Natural Sciences and Advanced Technology Education | *Volume 31 Number 3, 2022* | *Обучение по природни науки и върхови технологии*

https://doi.org/10.53656/nat2022-3.06

Experiments

IS IT SIMPLE TO EXPLAIN THE SIMPLE EXPERIMENTS? HOW DO SOLID BODIES FLOAT?

Dragia Ivanov, Stefan Nikolov
Plovdiv University "P. Hilendarski" (Bulgaria)

Abstract. This paper considers the floating of solid homogenous bodies with different simple shapes. The stable floating positions attained by the bodies are examined and some qualitative rules are derived to determine those positions. The sensitivity of those stable positions to the exact proportions of the bodies is shown.
Keywords: floating; hydrostatics; educational experiments

I. Introduction

The problems concerning the flotation of solid bodies have been considered since antiquity, mostly with regard to ships. Despite this, there still are interesting things to explore in this field (Bogner & Rüde 2013; Erdös, Schibler & Herndon 1992a; Erdös, Schibler & Herndon 1992b; Lauturp 2011; Wegner 2003), some of which are subject to exploration in this paper.

A stationary solid body submerged in water is acted upon by two forces – a gravitational force $\vec{G}$ and a buoyancy force $\overrightarrow{F_B}$. The gravitational force is a volume force, acting upon each little part of the body. The effective point of application (center of gravity – CG) of this force is fixed in a solid body and doesn't change irrespective of the position and orientation of the body in the liquid, its participation in any motion, whether it is homogenous or not. Buoyancy force is an electromagnetic surface force. It only acts upon the points of the body in contact with water. The net buoyancy force is a sum of the elementary force of normal reaction of the liquid acting upon the body. Every one of the elementary forces is determined by Pascal's law of hydrostatic pressure. For a static body in equilibrium the net buoyancy force is constant and does not depend on the orientation of the body in the liquid. It is also equal to the gravitational force ($F_B = G$). This is in fact the requirement for the flotation of stationary bodies.

The question about the effective point of application of the buoyancy force (center of buoyancy CB) turns out to be very important in practice. For a homogenous ($\rho = const$) static body the CG is always above the CB. The two centers are positioned on the same vertical line, the two forces compensate each other and the floating body is in equilibrium (Fig. 1).

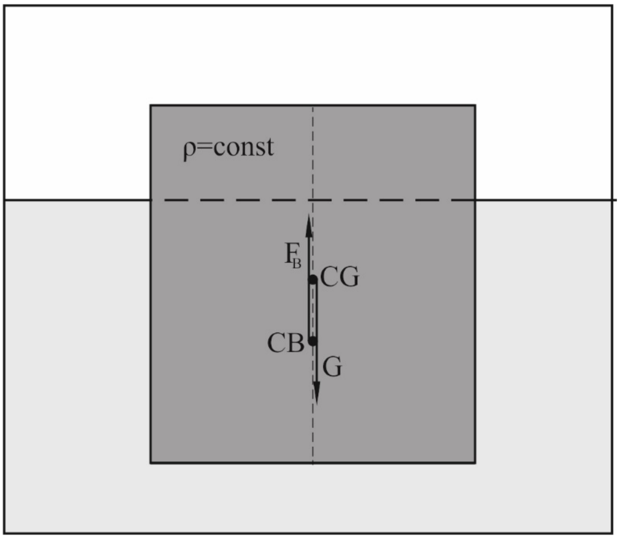

Figure 1. A homogenous floating body

If the body is not homogenous ($\rho \neq const$) it can happen that the CG is actually below the CB (Fig. 2). Like in the previous case, if the two forces act along the same vertical line the body will be in equilibrium.

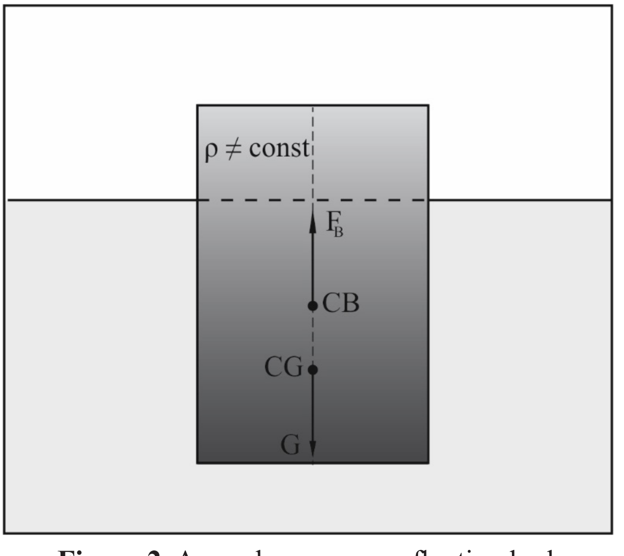

Figure 2. A non-homogenous floating body

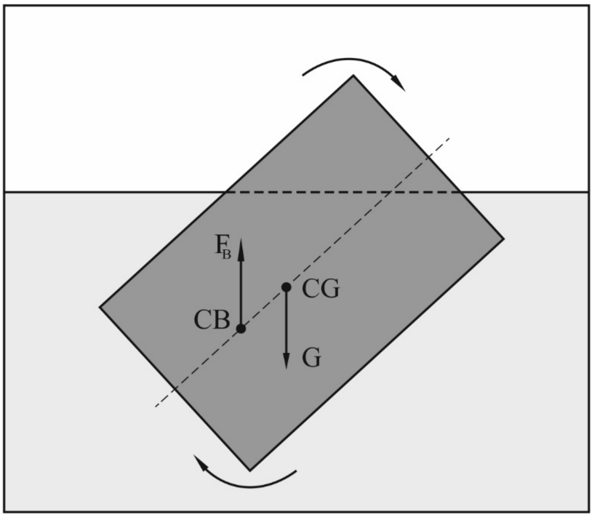

Figure 3. A floating body out of equilibrium

It is possible for a body to fulfill the flotation requirement but not to be in mechanical equilibrium. If the effective points of application of the two forces are not on the same vertical line the sum of their torques is not zero (Fig. 3). The body will rotate within the liquid until the two forces act along the same vertical line. Therefore, a more complete flotation requirement should also include a requirement for zero net torque.

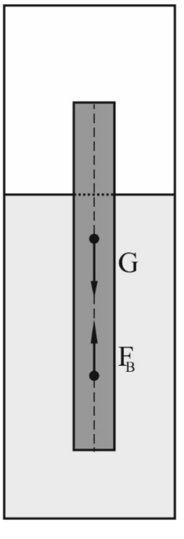

Figure 4.1. A long thin body in water is unstable when vertical

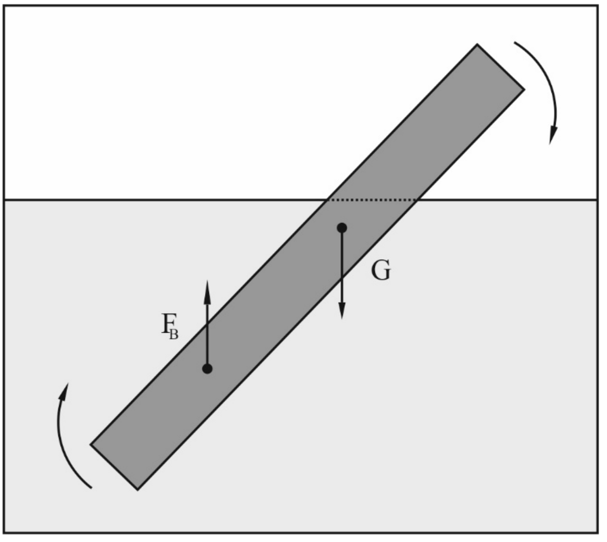

Figure 4.2. Once tilted, the long thin body rotates under the torque of the two forces

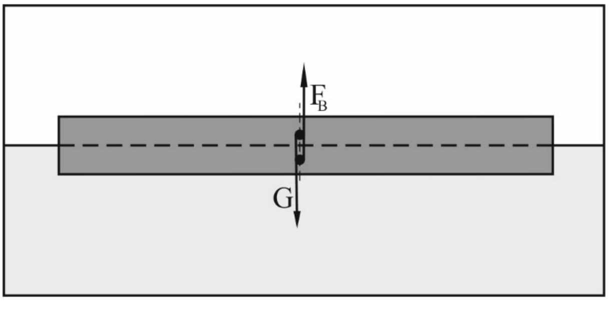

Figure 4.3. The long thin body is stable in a horizontal position

It is well known from mechanics that a body placed on a flat surface is more stable when its CG is lower and/or its supporting contact surface is larger. A similar situation happens when the body is immersed in a liquid. Let us, for example, immerse a relatively thin, long cylinder (a pencil) vertically in water (Fig. 4.1). The body quickly starts rotating under the action of the two forces. (Fig. 4.2) and transitions to a stable horizontal position (Fig. 4.3). It is obvious without the need for calculations that in the initial position the CG and CB are far apart and the “supporting” surface is very small. In the final position the distance between CG and CB is very small and the “supporting” surface is much larger. In both the initial and final state, the forces act along the same vertical line.

We can define the main rules for stable flotation of a solid body in a liquid:

1. The potential energy of the body-liquid system should be minimal
2. The center of gravity and center of buoyancy should be on the same vertical line
3. The distance between the two centers should be minimal
4. The maximal cross-section of the submerged part of the body should be horizontal

From the point of view of pure physics requirement 1 is sufficient to describe stable equilibrium but it is very difficult to apply in practice. The other rules may not always be true, especially for complex, non-homogenous bodies (like ships and boats) but for the simple homogenous objects we explore bellow they effectively equate to rule 1 while being easier to apply purely geometrically.

II. Rectangular prism

Let us consider a homogenous wooden rectangular prism with density $\rho_B \approx 0.5\ g/cm^3$ and sizes *a*=60mm, *b*=40mm, *c*=15mm (Fig. 5). We immerse in a vessel with water in an arbitrary orientation. Upon the action of the two forces the prism quickly rotates itself into its stable position. In this position the two largest faces of the prism are horizontal. (Fig. 6). The prism has two equivalent stable positions with either one of the two largest being the "bottom" while the other is the "top". Which one of the two stable positions is reached depends on the initial position of the prism and any initial motion the may have been imparted on it during the immersion.

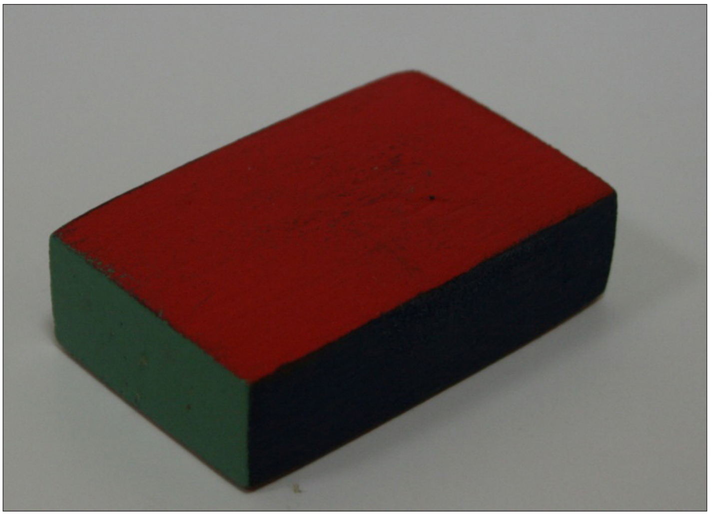

Figure 5. The rectangular prism

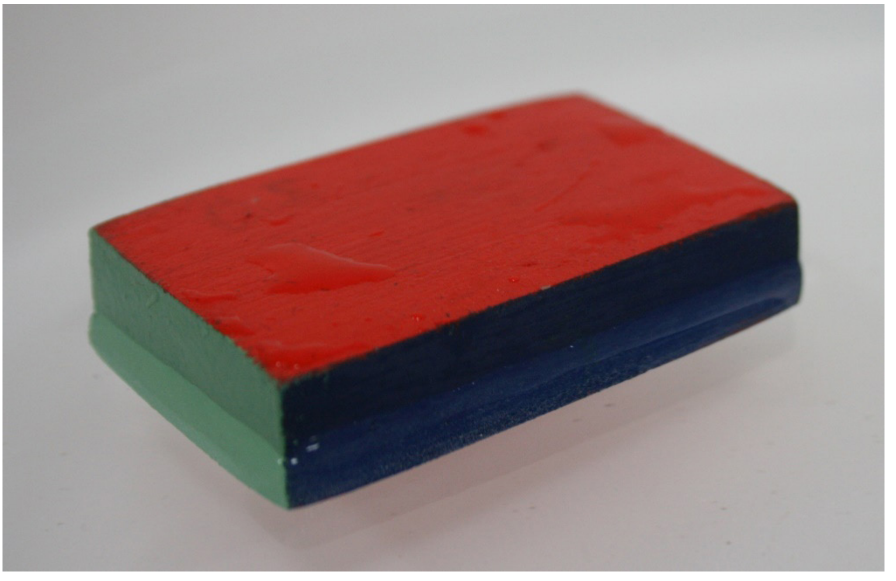

Figure 6. The rectangular floating in one of its stable positions

Let us determine the position of the CG and CB for three possible positions of the prism with a different pair of side horizontal (Fig. 7) starting with the one that the prism normally takes (Fig. 8). The CG is in the geometric center O of the figure, a distance $c/2$ from the base. This is the effective point of application of the gravitational force which has a magnitude $G = mg = V_B \rho_B g = abc\rho_B g$. The buoyancy force is applied in point A, in the geometric center of the submerged part of the body, which is a similar prism to the whole body but with the side c replaced with h. The buoyancy force is $F_B = V_W \rho_W g = hac\rho_W g$. In equilibrium $F_B = G$ which yields $h = \frac{\rho_B}{\rho_W} c$. The distance between the CG and CB is $x_1 = \frac{c}{2} - \frac{h}{2} = \frac{c}{2} - \frac{\rho_B}{\rho_W} . \frac{c}{2} = \frac{c}{2}\left(1 - \frac{\rho_B}{\rho_W}\right)$. Analogously we find the distances between the CG and CB for the other two cases. Ultimately

$$x_1 = \frac{c}{2}\left(1 - \frac{\rho_B}{\rho_W}\right),\ x_2 = \frac{b}{2}\left(1 - \frac{\rho_B}{\rho_W}\right) \text{ and } x_3 = \frac{a}{2}\left(1 - \frac{\rho_B}{\rho_W}\right), \text{ i.e. } x_1 < x_2 < x_3 .$$

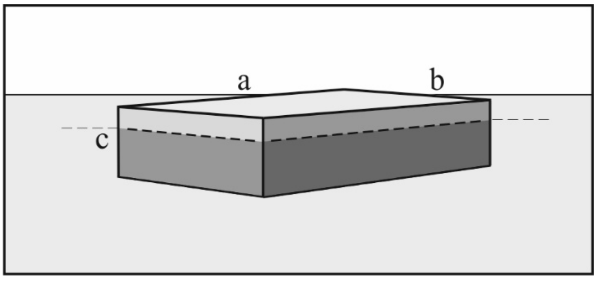

Figure 7.1. The prism in one possible position – large side up

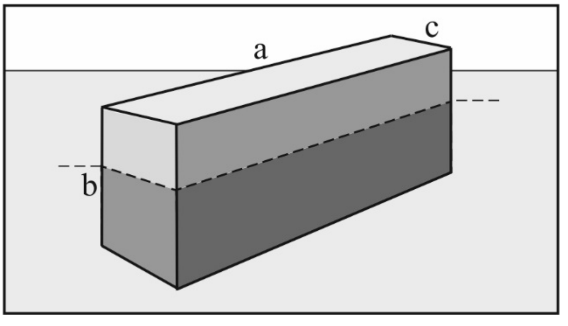

Figure 7.2. The prism in a possible position long thin side up

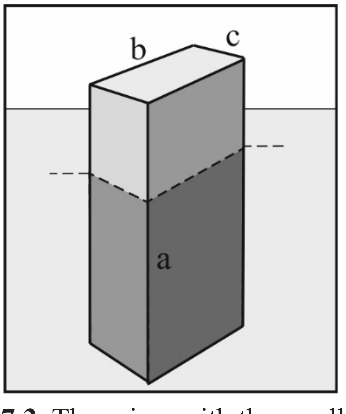

Figure 7.3. The prism with the smallest side up

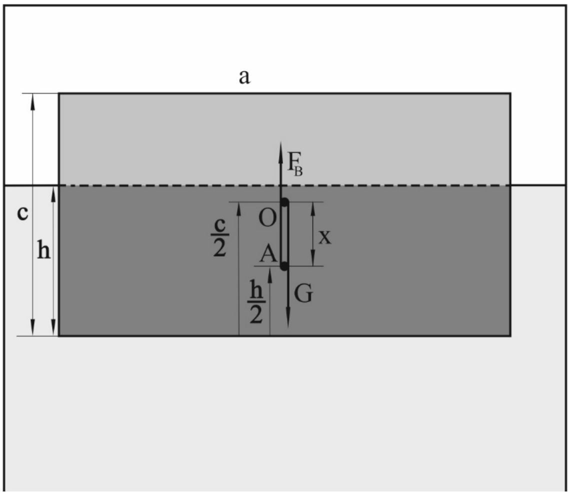

Figure 8. The floating prism in detail

The shortest distance between the CG and CB is attained in the case when the body is stable in the water. The respective horizontal cross-sections of the body have areas $S_1 > S_2 > S_3$.

We can make the following conclusions:

1. In the stable position of the body the gravity force and the buoyancy force are along the same vertical line with the point of application of the buoyancy force below the point of application of the gravity force.
2. The gravity force and the buoyancy force are equal in magnitude and act in opposite directions.
3. In the stable position of the body, the distance between the CB and CG is smaller than in all the unstable positions. Because of this the body-water system has its lowest potential energy in the stable position.
4. The largest side of the body is submerged and horizontal.

These considerations are made for a distance *c* that is small (the prism is relatively thin). The *a-b* side is thus practically parallel to the water surface. If this condition is not observed, there are other stable positions possible that are not symmetric relative to the water surface and no sides of the prism are horizontal. The exact mathematical treatment of these cases turned out to be very complicated. It turned out, however, that the conclusions hold in many other cases when the immersed body is homogenous, less dense than the liquid and geometrically convex (no dimples). The general rules defined above can be used to relatively easily predict and explain the results in a number of different specific cases that we will outline bellow.

III. Examples

In this section we will present the experimental study of the static stable positions of different solid bodies with well-known basis shapes immersed in water. The bodies are made out of wood with a density less than water ($\rho_B < \rho_W$). We can assume to a good approximation that the bodies are homogenous. We only explored geometrically convex bodies for which the CG is above the CB. We will check if the rules for stable equilibrium positions are applicable in each case. All bodies were painted in contrasting colors on neighboring sides for improved visibility in the pictures. The paint also protects the wood from soaking up water, which was a problem in some of our initial experiments.

III.1. Cylinders

We studied cylinders with the same base (diameter 33 mm) but different heights ranging from 15 mm to 52 mm (Fig. 9). They're all made of the same wood with $\rho \approx 0.5\, g/cm^3$. It turned out that their floating behavior is very different based on their height (Fig. 10). The shortest cylinder floats with its base practically horizontal and axis vertical. The next one tilts slightly. The third cylinder has its axis at almost 45° with the fourth one being tilted even more. The tallest cylinder is floating practically on its side, with the axis essentially horizontal and the bases perpendicular to the water. The tilt of each of the cylinders is determined by the rules defined above. The main requirement is for the potential energy of the system to be minimal, but the exact position for which this is achieved is different depending on the specific height-to-diameter ratio. Playing around with each of the cylinders, trying to change its tilt it can be visually confirmed that in each case the stable position provides for the largest submerged horizontal cross-section.

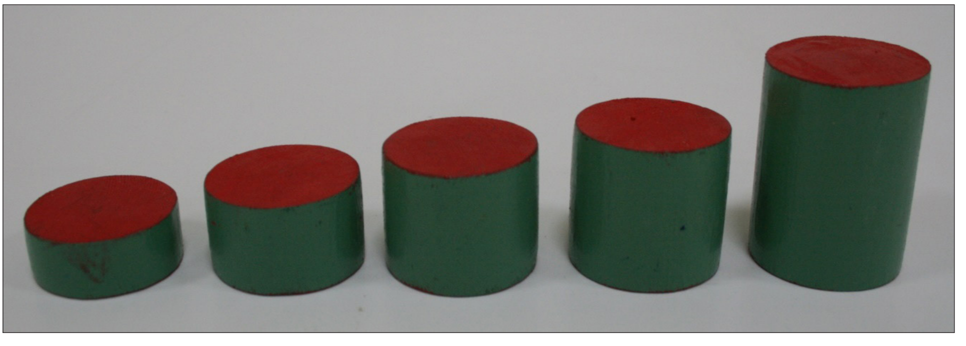

Figure 9. The cylinders used for the experiments

Figure 10.1. The stable floating position of the shortest cylinder

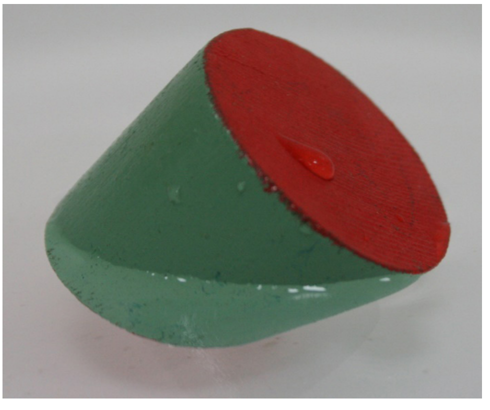

Figure 10.2. The stable floating position of the mid-height cylinder

Figure 10.3. The stable floating position of the tallest cylinder

III.2. Square prisms

A set of three homogenous wooden prisms with a density of $\rho \approx 0.5\, g/cm^3$, a square base with a 30 mm side and three different heights (15 mm, 30 mm, 50 mm). The middle on is actually a cube. The stable floating positions of each are shown on Fig. 12. The shortest one attains a position such that its square base is visually horizontal (Fig. 12.1). This prism has two distinct stable positions with each of the bases up. The second prism (cube) floats so that one of its vertices points up with the corresponding large diagonal close to vertical (Fig. 12.2). Three of the six sides of the cube show above the water symmetrically. The cube has eight stable positions with each of the vertices pointing up. The third, sufficiently tall prism attains a stable position such that one of its side/long edges points up, visually parallel to the water surface, and two of its larger side face above the water. This prism has four stable positions relative to each of the four side edges.

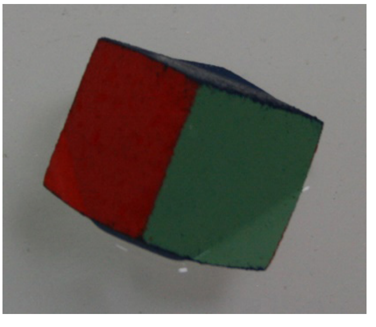

Figure 12.2. The stable floating position of the cube

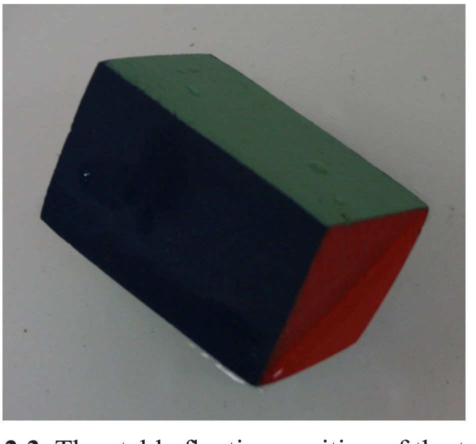

Figure 12.3. The stable floating position of the tall prism

While experimenting with other prisms in between these extreme cases of height-to-base ratios we also obtained stable floating positions that were different in-between versions of the three we presented above. The most interesting ones were those closest to the cube with a height-to-base ratio close to one. Those are also the hardest to analyze due to the rather complex geometrical situations that can arise.

III.3. Cones

We carried out the immersion experiments with a set of homogenous wooden cones (Fig. 13) with a density $\rho \approx 0.5\, g/cm^3$. They all have the same base with a diameter of 30 mm and progressively increasing heights of 25 mm, 33 mm and 48 mm. Upon immersion (Fig. 14) the shortest cone assumes a stable position with its axis visibly vertical, the tallest settles with its axis approximately horizontal and the middle-height cone has a stable position with its axis at an in-between angle. The short one could be put in a stable position both point-up and point-down but the point-down position was relatively easy to turn over with a little outside force while the point-up was much more stable in this sense. The in-between tilted position of the mid-height cone is with the point up, too, which also indicates that the point-up position is the more stable one. The exact angle of tilt for the mid-height cone depends on the precise height-to-diameter ratio.

Figure 13. The cones used for the experiments

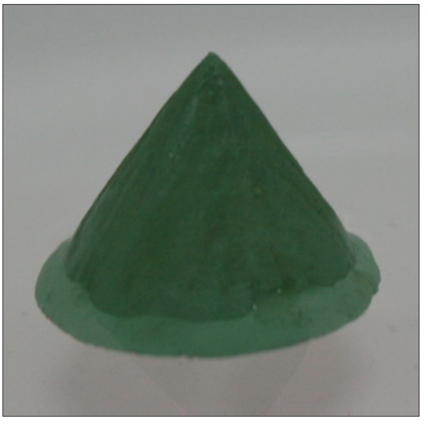

Figure 14.1. The point-up stable floating position of the short cone

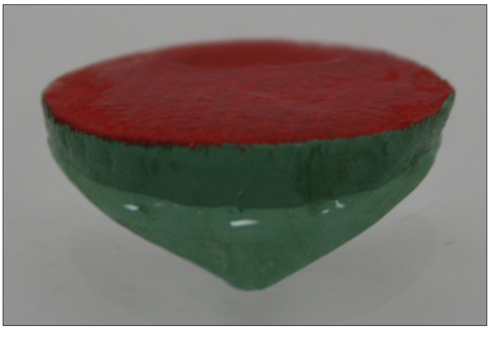

Figure 14.2. The point-down stable floating position of the short cone

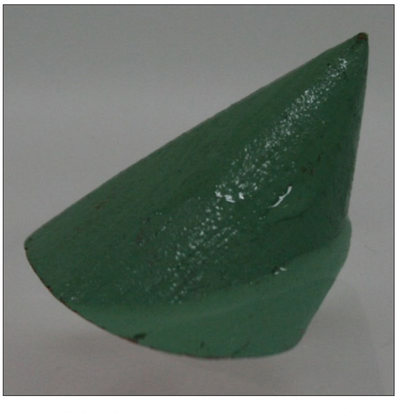

Figure 14.3. The stable floating position of the mid-height cone

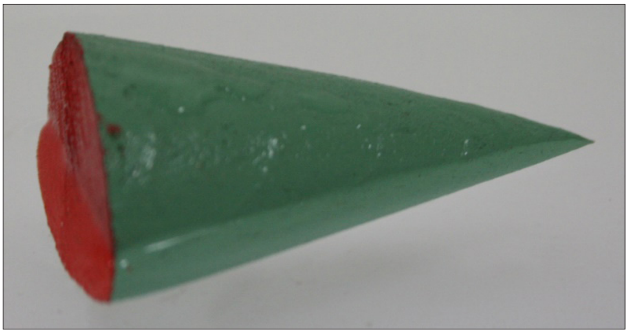

Figure 14.4. The stable floating position of the tall cone

III.4. Square pyramids

We carried out floating experiments with a set of wooden ($\rho \approx 0.5\, g/cm^3$) homogenous pyramids with a square base with a 35 mm side. The heights of the pyramids are 30 mm, 40 mm and 70 mm (Fig. 15). Upon immersion in water the pyramids attain one of their stable equilibrium positions (Fig. 16). The shortest pyramid is visibly upright, with its axis apparently vertical. Like with cones, a point-up and a point-down position are possible. The mid-high pyramid obtains a tilt in its axis and one of its side edges gets on top. Four such positions are possible, with each of the edges on top. The highest pyramid positions itself with its axis approximately horizontal and one of the edges on top for a set of four possible stable equilibrium states.

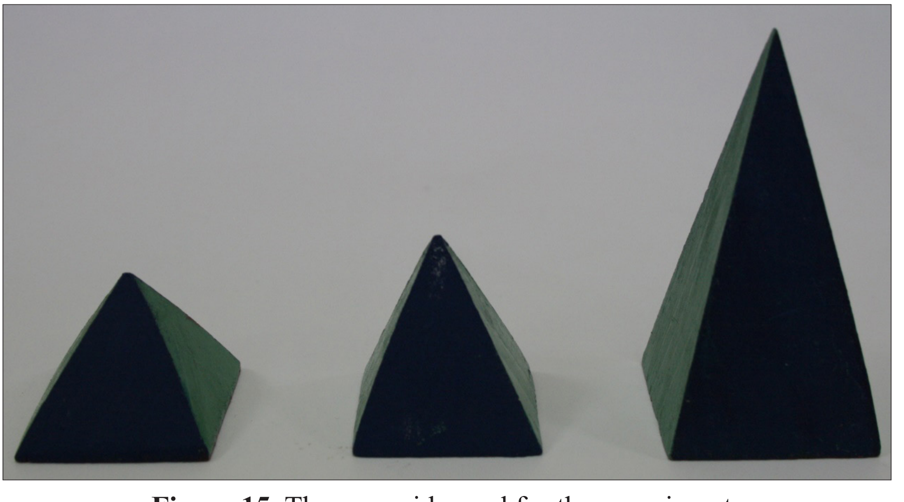

Figure 15. The pyramids used for the experiments

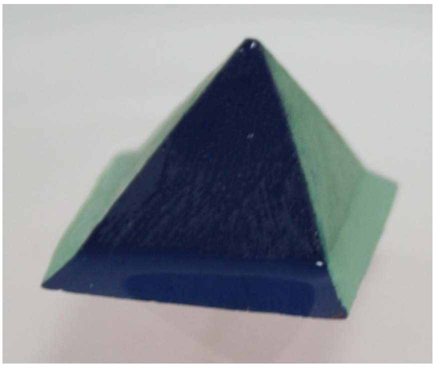

Figure 16.1. The point-up stable floating position of the short pyramid

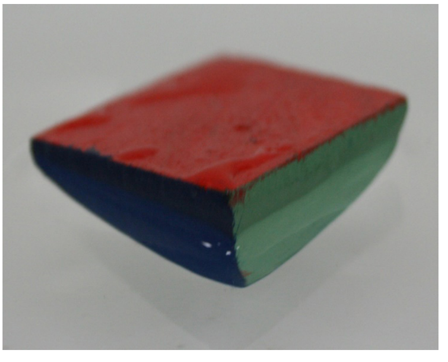

Figure 16.2. The point-down stable floating position of the short pyramid

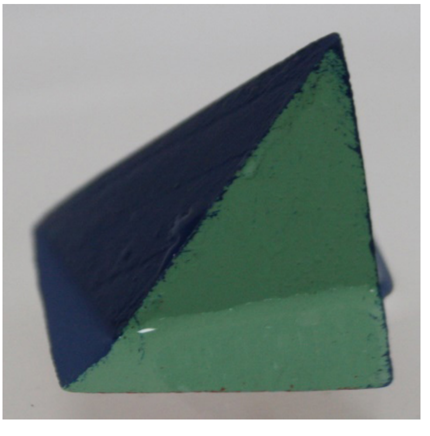

Figure 16.3. The stable floating position of the mid-height pyramid

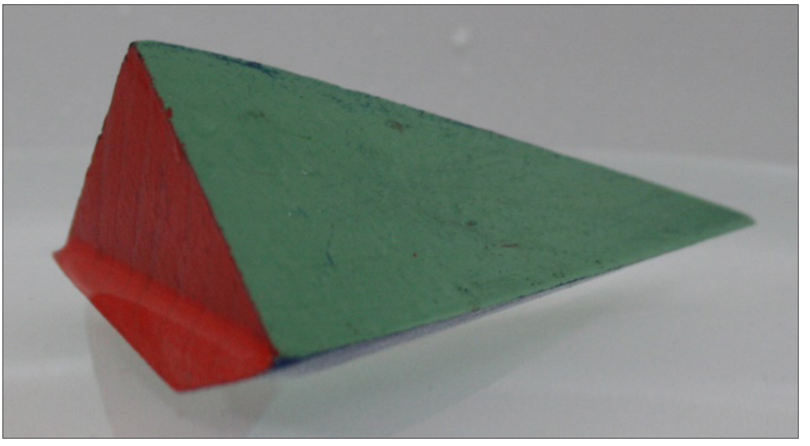

Figure 16.4. The stable floating position of the tall pyramid

III.5. Spherical caps

Figure 17 depicts a body formed by a sphere cut through with a plane (a spherical cap). No matter what part of the sphere the cap constitutes, upon immersion it always orients itself such that the flat part is parallel to the water surface. In this case there is an interesting peculiarity. Depending on the initial position of the body in the water it can orient itself flat side up or flat side down, i.e. it has two stable positions. The experiments showed that when we immerse the body at an angle to the water the critical angle α_c that separates these two states is more than 90° (Fig. 18.1). When the angle between the flat part of the body and the water is in the interval $0° < \alpha < \alpha_c$ the body tends toward the stable position, with the spherical part above water and the flat part below (Fig. 18.2, Fig. 19.1). When the angle is $\alpha_c < \alpha < 180°$ (Fig. 18.3) the spherical cap moves towards the stable position with the flat side on top (Fig. 18.4, Fig. 19.2). If we drop the spherical cap randomly in the water, the first position (flat side down) is more frequently obtained, it is in a way more stable. Both positions represent local minimums of potential energy of the body-water system, but the flat-side-down is the global minimum. Transition between the two positions requires an outside action since the intermediate positions are energetically unfavorable. An analogous behavior with two stable positions that are not equally stable was observed for the short cone and pyramid discussed above. It can also be obtained with a piece of cylinder cut along a plane parallel to its axis (Fig. 20).

Figure 17. The spherical cap

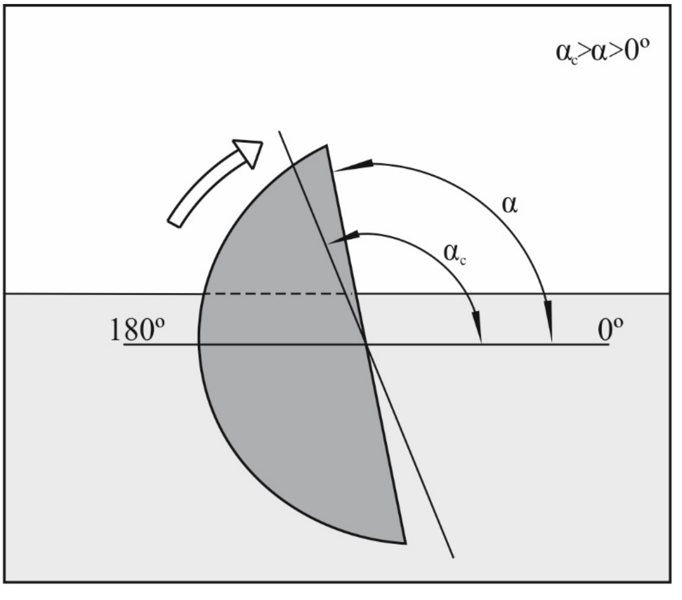

Figure 18.1. The cap at less than the critical angle

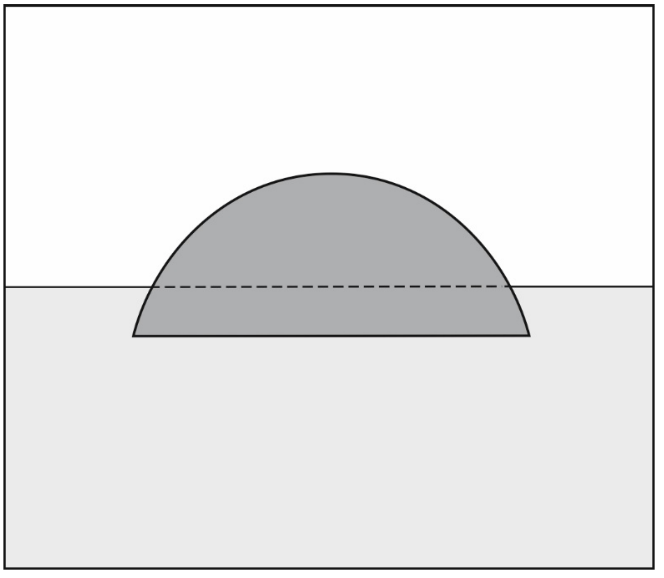

Figure 18.2. The flat-down position attained by the cap if at less than the critical angle initially

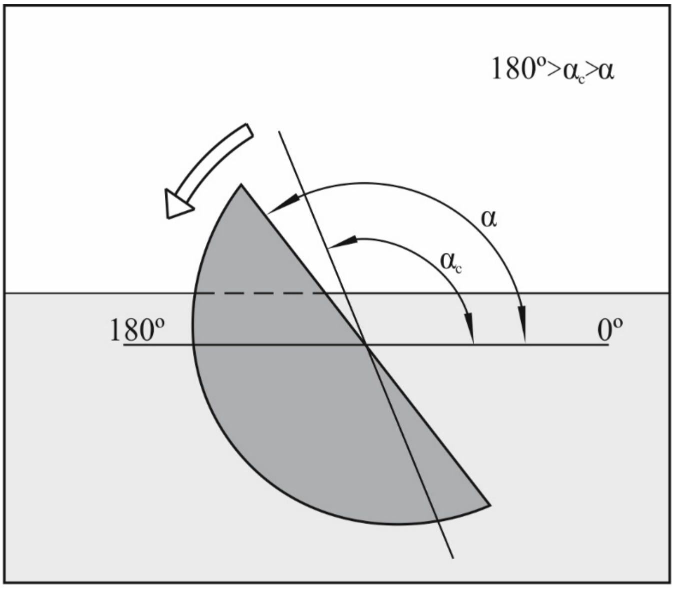

Figure 18.3. The cap at more than the critical angle

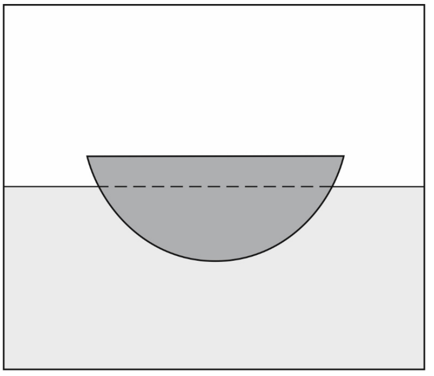

Figure 18.4. The flat-up position attained by the cap if at more than the critical angle initially

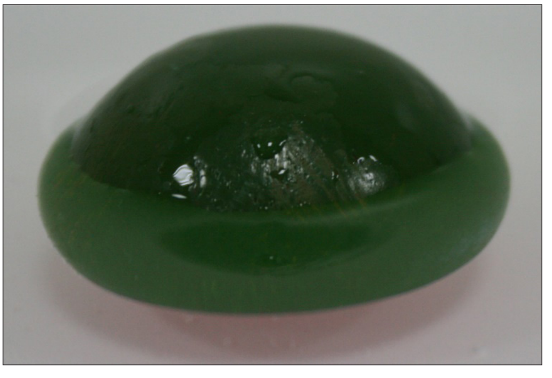

Figure 19.1. The cap in the flat-down stable floating position

Figure 19.2. The cap in the flat-up stable floating position

Figure 20.1. A half-cylinder in a stable floating position with the flat down

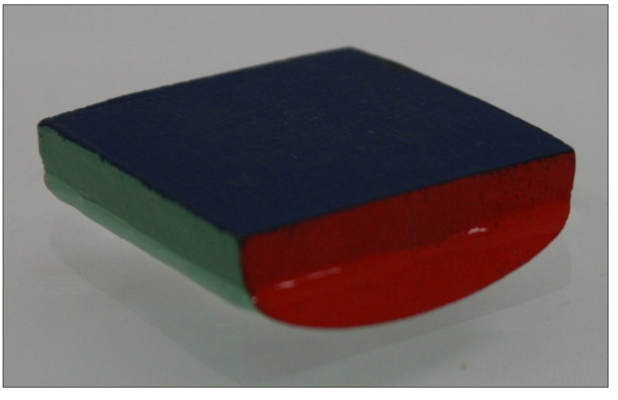

Figure 20.2. A half-cylinder in a stable floating position with the flat up

IV. Floating of light bodies

In the previous section we performed the experiments with bodies of relatively large density (still less than the water) so that a significant part of the body was under the water and the CG was at the water surface or below. It is possible to have many situations when the average density of the body is significantly lower than that of the liquid. Such bodies sink very little in the water and essentially float on top of it (Fig. 21). In those cases, the stability of the body can be more easily analyzed by treating it as if the body is just lying on a solid surface. Such experiments can be performed with bodies made out of expanded polystyrene, cork or other similar low-density materials.

V. Floating of non-homogenous bodies

In most practical situations the floating bodies are not solid homogenous objects. In those cases, it is possible for the CG to be both below and above the CB. This can be examined qualitatively with some simple experiments.

On a long thin stick (a pencil works well) we wound a piece of metal wire. Solder wire is very suitable as it is soft and can be wound tightly and won't spring back and loosen. At first, we position the metal winding close to the end of the pencil. It positions itself vertically (Fig. 22.1). Obviously, the CG of the overall body is below the CB. If the pencil is perturbed from the equilibrium position it quickly moves back as the torque by the pair of forces acting on the body is quite large.

We then move the metal winding up the pencil, closer to the middle. The stable position is tilted relative to the water surface (Fig. 22.2). The distance between the CG and CB is smaller. When the pencil is perturbed from the equilibrium it moves back more slowly, as the torque created by the two forces is less.

When we move the metal winding in the middle of the pencil, its equilibrium position is horizontal (Fig. 22.3). In this case the CG is actually above the CB and very close to it. In this case the pencil is slowest to return to equilibrium if it is perturbed.

Figure 21. A lightweight body made of expanded polystyrene floating as if on a solid surface

Figure 22.1. A pencil with a weight at the end floats vertically

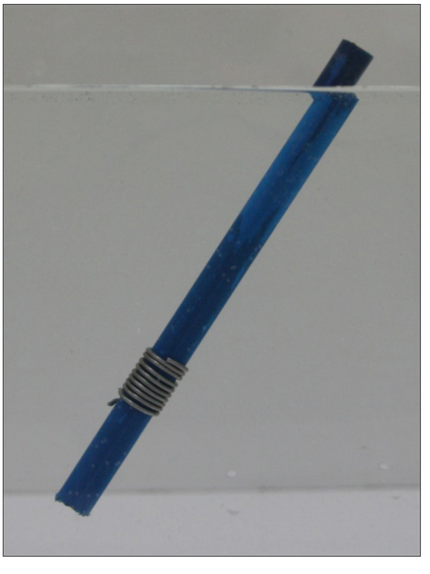

Figure 22.2. A pencil with the weight closer to the center can float stably at an angle

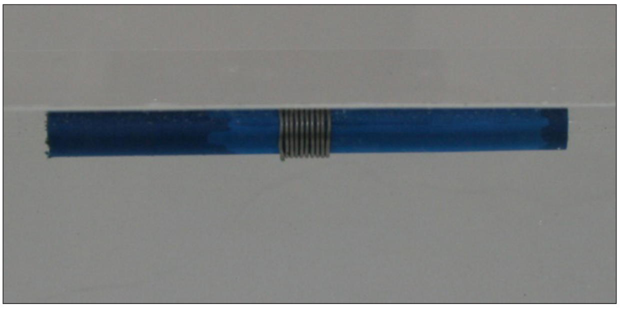

Figure 22.3. A pencil with the weight at about the middle floats horizontally

This experiment shows that a floating body is more stable when its CG is lower. This is very important for ship design and ship sailing. In fact, some cargo ships are dangerous to sail when empty because cargo normally brings the CG down and without it the CG move too high, making the ship very unstable. Some modern sailing boats have keels that protrude below the main hull for this exact purpose – to bring the CG lower. Some of them even have lead weights at the end of the keel. The keel also provides some hydrodynamic effects as the boat moves through the water but this is beyond the scope of this paper.

VI. Conclusions

The results of the numerous experiments performed confirm the stability rules formulated in the beginning of the paper. These rules can be used to predict some of the behavior of solid bodies floating in water and may even be used to perform some exact calculations for specific cases. The experiments can be used in an educational setting to start off the exploration of the topic of floating.

REFERENCES

BOGNER, S. & RÜDE, U., 2013. Simulation of floating bodies with the lattice Boltzmann method, *Computers & Mathematics with Applications*. **65** (6), 901 – 913, https://doi.org/10.1016/j.camwa.2012.09.012.

ERDÖS, P., SCHIBLER, G., & HERNDON, R. C., 1992a. Floating equilibrium of symmetrical objects and the breaking of symmetry. Part 1: Prisms, *American Journal of Physics*. **60**, 335 – 345. https://doi.org/10.1119/1.16877.

ERDÖS, P., SCHIBLER, G., & HERNDON, R. C., 1992b. Floating equilibrium of symmetrical objects and the breaking of symmetry. Part 2: The cube, the octahedron, and the tetrahedron, *American Journal of Physics*. **60**, 345 – 356, https://doi.org/10.1119/1.17130.

LAUTRUP, B., 2011. *Physics of Continuous Matter: Exotic and Everyday Phenomena in the Macroscopic World* (2nd ed.), Boca Raton: CRC Press. 41 – 55, https://doi.org/10.1201/9781439894200.

WEGNER, F., 2003. Floating Bodies of Equilibrium, *Studies of Applied mathematics*. **111**, 167 – 183, https://doi.org/10.1111/1467-9590.t01-1-00231.

✉ **Prof. Dr. Dragia Ivanov**
ORCID iD: 0000-0003-1721-1376
Faculty of Physics and Technology
Plovdiv University "Paisii Hilendarski"
24, Tsar Asen St.
4000 Plovdiv, Bulgaria
E-mail: draiva@uni-plovdiv.bg

✉ **Dr. Stefan Nikolov**
ORCID iD: 0000-0001-5498-9940
Faculty of Physics and Technology
Plovdiv University "Paisii Hilendarski"
24, Tsar Asen St.
4000 Plovdiv, Bulgaria
E-mail: stnikolov@uni-plovdiv.bg