EUROMATH SCIENTIFIC CONFERENCE

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Abstract. The paper is dedicated to a recent International Scientific Conference for secondary school students. Main parts of the Programme are described and one of the Workshops is presented with its methodological aspects.

Keywords: science, innovator, researcher, fame-lab, math-factor.

The 4th European Student Conference in Mathematics EUROMATH'2012 took place under the auspices of the Minister of Education, Youth and Science, Professor Sergei Ignatov, from March 21 to March 25, 2012 in Sofia. The organizers were the Cyprus Mathematical Society and the THALES Foundation (Cyprus) in cooperation with the European Mathematical Society, the VUZF University (Bulgaria) and the Union of Bulgarian Mathematicians. The Conference was attended by about 230 professionals, teachers and students of age 12-19 from European and International schools.

EUROMATH' 2012 answers the EU Member States concern about the young people' s lack of interest for Science and Technology and the need to equip them with skills and knowledge, needed to future responsible innovators, researchers and „science-active" citizens. The main aim is to develop and implement support actions in the field of Mathematics and Computer science education. This consists to update the style of teaching, promote best practices, develop new methods and train teachers in inquiry based teaching. Promoting excellence in education and skills development is one of the key elements within the „Innovation Union" Flagship Initiative under Europe 2020. An appropriate science teaching methodology such as the Inquiry Based Science Education can strongly contribute to the development of these skills. No doubt, the Conference will support actions to promote the more widespread use of problem and inquiry based science teaching techniques in secondary schools as well as actions to bridge the gap between the research community, science teachers and local actors (including providers of informal science education). The Conference actions will contribute for upgrading also the current school science curricula and promote European teachers' networks and collaboration.

Under the mottoes „Creativity and Innovation from early age", „Tomorrow' s Inventors", „Create-Exchange-Grow", the EUROMATH Conference Programme realized various initiatives. Seventy seven math projects were presented and included in an Abstract booklet. Young mathematicians were encouraged to work alone or with classmates to show their research abilities in front of an international group of students and teachers. At the same time they had a chance to write a full paper to be published in proceedings. Here are some of the workings out: „Combinatorics on the chessboard" by Artur Balabanov and Teodor Vakarelsky (Bulgaria); „Prisoner's dilemma and its' application in Economics" by Krasimira Trifonova (Bulgaria); „Mathematics in daily life" by Radoslav Dishev and Georgi Kirkov (Bulgaria); „The physical basis of scoring athletic performance" by Toshko Todorov (Bulgaria); „Mathematics and Music" by Dora Hipsa, Jelena Jaksic, Josip Kir Hromatko, Lara Rajkovic and Veronika Vrhovec (Croatia); „Mathematics in literature and philosophy" by Astero Constantinou, Giannis Koudounas, Ioanna Georgiou and Ioulia Efthyvoulou (Cyprus); „Third degree polynomials with rational characteristic points and their analysis" by Leon Harbecke (Germany); „Linear programming: a tool in the hands of business" by Eleftherios Mallios (Greece); „Three homeomorphic figures of P-Adic Topology" by Amir Hossein Ekhlasi (Iran); „Give me four colours and I' ll paint the world" by Debra Barki and Jasmine Blanga (Italy); „Mathematical Earthquake function" by Abduzhihat Bayanasov and Asset Yespolov (Kazakhstan); „Art and Mathematics" by Pirosca Radu and Mariana Nasui (Romania); „Multifractal approach to analysis of distribution of crater diameters" by Nikolay Klimkin (Russia); „Mathematical model of hologram" by Ksenija Butorac (Serbia); „Human being with golden ratio and Fibonacci sequence" by Bianka Malackanicova (Slovakia); „Fractals, natural and abstract" by Yue Wang Sabrina (Sweden); „Randomness" by Aliosha Pittaka (United Arab Emirates).

Mathematics Poster Design Competition for students was a part of the EUROMATH Conference activities attracting of nearly 50 posters. The students designed their mathematical thinking themselves, submitting posters of size A3 or A2 in colour or black/white. An International jury selected the best three and awarded: 1st Prize (400 euro) to Branimir Jungic (XV High School, Zagreb, Croatia); 2nd Prize (300 euro) to Veronika Vrhorec (XV High School, Zagreb, Croatia); 3rd Prize (200 euro) to Kyriaki Ioannou and Constantina Mikeou (The G. C. School of Careers, Cyprus); 3rd Prize (200 euro) to Tomas Sura, Matus Zeman, Leo Cunderlik and Samo Lihotsky (1st Independent High School, Slovakia).

A new initiative of the EUROMATH Conferences is the *MathFactor* Competition. Its goal is to prove that one can talk comprehensibly and with fun for Mathematics without compromising a corresponding scientific content. The theme could be an interesting theorem, a mathematical method, an application of Mathematics, but it has to be made simple so a non-expert can understand it, enjoy it and appreciate it, i.e. the task is to communicate Mathematics in 3 minutes to an open audience. There is no limitation for the presentation format. It could include a song, a dance, poesy or images but not a computer. The requirement is to be attractive only. This new competition resembles the well-known „Eurovision for Scientists" and „Laboratory *FameLab*". The second was started in 2005 in the UK by Cheltenham Science Festival and has quickly become established as a diamond model for successfully identifying, training and mentoring scientists and engineers to share their enthusiasm for their subjects with the public. *FameLab* is designed to inspire and motivate young people. It transcends cultures and languages. The robust training, coaching and recognition builds confidence and skills allowing to put into practice the skills in a wide variety of situation. But still, *FameLab* is directed to teachers and University lecturers, researchers and even University students. *MathFactor* gives opportunities to secondary school students, which makes the main difference with *FameLab*.

The International Jury of the EUROMATH' 2012 *MathFactor* Competition included Dr. Myrtani Pieri from the University of Cyprus. She is the 2011 *FameLab* International winner, marking the first time that a woman has won and also the first year that there were equal numbers of men and women in the final. She holds a PhD in molecular biology from Oxford University and is currently investigating why only some Cypriot families with a specific mutation go on to develop kidney failure. Inspired by a spate of pregnancies among her friends, in 2011 Dr. Pieri has delivered a polished winning presentation on the pregnancy paradox: how the maternal immune system tolerates the unborn child when half of the genes in the developing child came from „just some random guy". Her high criteria of content clarity and charisma helped the Jury to rank the *MathFactor* contestants: 1st Prize (400 euro) to Ljubica Vujovic (The first Grammar School in Kragujevac, Serbia) for „More honey, please"; 2nd Prize (300 euro) to Yue Wang (Malmo Borgarskola, Sweden) for „A brief overview and some useful applications of multivariable calculus"; 3rd Prize (200 euro) to Aleksandar Hrusanov (Bourgas High School of Mathematics and Science, Bulgaria) for „Is Fibonacci still alive?".

The presentation of the First prize winner Ljubica Vujovic was connected with a classical problem appearing in almost all Geometry textbooks: *Given are two points A and B in different semi-planes with respect to a given straight line l. Find a point D on l with the property that the sum $|AD|+|BD|$ is minimal.* This problem is ascribed to the Ancient Greek mathematician and engineer Heron of Alexandria (about 10–about 75 A.D.), known as the greatest experimentalist of Antiquity. The solution is quite instructive using axis symmetry with respect to l and connecting the image of A with B. Thus, D turns out to be the intersection point with l. What Ljubica Vujovic did with incorruptible charisma was to tell a funny love story of a bee, which was looking for the shortest way to its pet. Not less attractive was the presentation of the Bulgarian school boy Aleksandar Hrusanov dedicated to classic Mathematics too. In 1202 the Italian Leonardo Fibonacci (1170 – 1250), considered as the most talented western mathematician of the Middle Ages, introduced the following sequence in his book *Liber Abaci*: 0, 1, 1, 2, 3, 5, 8, 13, 21, … The main property of the sequence that each term (called Fibonacci number) equals the sum of the previous two, was presented by nuts. Aleksandar Hrusanov also succeeded to demonstrate attractively some of the Fibonacci numbers properties: they are intimately connected with the golden ratio and appear in biological settings, such as branching in trees, arrangement of leaves on a stem, etc.

Several workshops constituted an essential part of the EUROMATH Conference Programme. One of them is presented in the sequel, namely „The Computer as a heuristic tool in Mathematics investigations".

The software programme GOEMETER'S SKETCHPAD (GSP) has several instruments for the construction of plane geometry objects on the base of independent points, straight lines, circles and their intersections. On the other hand it allows us to create instruments by ourselves, which depend on preliminary chosen free points, lines, circles. Some of them are connected with remarkable points and sets of points in the triangle. For example: the centre of gravity, the orthocentre, the circum-centre and the circum-circle itself, the circle through the mid-points of the sides of the triangle – the so called Euler circle, the in-centre and the in-circle itself, the centres of the outer in-circles. Only those instruments are sufficient to discover remarkable points, lines and circles, which are connected with well-known geometric figures.

1. For an arbitrary triangle ABC consider the centre of gravity G, the orthocentre H and the circum-centre O simultaneously. Note that always the three points are collinear – the common line is the Euler line. Also, we have that $GH : GO = 2 : 1$. Thus, experimentally we establish the following:

Assertion 1.1. *The points G, H and O are collinear, verifying $GH : GO = 2 : 1$.* (Hitov, 1990)

In addition, if we construct the Euler circle, we notice that its centre is on the Euler line. Thus, we get:

Assertion 1.2. *The centre E of the Euler circle lies on the Euler line and it is the mid-point of the segment OH.* (Hitov, 1990)

2. Given is a quadrilateral $ABCD$. Consider the centres if gravity G_a, G_b, G_c and G_d of the triangles BCD, CDA, DAB and ABC, respectively. Connect the points A, B, C and D with the points G_a, G_b, G_c and G_d by straight lines, respectively.

Note that the connecting lines are concurrent. Denote the common point by ***G***. We have:

Assertion 2.1. *The lines* AG_a, BG_b, CG_c *and* DG_d *have a common point* G.

The point G is called centre of gravity of the quadrilateral $ABCD$.

A natural question is the following: What will happen if we connect the orthocentres H_a, H_b, H_c and H_d of the same triangles with the points A, B, C and D, respectively? We notice that those lines are not concurrent as it has happened in the previous case. Is it possible that there exist quadrilaterals for which the lines AH_a, BH_b, CH_c and DH_d are concurrent? Compare a triangle and a quadrilateral! What does a triangle possess more than a quadrilateral? EACH TRIANGLE HAS A CIRCUMCIRCLE, while for a quadrilateral such a fact is not always true. If $ABCD$ is inscribed in a circle with centre O, experiments with GSP show that:

Assertion 2.2. The lines AH_a, BH_b, CH_c *and* DH_d *are concurrent.*

Denote the common point by H. This point is called orthocentre of the inscribed quadrilateral $ABCD$.

By the help of GSP we notice more:

Assertion 2.3. *The points* G, H *and* O *are collinear, verifying* $GH = GO$.

The common line is called Euler line of the inscribed quadrilateral $ABCD$.

Construct the circle through G_a, G_b and G_c. Note that the point G_d lies on that circle too. The circle is called Euler circle of the inscribed quadrilateral $ABCD$. Something more, the centre of that circle lies on the Euler line. We have:

Assertion 2.4. *The centre of the Euler circle of the inscribed quadrilateral* $ABCD$ *lies on its Euler line.*

3. Consider now the in-centre of a triangle ABC. Construct the Euler lines of the triangles BCI, CAI and ABI. We notice, that:

Assertion 3.1. *The Euler lines of the triangles* BCI, CAI *and* ABI *are concurrent.*

The common point is called Schiffler point (according to the name of the German non-professional Kurt Schiffler (1896–1986), who proposed the above statement in 1985 as Problem 1018 of the Canadian Journal *Crux Mathematicorum* and the solvers of the problem named that remarkable point under his name).

What will happen if we construct the Euler lines of the triangles BCI_a, CAI_a and ABI_a, where I_a is the centre of the outer in-circle with respect to BC? We have:

Assertion 3.2. *The Euler lines of the triangles* BCI_a, CAI_a *and* ABI_a *are concurrent or parallel.*

What will happen if ΔABC is equilateral and P is an arbitrary point in the plane of the triangle? Observing the dislocation of the Euler lines of the triangles BCP, CAP and ABP, we establish:

Assertion 3.3. *The Euler lines of the triangles BCP, CAP and ABP are concurrent or parallel* (Dr. Svetlozar Doichev).

4. The in-centre of a given ΔABC is the intersection point of its bisectrices. By the bisectrices one could define a remarkable point transformation in the plane of ΔABC. Let P be an arbitrary point in this plane. By means of GSP we construct a line a, which is symmetric to AP with respect to the bisectrix of $\sphericalangle A$. Analogously, we construct the lines b and c, which are symmetric to BP and CP with respect to the bisectrices of $\sphericalangle B$ and $\sphericalangle C$, respectively. Note that the lines a, b and c are concurrent. Denote the common point by Q. The points P and Q are called isogonally conjugated with respect to ΔABC. Let l be an arbitrary line in the plane of ΔABC and P be an arbitrary point on l. Consider the isogonally conjugate point Q of P. Further, define the locus of the point Q, when P displaces on l. We notice that the point Q describes a conic. More precisely we establish, that:

Assertion 4.1. If *l is a line in the plane of ΔABC, which does not pass through the vertices of the triangle, then the set of the points isogonally conjugated to the points on l with respect to ΔABC, is a second degree curve l', circumscribed to ΔABC. The curve l' is an ellipse, a parabola or a hyperbola depending on whether l is non-secant, tangent or secant to the circum-circle Γ of ΔABC, respectively.*

Let ΔABC be equilateral and k be the circle through the centres of the outer in-circles of ΔABC. Find the locus of the isogonally conjugate point Q of P, describing k. We obtain a curve K of fourth degree. Further, construct the Euler lines of the triangles BCQ, CAQ and ABQ for an arbitrary point Q on K. We have:

Assertion 4.2. *The Euler lines of the triangles BCQ, CAQ and ABQ are parallel for any point Q on K.*

5. We have considered several cases when a point P is given in the plane of ΔABC and the Euler lines of the triangles BCP, CAP and ABP are concurrent. A natural question is the following: What will happen if the Euler lines are replaced by Euler circles? Observations by GSP lead to:

Assertion 5.1. *If A, B, C and P are 4 points in a plane and no 3 of them are concurrent, then the Euler circles of the triangles ABC, BCP, CAP and ABP are concurrent.*

Pay attention to the following. The Euler circle contains the intersection points of the lines AH, BH and CH with the lines BC, CA и AB, the mid-points of the segments AH, BH and CH, also the mid-points A_0, B_0 and C_0 of BC, CA and AB, respectively (according to the already used notations). Let P be an arbitrary point in the plane of ΔABC. Consider the intersection points of AP, BP and CP, the mid-points of the segments AP, BP and CP. Consider also a second degree curve $\Omega(P)$ through 5 of those 6 points. Note that the sixth point lies on $\Omega(P)$ too. Even more, $\Omega(P)$ contains also the points A_0, B_0 and C_0. Consequently, $\Omega(P)$ is a generalization of the Euler circle, depending on an arbitrary point P. Construct the Euler curve of P from the configuration, which participates in the last assertion. We have:

Assertion 5.2. *The Euler curve $\Omega(P)$ of the point P passes through the common point of the Euler circle of the triangles ABC, BCP, CAP and ABP.*

6. Consider the circum-circles k_a, k_b and k_c of the triangles BCH, CAH and ABH, respectively (H is the orthocentre of ΔABC). Take a line l through H. Consider the tangents to k_a, k_b and k_c through the second intersection points of l with the corresponding circles. Denote by $A_1B_1C_1$ the triangle defined by those tangents. Finally, construct the circum-circle Γ of ΔABC and the in-circle Γ_1 of $\Delta A_1B_1C_1$. Observations of the mutual dislocation of those circles show that:

Assertion 6.1. *The circles Γ and Γ_1 are tangent.*

What will happen if the orthocentre H is replaced by any other point P from the plane of ΔABC? Observations by GSP show that assertion 6.1 remains true.

All the above observations were realized together with the participants in the Workshop. Strict mathematical proofs were not discussed. It is important to note that the listed assertion are proved by the authors in various articles of them using the technique from (Pascalev & Chobanov, 1985) and (Grozdev & Nenkov, 2010). The last assertion is in process of publishing.

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Научна конференция EUROMATH

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