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*Research and Paradigms*
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# EXPLAINING THE COMPONENTS OF REFLECTIVE THINKING BASED ON FUZZY TOPSIS ANALYSIS

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**Abstract.** Reflective thinking paradigm contains a set of trainings based on the pattern of mental tools and it helps its users to be able to present the educational materials in a framework that embraces the nature of thinking, various learning activities and the learning based on reflective, creative and critical forms of thinking. Reviewing of the related literature shows that most of the challenges are due to learning and learning-related factors. Accordingly, the present study examines the thinking level of senior postgraduate engineering students of Shahr-e-Rey University. For this purpose, the Reflective Thinking Questionnaire (RTQ) developed by Kember et al. (2000) was used. TOPSIS method was used to evaluate the thinking level of the subjects. The results showed that habitual action, which is one of the skills of reflective thinking, is ranked first and perception is ranked last, which means that the senior postgraduate engineering students learn superficially.

It is suggested to ask professors and teachers to hold classes that train reflective thinking skills, so that they can be a good and appropriate model of using reflective thinking skills for their students. So that students can improve their thinking levels and succeed in their math problems solving and these learners have good academic progress.

*Keywords*: critical thinking; reflective thinking; habitual action; understanding; reflection; critical reflection

## Introduction

The biggest mathematical challenge that students face is in the area of arithmetic and its related courses such as calculus. This challenge is due to the common teaching methods in universities, because the strategies of teaching arithmetic are mostly based on algebraic manipulation methods (Turker & Letzal 1995).

Such teaching methods are mostly based on procedural methods, rather than focusing on understanding the arithmetic concepts (Alex 2004). Therefore, to provide an effective teaching, education curricula should focus on the concepts of arithmetic. To achieve this aim, graphic, numerical, algebraic and verbal methods should be used (Gordon 2004; Alex 2006), because graphical visualization and thinking will be more effective in improving educational approaches (Hughes-Hallett 1996).

Today, thinking is considered vital in the field of education and the research related to this issue show its importance in the teachers' professional concerns (Clara 2015).

There are some skills for thinking. Reasoning, information, and critical thinking are some types of thinking skills (Lippmann 1998). Critical thinking is one of the highest levels of thinking that is used in mathematical problem solving and mathematical analysis (Kim 2009). It can be stated that critical thinking is one of the important components in abstract issues (Kaddoura 2013). In other words, recognizing and strengthening critical thinking as a valuable and relevant outcome of any program in higher education, is considered as an abstract mathematical problem (Gul et al. 2010; Kaddoura 2013).

Given the importance of the relationship between thinking skills and their relationship with problem solving, here we refer to another higher level thinking skill called reflective thinking skill. Reflective thinking is a mental engagement in cognitive processes to understand the opposing factors in the real environment, which is an essential element in the learning process. This mental engagement leads learners to actively build knowledge about a situation in order to form and develop a strategy for dealing with that situation (Porentavikol et al. 2016). In fact, reflective thinking is the same strategy that is applicable in Mathematics, so it is safe to say that RT (reflective thinking) skill is a basis of mathematics (Cutts 2018).

Using the reflective thinking skill and by focusing on solving mathematical problems, learners analyzed the mathematical contents (Mezirow 1991). Use of reflective thinking skill allows learners to fully understand the mathematical content (Schön 1987) and then analyze the mathematical content and therefore, learn it meaningfully (Inoue & Buczynski 2011; Eryaman 2007; Moon 2008).

According to the previous descriptions, it can be said that there is a connection and relationship between reflective thinking and critical thinking. The study on graduated students fully admits this relationship in that fostering reflective thinking enhances the level of critical thinking which is very important for the students. Reflective thinking is presented as a careful, patient, and active consideration of any belief or any supported form of knowledge in a supported context and in line with the obtained results. Students should act in a way that leaves them a place for openness and flexibility (Shabani 2014). Users need to reflect on their mental and intellectual processes and get benefit from the past and present experiences in order to intelligently select, design and prepare for their mind maps. Designing a reflective model, flexible thinking and self-regulating features of students, motivates them and makes reflective thinking pattern to be manifested in a planned and automatic way in the professors' teaching process and methods, so that the students will consciously participate the physical environments and classrooms more efficiently and in a timely manner (Aghazadeh 2014).

**Framework**

Kember et al. (2000) described reflective thinking skills in four levels of habitual action, understanding, reflection, and critical reflection. The first two levels of habitual action and ordinary understanding are considered unreflective, while the levels of reflection and critical reflection are considered reflective. Habitual action refers to activities that take place with little thought, so that learners follow the steps without thinking about what is going on. In other words, habitual action is another approach to the levels of reflective thinking for the learning purpose which involves the lowest level of reflection and minimal thought and conflict (Biggs et al. 2001; Leung & Kember, as cited by Peltier 2005).

After the level of habitual action, there is the level of understanding at which learners do their personal experiences and learning without thinking. Although at the level of understanding, learner seeks the basic meaning, there is still no reflection.

At the reflection level, learners not only gain an accurate understanding, but also can reflect on their personal experiences. Finally, as the highest level of reflective thinking, critical reflection implies a change in students' attitudes toward the fundamental belief in understanding a key concept or phenomenon (Kember et al. 2000).

Several studies have been conducted in this area, some of which are mentioned below:

In a study by Toraman (2020), it was found that there is a significant relationship between reflective thinking, problem solving, and metacognition.

Deringö (2019) showed that the reflective thinking skill of fourth grade elementary school students is at a high level and female students have a higher level of reflective thinking than male students and there is a significant relationship between students' reflective thinking skills and their academic achievement in mathematics.

In a study by Erdoğan (2020), it was found that there is a positive and significant relationship between critical thinking skills and reflective thinking skills of in-service teachers and critical thinking skill of in-service teachers is a significant predictor of their reflective thinking skills.

Erdogan (2019) found that the cooperative learning which takes place through the reflective thinking activities, will have a positive impact on students' critical thinking skills.

Ural & Dadli (2020) investigated the impact of seventh grade students' reflective thinking skills on environmental attitudes, and problem-based learning (PBL). The results of their study showed that PBL had a significant effect on seventh grade secondary school students' environmental knowledge. At the same time, environmental attitudes did not have a significant effect on students' reflective thinking skill.

Evin Gence (2018) investigated the effect of curriculum on the in-service teachers’ level of reflective thinking and self-directed learning. He concluded that curriculum has a positive effect on the level of reflective thinking and self-directed learning.

According to the literature review, most of senior postgraduate engineering students have a low level of analysis in solving math problems; since their academic achievement and progress in mathematics is not desirable. For this purpose, in this study we intend to examine the senior post graduate students’ level of reflection thinking.

The research question of this study is as follows:

What is the postgraduate students’ level of thinking in terms of reflective thinking skills?

**Research Method and Instrument**

The present study is a descriptive study with survey method. In this study, based on theoretical foundations and by relying on the main and secondary aims of the study, the standard questionnaire of reflective thinking levels developed by Kember et al. (2000) was used, which comprises of 16 items and 4 components of habitual action, understanding, reflection and critical reflection. Each component consists of 4 items and measures the students’ levels of reflective thinking in 5 point Likert scale, including strongly agree, agree, no opinion, disagree and strongly disagree.

**Reliability and validity of the questionnaire**

At first, the researcher, after selecting 35 students, administered the pre-test and post-test to the selected sample, and then using content validity, the opinions of experts in the field of mathematics education were recorded and the questionnaire items were confirmed, and then based on data analysis with Cronbach's alpha, in two test repetitions at regular intervals, an acceptable value above 70% was obtained to establish reliability.

Moreover, the Cronbach's alpha based on what was obtained in the study by Azimi &Taghizadeh (2019) was as follows: normal action: 65%, understanding: 74%, reflection: 81%, critical thinking: 87%, which in comparison with the work of Kember (2000), each of them has a good reliability. The overall reliability of this questionnaire is 73%.

**Participant**

The statistical population included 25 senior post graduate engineering students of Islamic Azad University of Shahr-e-Rey Using purposive sampling method, 15 students (male and female) were considered as a study sample during the educational year 2019 – 2020. Questionnaire was administered to senior engineering students who had taken theoretical course of general mathematics. General mathematics is a prerequisite for senior courses.

**Findings**

The TOPSIS method was used to estimate the postgraduate engineering students' level of reflective thinking skill.

Findings obtained from TOPSIS method are as follows:

1. Prioritizing the main features through fuzzy TOPSIS technique, based on expert opinions.

The fuzzy numbers and verbal expressions used in this research are shown in Table 1.

Based on the above mentioned fuzzy numbers and verbal expressions, the average fuzzy scores of the experts' opinions regarding the options available in this research are shown in Table 1.

**Table 1.** Fuzzy numbers and verbal expressions

| Fuzzy number | verbal expression |
|---|---|
| (1,1,3) | Very poor |
| (1,3,5) | Poor |
| (3,5,7) | Average |
| (5,7,9) | Good |
| (7,9,11) | Very good |

**Converting fuzzy numbers to verbal expressions**

The method of converting fuzzy numbers to verbal expressions used in this software is based on the method proposed by Ching in his study. In order to determine the verbal expression of the fuzzy number related to option "A1", we must first calculate the distance of this number from each of the verbal expressions in the figure above. After determining the distances, we determine the appropriate verbal expression according to the minimum distance between the fuzzy number of the relevant option and the verbal expressions.

$$\widetilde{R} = [\widetilde{r}_{ij}]_{m\times n} \quad i = 1,2,...,m \quad j = 1,2,...,n \qquad (1)$$

The following formula is used to calculate the distance between two fuzzy numbers:

Suppose $\widetilde{A}$ and $\widetilde{B}$ are two fuzzy numbers with the following values:

$$\widetilde{B} = (a_2, b_2, c_2) \quad \widetilde{A} = (a_1, b_1, c_1) \tag{2}$$

The distance between $\widetilde{A}$ and $\widetilde{B}$ is then calculated as follows:

$$D(\widetilde{A}, \widetilde{B}) = \sqrt{\frac{1}{3}[\, a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2\,]} \tag{3}$$

According to the above formula, the distance between the fuzzy numbers of each option with each of the verbal expressions is as described in Table 1:

Step 2: Descaling the decision matrix: In this step we need to convert the fuzzy decision matrix for the evaluation of options into a fuzzy non-scale matrix ( $\widetilde{R}$ ). The following equations are used to obtain the matrix:

m: number of options n: number of experts (4)

If the fuzzy numbers are (a, b, c), $\widetilde{R}$ which is an unscaled (normalized) matrix will be obtained as follows:

$$\widetilde{r}_{ij} = (\frac{a_j}{c_j^*}, \frac{b_j}{c_j^*}, \frac{c_j}{c_j^*}) \tag{5}$$

In this equation, $c_j^*$ is the maximum of the value c in expert jth among all options. The equation (4-2-4) indicates it:

$$c_j^* = \max_i c_j \tag{6}$$

Step 3: Creating a fuzzy unscaled weighted matrix)

$$\widetilde{V} = [\widetilde{v}_{ij}]_{m \times n} \quad i = 1,2,\ldots,m \quad j = 1,2,\ldots,n \tag{7}$$

$$\widetilde{v}_{ij} = \widetilde{r}_j \otimes \widetilde{w}_j \tag{8}$$

In this equation, the unscaled matrix is obtained from the second step.

Note that here by weight we mean the weight of experts' opinions, which is considered the same.

Step 4: Determining the fuzzy positive ideal $(FPIS, A^+)$ and the fuzzy negative ideal $(FPIS, A^-)$.

$$A^+ = (v_1^*, v_2^*, \ldots, v_n^*) \tag{9}$$

$$A^{-} = (v_1^{-}, v_2^{-}, \ldots, v_n^{-}) \quad (10)$$

In this software, the fuzzy positive ideal values and fuzzy negative ideal values introduced by Chen are used.

These values are as follows:

$$v_j^{*} = (1,1,1) \quad (11)$$

$$v_j^{-} = (0,0,0) \quad (12)$$

Step 5: Calculating the sum of the distances of each of the options from the fuzzy positive ideal and the fuzzy negative ideal:

If $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers as shown below, then the distance between these two fuzzy numbers will be obtained by the relation (13).

$$\tilde{A} = (a_1, a_2, a_3) \quad (13)$$

$$\tilde{B} = (b_1, b_2, b_3) \quad (14)$$

$$D(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2]} \quad (15)$$

According to the above explanations on how to calculate the distance between two fuzzy numbers, we obtain the distance of each component from the positive ideal and the negative ideal:

Step 6: Calculating the relative proximity of option ith from the ideal solution. This relative proximity is defined as follows:

$$CC_i = \frac{d_i^{-}}{d_i^{*} + d_i^{-}} \qquad i = 1,2,\ldots,m \quad (16)$$

Step 7: Ranking the options: the problem or item's options can be ranked in descending order. Any option with a higher CC is better.

**Ranking the components of research variables through TOPSIS method**

According to the research question (What is the postgraduate engineering students' level of thinking in terms of reflective thinking skills?), the TOPSIS method was used to answer and prioritize the components of reflective thinking (habitual action, understanding, reflection and critical reflection). Through fuzzy analysis, we showed each participant's questionnaire in the form of a fuzzy table and rank the components of the 15 respondents. The seven steps of TOPSIS were implemented and the last two results are as follows:

**Table 2.** A results sample of the positive and negative ideals of students' reflective thinking

| Postgraduate engineering students | Positive ideal ($d_i^-$) | | | | Positive ideal ($d_i^+$) | | | |
|---|---|---|---|---|---|---|---|---|
| | Habitual action | understanding | Reflection | Critical reflection | Habitual action | understanding | Reflection | Critical reflection |
| Student≠1 | 0.20 | 0.20 | 0.16 | 0.18 | 0.79 | 0.79 | 0.84 | 0/88 |
| Student≠2 | 0.14 | 0.19 | 0.14 | 0.14 | 0.86 | 0.80 | 0.86 | 0.86 |
| Student≠3 | 0.19 | 0.19 | 0.19 | 0.19 | 0.89 | 0.84 | 0.84 | 0.89 |
| Student≠4 | 0.11 | 0.20 | 0.11 | 0.11 | 0.80 | 0.80 | 0.80 | 0.80 |
| Student≠5 | 0.19 | 0.19 | 0.19 | 0.19 | 0.88 | 0.79 | 0.88 | 0.88 |
| Student≠6 | 0.18 | 0.18 | 0.18 | 0.18 | 0.82 | 0.82 | 0.82 | 0.82 |
| Student≠7 | 0.14 | 0.19 | 0.14 | 0.14 | 0.86 | 0.80 | 0.86 | 0.86 |
| Student≠8 | 0.19 | 0.19 | 0.19 | 0.19 | 0.80 | 0.80 | 0.80 | 0.80 |
| Student≠9 | 0.16 | 0.20 | 0.16 | 0.16 | 0.84 | 0.79 | 0.84 | 0.84 |
| Student≠10 | 0.11 | 0.20 | 0.11 | 0.11 | 0.88 | 0.79 | 0.79 | 0.84 |
| Student≠11 | 0.09 | 0.14 | 0.19 | 0.04 | 0.91 | 0.86 | 0.80 | 0.91 |
| Student≠12 | 0.16 | 0.20 | 0.20 | 0.20 | 0.84 | 0.79 | 0.79 | 0.79 |
| Student≠13 | 0.14 | 0.19 | 0.19 | 0.14 | 0.86 | 0.80 | 0.80 | 0.86 |
| Student≠14 | 0.12 | 0.18 | 0.12 | 0.18 | 0.89 | 0.82 | 0.89 | 0.82 |
| Student≠15 | 0.07 | 0.20 | 0.16 | 0.11 | 0.93 | 0.79 | 0.84 | 0.88 |

**Table 3.** Results of the relative proximity of CCi (weight) of students' reflective thinking

| Items | Distance from positive ideal ( $d_i^+$ ) | Distance from negative ideal ( $d_i^-$ ) | Relative proximity of CCi (weight) | Rank |
|---|---|---|---|---|
| Habitual action | 12.83 | 2.293 | 13.83 | 1 |
| Understanding | 12.12 | 2.96 | 13.12 | 4 |
| Reflection | 12.48 | 2.62 | 13.48 | 3 |
| Critical reflection | 12.71 | 2.46 | 13.71 | 2 |

Based on the results of fuzzy TOPSIS and estimating the rankings of the components of reflective thinking, we can claim that habitual action provides a strong explanation of the students' reflective thinking. In fact, considering the issue of reflective thinking, students have the ability to act habitually compared to other authors. Furthermore, in this ranking, it is clear that students' ability to understand is less intense than other components of reflective thinking. Moreover, in measuring students' reflective thinking, we can pay more attention to habitual action compared to other components and consider this component as a strong predictor of their reflective thinking.

**Discussion**

Learning mathematics requires to be armed with the thought weapon. Academic achievement and learning and teaching processes are influenced by various factors, the most important of which is reflective thinking. On the other hand, reflective thinking requires skills such as habitual action, understanding, reflection, and critical reflection, to analyze mathematical problems.

According to what have been stated above, ordinary action is one of the lowest levels of reflective thinking. Learners who, in fact, tend to approve their performance and provoke the favorable judgment of others about their personal performance, and their goal is to be better than others, and also those who the content analysis and mastery of assignments does not have the main place in their view; their learning process is less thoughtful and they represent a weaker performance. These people with their low-level beliefs are passive in their learning process, and they just memorize the content. In contrast, the learner who participates in class discussions and engages in doing class assignments has a higher level of reflective thinking.

The reflection skill involves a constant and active attention towards any type of idea or belief that leads to a deeper thinking. Finally, critical thinking skill involves awareness of issues in the sense of why we understand them and how we feel them. This group of people with high level of epistemological beliefs are in a learning environment with deep approaches and learning takes place in them with a reflective and critical view of problems and issues.

Various studies in the field of mathematical learning show that the main causes of students' difficulties in mathematical learning and making progress in this course of study, are often due to the repetitive and habitual learning, inappropriate strategies and superficial mathematical reasoning. This issue has led them to approach the mathematical problems and subjects superficially and inaccurately and deal with mathematical problems with apathy (poor personal sense of achievement in academic and educational matters).

Another shortcoming is the students' lack of information about multiple learning sources. Multiple learning sources allow students to interact with the content, information and knowledge needed to fully interact with interactive thinking. When these elements are present, students may engage in reflective thinking processes and thus reach the deeper levels of learning. In other words, it can be said that reflective thinking represents the deep learning, and where there are no teaching-learning activities such as reflective thinking, only superficial learning can be achieved (West 1996).

Another challenge that students face is that they are not at high levels of learning and are not active learners because those with high levels of learning are self-directed active learners who have a strong desire to learn, use problem-solving skills, have the necessary capabilities in doing independent learning activities and independently manage their learning (Yousefi, 2011).

Finally, it can be said that the goal is to have professors guide students towards organizing the content, facilitating more complex processing, and elevating the thinking to a higher level. Accordingly, after administrating a pre-test to the senior post-graduate students in general mathematics course, researchers concluded that they were not in the desired condition.

Now the question is that, what is the senior post graduate engineering students' level of reflective thinking which has made them not to be in a favorable condition regarding the academic achievement and progress?

**Results**

In this study, the questionnaire developed by Kember et al. (2000) was used to examine the thinking level of senior postgraduate engineering students. After collecting this questionnaire from students, the researcher examined the level of reflective thinking of senior engineering students of Islamic Azad University of Shahr-e-Rey. The results showed that habitual action was ranked first and understanding was ranked fourth in this study and critical reflection and reflection were ranked third and fourth. This issue shows that habitual action is a strong explainer of students' reflective thinking. In fact, students have the ability to act habitually compared to other components considering the issue of reflective thinking. Moreover, in this ranking, it is clear that students' understanding ability is less intense than other components of reflective thinking. Furthermore, in measuring students' reflective thinking, we can pay more attention to habitual action than other components and consider this component as a strong predictor of their reflective thinking. It can be said that according to the definitions of the four components provided by Kember et al. (2000), post graduate engineering students have a high level of habitual action. Therefore, these students learn mathematics superficially, since habitual action and understanding are at the lower levels of reflective thinking, while critical reflection and reflection are at the higher levels of reflective thinking.

It is suggested to provide in-service teachers at all levels from elementary school level to high school level with various trainings about thinking methods and the use of reflective thinking to solve mathematical problems. So that these teachers can display reflective thinking levels well and so that they can be a good role model for their learners. It is also suggested to do these evaluations at different levels such as undergraduate, high school and other educational levels so that curriculum planners and administrators can adopt adequate programs for students, so that by improving knowledge they can be effective in providing better scientific quality and leading the students to get academic achievement in their courses.

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