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FINANCIAL PROJECTS AND LOANS WITH DIFFERENT, DECURSIVE ANNUITIES, INTEREST RATES AND CAPITALIZATION PERIODS WITH THE USE OF EXCEL

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Abstract. The authors of the paper have innovated and explored the theory of size relations of a financial project, the duration of its creation, the determination of its price, the duration of use and financing of the price of a project. The duration of use of a financial project has two planned time intervals. The first time interval is planned for repayment of the loan amount of the project price, and the second one is for acquisition of the planned profit. A new model of loan repayment is used – with different; decursive annuities, interest rates and periods of capitalization. The introduced theory was applied to calculations in the form of a financial project model using Excel.

Keywords: financial project; loan; annuities; interest rates; capitalization

Introduction

Projects with planned duration, price, price financing and duration of use are financial projects, projects in general. Project holders are mostly legal entities or individuals. Project funding can be self-financing, loan (credit), and a combination of both. Project management and administration is mainly performed by project management with associates.

The topic of the paper is innovation and research of the theory for the introduction of a model of a financial project, with loan financing of its price, which can be supported by spreadsheets (MS Excel). Among other things, the theory will be based on the application of complex, conformal capitalizations, by discounting and discounting the amounts of the present term to the amounts of earlier and later terms of the same time course.

1. Planned loan financing of the project price amount by tranche payments

Loan financing of the project price can be done by paying one tranche at the beginning of the project or by paying two or more tranches in the time interval of project development. The general theory will be introduced for *m* different tranche amounts with different payment terms, different interest rates and different lengths of capitalization period.

Sizes with introduced marks for the 1st capitalization period:
(1) Amount of the 1st tranche, TR_1
(2) Payment term of the 1st tranche, TE_0 in format dd.mm.yyyy hh: mm: ss
(3) Length of the 1st capitalization period, in days, d_1
(4) Length of the 1st capitalization period, in years, $g_1 = \frac{d_1}{365}$
(5) Amount of the annual interest rate, P_1
(6) Amount of the annual interest factor, $R_1 = 1 + P_1$

Sizes with introduced marks for the 2nd capitalization period:
(1) Amount of the 2nd tranche, TR_2
(2) Payment term of the 2nd tranche, $TE_2 = TE_0 + d_1$ in format dd.mm.yyyy hh: mm: ss
(3) Length of the 2nd capitalization period, in days, d_2
(4) Length of the 2nd capitalization period, in years, $g_2 = \frac{d_2}{365}$
(5) Amount of the annual interest rate, P_2
(6) Amount of the annual interest factor, $R_2 = 1 + P_2$.

Sizes with introduced marks for *m* capitalization period:
(1) Amount *m* of the tranche, TR_m, for $m \in \{3,4, ...,M\}$
(2) Term of payment *m* tranche, $TE_m=TE_{m-2}+d_{m-1}$
(3) Length *m* of the capitalization period, in days, d_m
(4) Length *m* of the capitalization period, in years, $g_m = \frac{d_m}{365}$
(5) Amount of the annual interest rate, P_m
(6) Amount of the annual interest factor, $R_m=1+P_m$
(7) Term end *m* of the capitalization period, $TE_m=TE_{m-1}+d_m$
(8) Amount of self-financing for term TE_m, *SF*

Input values for calculations are: tranche amounts, payment term of the 1st tranche (TE_0), length of capitalization period in days, annual interest rate amounts and self-financing amount (*SF*).

Some or all of the tranche amounts, capitalization periods and interest rates may have equal values.

Price of the project and length of its development:
(1) Date of completion of the project, $TE_m=TE_{m-1}+d_m$,
(2) Discounted amount of the first tranche ETR_1 for the date of completion of the project, $ETR_1 = TR_1.R_1^{g1}.R_2^{g2} ...R_m^{gm}$

(3) Discounted amount of the 2nd tranche ETR_2 for the date of completion of the project, $ETR_2 = TR_2.R_1^{g1}.R_2^{g2} ...R_m^{gm}$

(4) Discounted amount m of the tranche ETR_m for the project completion date, $ETR_m = TR_m.R_m^{gm}$,

(5) Amount of the project cost G_0; the sum of discounted tranche amounts, reduced by the amount of self-financing, $G_0=ETR_1+ETR_2+ \ldots +ETR_m - SF$

(6) Length of the project development period, in days, $Rd=d_1+d_2+\ldots +d_m$ or $Rd=TE_m - TE_0$

(7) Length of the project development period, in years, $Rg = \frac{Rd}{365}$

In addition to the price and length of the development period, the project has a planned duration, which begins with the deadline for the development of the project. The duration of the project has n internal time intervals for decursive presentation of the amount in Bosnia and Herzegovina convertible marks (KM) of the two sides of the business; DEBT (expenditures, expenses) and DEMAND (receipts, revenues). The amounts of the DEBT side Du are negative or zero ($Du \leq 0$), and the amounts of the DEMAND side Po are positive or zero ($Po \geq 0$), at the end of the time interval with the internal rate p. The sum of the amount DEBTED and the amount RECEIVED is BALANCE S, i.e. $S = Du + Po$, for the same term of the time interval of the internal rate p.

The amount of the balance S in relation to the amounts owed Du and claimed Po, has the following relations:

(1) $S > 0S > 0$ for $Po > |Du|Po > |Du|$,

(2) $S = 0S = 0$ for $Po = |Du|Po = |Du|$,

(3) $S < 0S < 0$ for $Po < |Du|Po < |Du|$.

In general, the discounted amount DS of the balance S in relation to the project price amount G_0 and its term, determines the absolute payback AO of the project price for the internal rate *p of the* time interval.

The relative payback RO of the project price amount is the quotient of the absolute payback amount and the project price amount $RO = \frac{AO}{G_0}$ or expressed as a percentage, $RO\% = \frac{AO}{G_0}.100\%$.

A series of balance amounts, $S_1, S_2, \ldots, S_n$ has a number of discounted amounts, $DS_1, DS_2, \ldots, DS_n$ for the start date of the project. There are a number of discounted amounts through partial sums DS_1; $DS_1 + DS_2$; …; $DS_1 + DS_2 + \ldots + DS_n$, each of which, starting from the first, can repay the amount of the project price G_0, if:

$$SUM_1 = DS_1 \geq G_0;\ SUM_2 = DS_1 + DS_2 \geq G_0;\ SUM_n = DS_1 + DS_2 + \ldots + DS_n \geq G_0.$$

By definition, the partial sums are absolute repayments, i.e.:

$SUM_1 = AO_1$, $SUM_2 = AO_2$, …, $SUM_n = AO_n$.

If within the planned duration, the project is not in operation for some reason, then all partial sums of the discounted balance amounts are equal to zero. In this case, it is necessary to calculate the increased project price amounts for the amounts of compound interest by capitalization periods. For negative balance amounts, the project beneficiary has losses with adequate, new loan amounts.

The best option is to repay the project price with the first partial amount $SUM_1 = DS_1 \geq G_0$), that is, the discounted amount of the first balance S_1.

The worse option is to repay the project price with the last partial amount, $SUM_n = DS_1 + DS_2 + \dots + DS_n \geq G_0$.

The last partial amount cannot repay the amount of the project price, if it is:

$DS_1 + DS_2 + \dots + DS_n \leq G_0$.

It is desirable that all amounts of the balance be positive, that is $S_1 > 0$, $S_2 > 0$, …, $S_n > 0$ and that their discounted amounts can repay the amount of the project price up to and including the time interval of the ordinal number k for $1 \leq k \leq n$. Some balance amounts may be equal to zero or negative amounts, with the proviso that all discounted balance amounts may repay the project cost. Therefore, a loan with k different, decursive annuities that are successively equal to the balance amounts has been contracted to repay the project price: $a_1 = S_1$, $a_2 = S_2$, … $a_{k-1} = S_{k-1}$, $a_k = S \leq S_k$.

The last amount of the annuity a_k is to cover the last balance of the debt. At the end of the time interval of the ordinal number k, the project price was repaid, and the rest of the balance $S_k - a_k = P_1 \geq 0$ was the first amount of profit $P_1 \geq 0$. The following balance amounts, S_{k+1}, S_{k+2}, …, S_{n-k}, get the role of the amount of profit, $P_2 = S_{k+1}$, $P_3 = S_{k+2}$, …, $P_{k+1} = S_{n-k}$. Like balance amounts, profit amounts can be positive (greater than zero), negative (less than zero), and can have an amount equal to zero.

In general, in the loan repayment calculations, the amount of the 1st profit P_1 presents the financial condition of the project beneficiary after the loan repayment, based on:

(1) For $P_1 > 0$, it follows that the loan has been repaid and the amount of the new balance is profit;

(2) For $P_1 = 0$, it follows that the loan has been repaid and the amount of the new balance is profit;

(3) For $P_1 < 0$, it follows that the loan has been repaid and the amount of the new balance is profit or the amount of new debt, $G_{0_0} = |P_1|$.

The amount of the new debt will be paid by the project beneficiary at the time of its arrival or will contract a new loan with the same or another creditor.

The following theory is based on the planned equalization of internal rates of return for balance amounts, with interest rates for annuity amounts, for periods of capitalization of ordinal numbers from 1 to k.

2. Sizes and their relations for project cost repayment periods

During the duration of the project, the repayment of the project price amount is planned with the balance amounts in the first k capitalization periods of different durations with different interest rates.

Sizes of the 1st capitalization period:

(1) Start date of the 1st capitalization period, $T_0 = TE_m$ in format dd.mm.yyyy hh: mm: ss

(2) Amount of the project cost, G_0

(3) Amount of the 1st balance; amount of 1st annuity, $S_1 = a_1$,

(4) Annual interest rate, p_1

(5) Annual interest rate factor, $r_1 = 1 + p_1$

(6) Length of the 1st capitalization period, in days, x_1

(7) Length of the 1st capitalization period, in years, $y_1 = \frac{x_1}{365}$

(8) Amount of the 1st discounted balance; the amount of the absolute repayment, for the term T_0, $DS_1 = SUM_1 = AO_1 = \frac{S_1}{{r_1}^{y_1}}$

(9) Relative payback of the project price, in percentage, $RO_1 \frac{AO_1}{G_0}.100\%$

Sizes of the 2nd capitalization period:

(1) Start date of the 2nd capitalization period, $T_1=T_0+x_1$

(2) Amount of 2nd balance, amount of 2nd annuity, $S_2 = a_2$

(3) Annual interest rate, p_2,

(4) Annual interest factor, $r_2=1+p_2$,

(5) Length of the 2nd capitalization period, in days, x_2,

(6) Length of the 2nd capitalization period, in years, $y_2 = \frac{x_2}{365}$

(7) Amount of the 2nd discounted balance for the term T_0, $DS_2 = \frac{S_2}{{r_1}^{y_1}.{r_2}^{y_2}}$

(8) Sum of the 1st and 2nd discounted balance; the amount of the 2nd absolute payback, for the term T_0, $SUM_2 = AO_2 = DS_1 + DS_2 = \frac{S_1}{{r_1}^{y_1}} + \frac{S_2}{{r_1}^{y_1}.{r_2}^{y_2}} \leq G_0$

(9) Relative payback of the project price, in percent, $RO_2 \frac{AO_2}{G_0}.100\%$

…

Sizes kth of the capitalization period:

(1) Term of the beginning *k*th of the period of capitalization, $T_k=T_{k-2}+x_{k-1}$,

(2) Relation of the amount of the *k*th balance, *k*th annuity and the amount of the first profit, $S_k = a_k + P_1$

(3) Annual interest rate, p_k

(4) Annual interest rate factor, $r_k = 1+p_k$

(5) Length of the *k*th capitalization period in days, x_k

(6) Length of the *k*th capitalization period in years, $y_k = \frac{x_k}{365}$

(7) Amount of the k^{th} discounted balance, for term T_0, $DS_k = \frac{S_k}{r_1^{y_1}.r_2^{y_2} \ldots .r_k^{y_k}}$

(8) Sum of the 1^{st} till k^{th} discounted balance; the amount of absolute repayment, for the term T_0, $SUM_k = AO_k = DS_1 + DS_2 + \cdots . + DS_k =$ $SUM_k = AO_k = DS_1 + DS_2 + \cdots . + DS_k \quad \frac{S_1}{r_1^{y_1}} + \frac{S_2}{r_1^{y_1}.r_2^{y_2}} + \cdots + \frac{S_2}{r_1^{y_1}.r_2^{y_2} \ldots r_k^{y_k}} \le G_0$

(9) Relative payback of the project cost, in percent, $RO_k \frac{AO_k}{G_0} . 100\%$,

(10) Term end of the k^{th} capitalization period, $T_k = T_{k-1} + x_k$

(11) Length of the k^{th} capitalization period in days, $Dx = x_1 + x_2 + \ldots + x_k$, or $Dx = T - T_0$

(12) Length of the k^{th} capitalization period in years, $Dy = \frac{Dx}{365}$

The annuity a_k for the term T_0 has a discounted amount

$Da_k = G_0 - (DS_1 + DS_2 + \ldots + DS_{k-1}$.

The amount of the annuity a_k determines the discounted amount of its discounted amount for the term T_k, i.e. $a_k = D_{a_k} . r_1^{y_1} . r_2^{y_2} . \ldots . r_2^{y_2}$

The first amount of profit P_1 in the k^{th} period of capitalization determines $P_1 = S_k - a_k$.

Therefore, it is planned to repay the project price with a loan amount G_0 with interest amounts K_i, planned different annuity amounts a_i, different lengths of capitalization periods x_i)and different interest rates p_i, for $i \in \{1,2, ..., k\}$. Some or all of the stated amounts may be the same.

What follows is the introduction of the theory for making a repayment plan for this new loan repayment model.

3. Loan of various annuities, interest rates and capitalization periods

Primary loan amounts at the beginning and end of the 1st capitalization period:

(1) Amount of the loan at the beginning of the 1^{st} capitalization period, G_0

(2) Term of the beginning of the loan capitalization, T_0 in format dd.mm.yyyy hh: mm: ss

(3) Planned amount of annuities at the end of the 1^{st} capitalization period, $a_1 = S_1$

(4) Amount of the remaining debt at the end of the 1^{st} capitalization period, $G_1 = G_0 . r_1^{y_1} - a_1$

(5) Amount of interest at the end of the 1^{st} capitalization period, $K_1 = G_0 . (r_1^{y_1} - 1)$

(6) Repayment amount at the end of the 1^{st} capitalization period, $b_1 = a_1 - K_1$

(7) Amount repaid at the end of the 1^{st} capitalization period, $O_1 = b_1$

(8) Amount of the loan price at the end of the 1st capitalization period, $C_1 = G_0.(r_1^{y_1} - 1)$

Primary loan amounts at the beginning and end of the 2nd capitalization period:

(1) Start date of the 2nd capitalization period, $T_1 = T_0 + x_1$

(2) Planned amount of annuities at the end of the 2nd capitalization period, $a_2 = S_2$

(3) Amount of the remaining debt at the end of the 2nd capitalization period, $G_2 = G_1.r_2^{y_2} - a_2$

(4) Amount of interest at the end of the 2nd capitalization period, $K_2 = G_1.(r_2^{y_2} - 1)$

(5) Repayment amount at the end of the 2nd capitalization period, $b_2 = a_2 - K_2$

(6) Amount repaid at the end of the 2nd capitalization period, $O_2 = O_1 + b_2$

(7) Amount of the loan price at the end of the 2nd capitalization period, $C_2 = G_0.(r_1^{y_1} r_2^{y_2} - 1)$

…

Primary loan amounts at the beginning and end of the *k*th capitalization period:

(1) Beginning date of the *k*th capitalization period, $T_{k-1} = T_{k-2} + x_{k-1}$

(2) Term of the end of the *k*th capitalization period, $T_k = T_{k-1} + x_k$

(3) Amount of the balance at the end of the *k*th capitalization period, $S_k \geq a_k$

(4) Amount of annuity at the end of the *k*th capitalization period, $a_k = S \leq S_k$

(5) Amount of the 1st profit at the end of the *k*th capitalization period, $P_1 = S_k - a_k$

(6) Amount of the remaining debt at the end of the *k*th capitalization period, $G_k = G_{k-1}.r_k^{y_k} - a_k = 0$

(7) Amount of interest at the end of the *k*th capitalization period, $K_k = G_{k-1}.(r_k^{y_k} - 1)$

(8) Amount of repayment at the end of the *k*th capitalization period, $b_k = a_k - K_k$

(9) Amount repaid at the end of the *k*th capitalization period, $O_k = O_{k-1} + b_k$

(10) Amount of the loan price at the end of the *k*th capitalization period, $C_k = G_0.(r_1^{y_1} r_2^{y_2} \dots r_k^{y_k} - 1)$

The introduced theory is supported by spreadsheets (MS Excel) data processing without additional programming, shown on the example of one financial project.

4. Application of Excel in making an example of a financial project

Example 1:

With the term TE_1 = 24.03.2017 09:54:22, the preparation of the financial project of loan financing began with the payment of the 1st tranche of the amount,

TR_1 = 35,800 KM. After d_1 = 300 days, the 2nd tranche of the amount was paid, TR_2 = 29,000 KM. After the next d_2 = 220 days, the 3rd tranche of the amount TR_3 = 31,200 KM was paid when the financing of the project was completed, and, after d_3 = 380 days, its development was completed. There was no self-financing of project development costs, to reduce the loan amount.

Annual interest rates have been agreed for the capitalization of tranches and the calculation of the project price, successively: P_1 = 4.2%, P_2 = 4.5% and P_3 = 4.9%. During the duration of the project, 4 time intervals are planned: x_1 = 150 days, x_2 = 235 days, x_3 = 310 days and x_4 = 255 days, to repay its price G_0, planned decursive balance amounts; S_1 = -7.420 KM, S_2 = 0.000 KM, S_3 = 59.530 KM, and the amount $S \leq S_4$ = 69.845 KM. Successive annual internal rates are planned: p_1 = 5.1%, p_2 = 5.3%, p_3 = 5.5%, and p_4 = 5.9% for discounting and discounting the balance amount.

Using spreadsheet processor, calculate: the amount of the project price and the length of its development, the absolute and relative repayability of the project price amount with partial amounts of discounted balance amounts, the amount of the balance, covering the last remaining loan debt, and the amount of the 1st profit. Make a loan repayment plan for the project cost amount, with different annuity amounts, equal to different balance amounts, for planned equal internal balance rates and interest rates for annuities with different capitalization period lengths. Present the amounts of the primary loan amounts graphically.

The representation of the example model was done by the use of MS Excel data processor, through a spreadsheet containing 4 sheets. The cells with input values are highlighted in yellow color, the cells, containing nested and cloned values are white, and cells, presenting the output amounts are in green color.

In the 1st spreadsheet (Figure 1), the tranches were capitalized with the calculation of the project price amount G_0 = 104.062,292 KM and the length of the project development period Rd = 900 days or Rg = 2,465753425 years.

Table 1: Planned financing of the project price amount with a loan in tranches:		
Amount of the first tranche, TR1=	35.800,000	KM
Deadline for payment of the first tranche, TE1=	24.03.2017 09:54:22	
Length of the first capitalization period in days, d1=	300	days
The length of the first capitalization period in years, g1=d1/365=	0,821 917 808	years
Annual interest rate for the first capitalization period, P1=	0,042 000 000	4,200%
Annual interest rate factor for the first capitalization period, R1=1+P1=	1,042 000 000	
Amount of the second tranche, TR2=	29.000,000	KM
Term of payment of the second tranche, TE2=TE1+d1=	18.01.2018 09:54:22	
The length of the second capitalization period in days, d2=	220	days
The length of the second capitalization period in years, g2=d2/365=	0,602 739 726	years
Annual interest rate for the second capitalization period, P2=	0,045 000 000	4,500%
Annual interest rate factor for the second capitalization period, R2=1+P2=	1,045 000 000	
The amount of the third tranche, TR3=	31.200,000	KM

The deadline for payment of the third tranche, TE3=TE2+d2=	26.08.2018 09:54:22	
The length of the third capitalization period in days, d3=	380	days
The length of the third capitalization period in years, g3=d3/365=	1,041 095 890	years
Annual interest rate for the third capitalization period, P3=	0,049 000 000	4,900%
Annual interest rate factor for the third capitalization period, R3=1+P3=	1,049 000 000	
Deadline for completion of financing the planned project price, TE4=TE3+d3=	10.09.2019 09:54:22	
Discounted amount of the first tranche for the term(TE4), ETR1=TR1*R1^g1*R2^g2*R3^g3=	39.968,721	KM
Discounted amount of the second tranche for the term (TE4), ETR2=TR2*R2^g2*R3^g3=	31.300,366	KM
Discounted amount of the third tranche for the term (TE4), ETR3=TR3*R3^g3=	32.793,205	KM
Amount of self-financing (SF), for the term (TE4), SF=	0,000	KM
The amount of discounted tranche amounts with the amount of self-financing, G0=ETR1+ETR2+ETR3-SF=	104.062,292	KM
Length of project development deadline in days, Rd=d1+d2+d3=	900	days
Length of project development deadline in years, Rg=Rd/365=	2,465 753 425	years

Figure 1. Static form of he spreadsheet 1

In the 2nd spreadsheet (Figure 2), the decursive amounts of the balance were discounted for the period of the beginning of the duration and repayment of the project price. Calculations of the amount of absolute and relative payback of the project price for the first 4 capitalization periods were performed:

$AO_1 = AO_2 = -7.269{,}124$ KM; $RO_1 = RO_2 = -6{,}986\%$
$AO_3 = 46.640{,}124$ KM; $RO_3 = 44{,}819\%$
$AO_4 = 107.488{,}520$ KM; $RO_4 = 103{,}292\%$.

Table 2: Determining the amount of project cost coverage and the amount of the first profit		
The start date of the lst project price repayment period, TO=TE4 =	10.09.2019 09:54:22	
Planned project cost, G0 =	104.062,292	KM
The amount of the l st balance at the end of the l st project price repayment period, Sl =	-7.420,000	KM
Annual interest rate of the l st project price repayment period, pl =	0,051 000 000	5,100%
Annual interest factor of the l st project price repayment period, rl=l+pl =	1,051 000 000	
Length of the l st project price repayment period, in days, xl =	150	days
Length of the l st project price repayment period, in years, yl=xl/ 365 =	0,410 958 904	years
Discounted amount of the l st balance for the beginning of the l st period, DSl=SUMl=AOl=Sl/rl^yl =	-7.269,860	KM;AOl
Relative payback of the project price, in percent, ROl=(SUM1/G0)*l00% =	-6,986	%
The start date of the 2nd project price repayment period, Tl =T0+xl =	07.02.2020 09 :54:22	
The amount of the 2nd balance at the end of the 2nd project price repayment period, S2 =	0,000	KM
Annual interest rate of the 2nd project price repayment period, p2 =	0,053 000 000	5,300%
Annual interest factor of the 2nd project price repayment period, r2=1+p2 =	1,053 000 000	
Length of the 2nd project price repayment period, in days, x2 =	235	days
Length of the 2nd project price repayment period, in years, y2=x2/ 365 =	0,643 835 616	years
Discounted amount of the 2nd balance for the beginning of the l st period, DS2=S2/(r1^yl *r2^y2) =	0,000	KM
Amount discount. balance amount; absolute payback of the project cost, SUM2=AO2=DS1+DS2 =	-7.269,860	KM
Re lative payback of the project price, in percent, RO2=(SUM2/ G0)*100% =	-6,986	%
The start date of the 3rd project price repayment period, T2=Tl+x2 =	29.09.2020 09 :54:22	
The amount of the 3rd balance at the end of the 3rd project price repayment period, S3 =	59.530,000	KM
Annual interest rate of the 3rd project price repayment period, p3 =	0,055 000 000	5,500%
Annual interest factor of the 3rd project price repayment period, r3=1 +p3 =	1,055 000 000	
Length of the 3rd project price repayment period, in days, x3 =	310	days
Length of the 3rd project price repayment period, in years, y3=x3/ 365 =	0,849 315 068	years
Discounted amount of the 3rd balance for the beginning of the l st period, DS3=S3/(r1^yl *r2^y2*r3^y3) =	53.909,984	KM

Amount of discounted balance amounts; absolute payback, SUM3=AO3=DS1+DS2+DS3 =	46.640,124	KM
Relative payback of the project price, in percent, RO3=(SUM3/ G0)*100% =	44,819	%
The start date of the 4th project price repayment period, T3=T2+x3=	05.08.202109:54:22	
The amount of the 4th balance at the end of the 4th project price repayment period, S4=	69.845,000	KM
Annual interest rate of the 4th project price repayment period, p4=	0,057 000 000	5,700%
Annual interest factor of the 4th project price repayment period, r4=1+p4=	1,057 000 000	
Length of the 4th project price repayment period, in days, x4=	255	days
Length of the 4th project price repayment period, in years, y4=x4/ 365=	0,698 630 137	years
Discounted balance amount for the beginning of the l st period, DS4=S4/(r1^yl *r2^y2*r3^y3*r4^y4)=	60.848,396	KM
The amount of the discounted balance amount; absolute payback, SUM4=AO4=DS1+DS2+DS3+DS4=	107.488,520	KM
Relative payback of the project price, in percent, RO4={SUM4/ G0)*100%=	103,292	%
The end date of the 4th project price repayment period, T4=T3+x4=	17.04.2022 09:54:22	
Discounted amount of the project price coverage balance, DS=G0-{DS1+DS2+DS3)=	57.422,168	KM
The amount of the project cost coverage balance; amount 4. annuity, S=a4=S4-Pl=	65.912,195	KM
Discounted amount of the lst profit from the 4th capitalization period, DP1=SUM4-G0=	3.426,227	KM
The amount of the lst profit from the 4th period of capitalization, Pl=DPl * rl^yl * r2^y2*r3^y3*r4^y4=	3.932,805	KM

Figure 2. Static form of spreadsheet 2

In the 3rd spreadsheet (Figure 3), the calculations of the amounts of the primary loan were performed, for the primarily known annuity amounts $a_1 = S_1 = -7.420$ KM, $a_2 = S_2 = 0.000$ KM, and $a_3 = S_3 = 59.530$ KM. The amount of coverage annuity was calculated: $a_4 = S = 65.912,195$ KM for the rest of the loan debt, and the amount of the 1st profit $P_1 = 3.932,805$ KM.

The loan repayment period is $Rx = 950$ days or $Ry = 2,603$ years.

Table 3: Loan with different annuity amounts, interest rates and capitalization periods		
Table 3.1: Amounts of primary amounts at the beginning and end of the 1st loan capitalization pe:riod		
Loan amount (project price), G0=	104.062,292	KM
Loan capitalization start date, T0=	10.09.2019 09:54:22	
The end date of the lst capitalization period, Tl=T0+xl=	07.02.2020 09:54:22	
lst annuity amount (lst balance amount), al=Sl=	-7.420,000	KM
Amount 1. of the remaining debt, Gl =G0*rl ^ yl-al=	113.631,420	KM
lnterest amount, Kl =G0*(rl ^ yl-1)=	2.149,128	KM
Amount of lst repayment, bl=al-Kl =	-9.569,128	KM
Loan repayment amount, Ol=bl=	-9.569,128	KM
Loan price amount, Cl=G0*(rl ^ yl-1)=	2.149,128	KM
Table 3.2: Amounts of primary amounts at the end of the 2nd loan capitalization period		
The end date of the 2nd capitalization period, T2=Tl +x2=	29.09.2020 09:54:22	
2nd annuity amount (2nd balance amount), a2-S2=	0,000	KM
Amount 2. of the remaining debt, G2 = G1 * r2 ^ y2-a2=	117.473,151	KM
lnterest amount, K2=Gl *(r2 ^ y2-1)=	3.841,731	KM
Amount of 2nd repayment, b2=a2-K2=	-3.841,731	KM
Amount repaid, O2=O1+b2=	-13.410,859	KM
Loan price amount, C2 = G0*(rl ^ yl * r2 ^ y2-1)=	5.739,999	KM
Table 3.3: Amounts of primary amounts at the end of the 3rd loan capitalization period		
Term of the end of the 3rd loan capitalization period, T3=T2+x3=	05.08.2021 09:54:22	
Third annuity amount (third balance amount), a3=S3 =	59.530,000	KM
Amount 3. residual debt, G3 = G2* r3 ^ y3-a3=	63.408,323	KM
Amount of 3rd interest, K3=G2*(r3 ^ y3-1)=	5.465,171	KM
Amount 3. repayment, b3=a3-K3=	54.064,829	KM
Amount repaid, O3=O2+b3=	40.653,969	KM
Loan price amount, C3=GO*(rl ^ yl * r2 ^ y2* r3 ^ y3-1)=	10.848,301	KM

Table 3.4: Amounts of primary amounts at the end of the 4th loan capitalization period, profit amount (Pi) and loan repayment period		
The end date of the 4th loan capitalization period, T4=T3+x4=	17.04.2022 09:54:22	
The amount of the fourth balance, S4=a4+Pl=	69.845,000	KM
The amount of the lst profit from the 4th period of loan capitalization, Pl=S4-a4=	3.932,805	KM
Amount of the fourth annuity (amount of the project price coverage balance), a4=S=S4-Pl =	65.912,195	KM
Amount 4. remaining debt, G4=G3* r4 ^ y4-a4=0=	0,000	KM
Interest amount, K4=G3*(r4 ^ y4-l)=	2.503,873	KM
Repayment amount, b4=a4-K4=	63.408,323	KM
Amount repaid, O4=O3+b4=	104.062,292	KM
Loan price amount, C4=G0*(rl ^ yl * r2 ^ y2* r3 ^ y3* r4 ^ y4-1)=	15.385,899	KM
Loan repaymen period, in days, Rx=T4-T0=	950,000	days
Loan repayment period, in years, Ry=Rx/ 365=	2,603	years

Figure 3. Static form of spreadsheet 3

In the 4th spreadsheet (Figure 4) there is a loan repayment plan with primarily known different, decursive annuity amounts, for 4 capitalization periods of different lengths and different interest rates. With the amounts of the primary values of the loan repayment plan, their graphic overview is harmonized, (Figure 4).

Table 4: Load repayment plan of different annuities, interest rates and capitalization periods

Ordinal number (i)	Terms (Ti)	Remains of debt (Gi-1)	Annuities (ai)	Interests (Ki)	Repayments (bi)	Amounts repaid (Oi)	Loan prices (Ci)
0	10.09.2019 09:54:22	104.062,292					
1	07.02.2020 09:54:22	113.631,420	-7.420,000	2.149,128	-9.569,128	-9.569,128	2.149,128
2	29.09.2020 09:54:22	117.473,151	0,000	3.841,731	-3.841,731	-13.410,859	5.739,999
3	05.08.2021 09:54:22	63.408,323	59.530,000	5.465,171	54.064,829	40.653,969	10.848,301
4	17.04.2022 09:54:22	0,000	65.912,195	2.503,873	63.408,323	104.062,292	15.385,899
SUM			118.022,195	13.959,903	104.062,292		

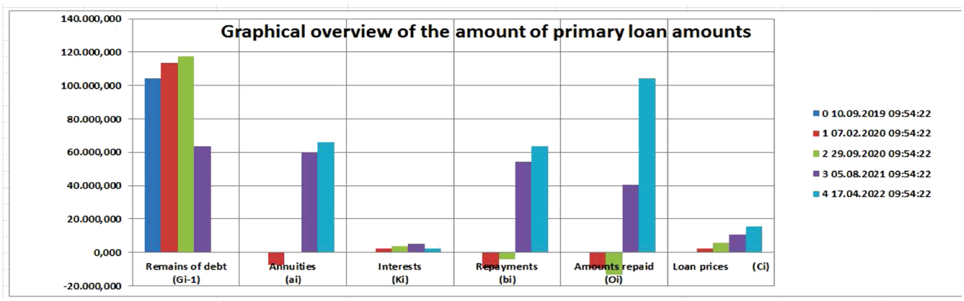

Figure 4: Static form of spreadsheet 4 copied from List C

Presented example shows a way in which the data processors could support the implementation of one model, based on the introduced theory, of financial project.

The solution is working even if, during the project (as it happen in practice) there are changes in all or some parameters.

Conclusion

In the used literature on the topic of the model of making and loan repayment of the price of making a financial project, the starting point is the relation

$G_0 = \frac{a_1}{r} + \frac{a_2}{r^2} + \cdots + \frac{a_k}{r^k}$. In this relation, the loan amount G_0 is equal to the sum of the discounted amounts k of annual, decursive annuities, for the term of the beginning of loan capitalization, and r is the annual, decursive interest factor. In this case all annual, decursive annuities $a_1, a_2, \ldots, a_k$ are higher than zero and are not related to the planned amounts of the balance, i.e. to the borrowing capacity of the project beneficiary, within its duration. The repayment of the loan is generally equal, annual, decursive annuities or equal repayments, which is not in line with the amounts of the balance, and especially with its negative amounts or amounts equal to zero.

The authors of the paper performed the necessary research and introduced the theory of a new model of loan financing of the amount of the price of a financial project. The new loan model is with different annuity amounts, different lengths of capitalization period and different interest rates. Annuity amounts are aligned with balance amounts, which means that annuities can have negative amounts and amounts, equal to zero. The last annuity of the loan repayment plan is the annuity to cover the last balance of the debt, which, with the last amount of the balance, determines the amount of the first profit from the project at the end of the loan repayment period. The negative amount of the first profit represents the amount of debt for the term of the end of the loan repayment period. The introduced theory was demonstrated by the use of MS Excel spreadsheet by representation of calculations, related to the Example 1. For each change of one, more or all input amounts, the spreadsheet automatically creates a new loan repayment plan with a graphical overview of the amounts of primary sizes.

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