some of the future research in this field should include larger sample and, if possible, different high schools and vocational schools. Also, there are some limitations in the application of this approach. It should consider the equipment of the school, teachers' and students' digital competencies, as well as the curricula. In this research, the students could use their smartphones, which made the modelling process somewhat slower. Computers are more appropriate for this approach, primarily because of the screen size.

Finally, it can be concluded that mathematical modelling within a computer-supported collaborative environment improves the teaching and learning the application of definite integrals.

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