Educational Matters
Въпроси на преподаването

ERRORS RELATED TO TOPICS IN GEOMETRY, DATA REPRESENTATION AND ANALYSIS MADE BY FIFTH GRADE STUDENTS IN THE REPUBLIC OF MACEDONIA

[1]Metodi Glavche, [2]Risto Malčeski, [3]Katerina Anevska
[1]Ss. Cyril and Methodius University – Skopje
[2,3]FON University – Skopje

Abstract. In the learning process it is natural for omissions to occur and if they are not identified and eliminated in a timely manner, they are multiplied further on in the course of education. Therefore, one of the tasks of the mathematics instruction is to detect the errors made by the students and to eliminate them on time. Errors that have made students from fifth grade in arithmetic and algebra are processed in (Glavche, Malčeski & Anevska, 2015). In this paper the results of the research are presented about some of the most prominent errors that fifth grade students in the Republic of Macedonia make. Additionally, the paper offers a systematic overview of the identified errors and the reasons for their occurrence. It also offers procedures for elimination of the identified errors.

Keywords: data, error, fifth grade, geometry, student

1. Introduction

It is expected and natural that students might make errors when learning certain areas of the material envisioned. The detection of these errors and their timely correction is an ongoing and continuing process carried out by the teacher, which is crucial for acquiring permanent and applicable knowledge and skills. Consequently, the subject of the present paper is the detection and correction of the errors related to topics in geometry, data representation and analysis, that fifth grade students in the initial education make. In order to detect the errors, we have carried out a research which is based on a sample of 500 students. We also have used information gathered from the teaching observation of the subject Methodology of teaching mathematics, carried out with the students from the Faculty of Pedagogy in Skopje. Taking into consideration that different mathematics textbooks are used in the schools in the

Republic of Macedonia, we have used tests of knowledge prepared by the teachers in order to render the research objective. Although they implement the same syllabus, the results of the instruction are, among other things, influenced directly by the textual didactic means, i.e. the textbook used in a particular school.

Further on, we identify the causes for the occurrence of each group of errors, and we offer a method and a procedure for their correction, which can be used to prevent the occurrence of these and similar errors that students of the future generations may make. We can have an extensive discussion about the need for a timely detection and correction of the errors made by students in mathematics instruction. However, without scrutinizing this matter, we feel that it is enough to emphasize that the timely detection and correction of the omissions in the mathematical knowledge is important because of:

– the concentric circles, which mathematics instruction is realized in, meaning that the errors, which are not corrected on time, are multiplied when learning new material, and

– the gaps in the mathematical knowledge are a great obstacle in the integration of the overall instruction in primary education, and especially the integration of the technical and the scientific instructional disciplines.

2. Research Design

The above stated defines the subject of the research, which is quite complex and covers:

– detecting errors made by the fifth grade students concerning topics in data representation, analysis and geometry,

– categorization of the errors,

– detecting the reasons for the errors and

– suggesting methods and procedures for elimination of the errors that students make.

Taking into consideration the subject of the research we formulate one main and three auxiliary hypotheses, which we think are the result of the education process nature:

a) Main hypothesis: *The errors that students make are accidental.*

b) First auxiliary hypothesis: *The errors are the result of the mathematics syllabus.*

c) Second auxiliary hypothesis: *The errors are the result of the development and presentation of the syllabus in the textbooks.*

d) Third auxiliary hypothesis: *The errors are the result of the realization of the syllabus by the teachers.*

In the course of the research, which was carried out on a random sample of 500 students from 23 primary schools, we used the following scientific methods to test the main and auxiliary hypotheses:

– *the statistical method* for testing the main hypothesis,

– *comparative analysis* for testing the auxiliary hypotheses and

– *systematization and analytic-synthetic method,* which is used in the process of systematization of the errors made by the students, as well as developing procedures for their elimination.

We used a *Questionnaire for the Teachers* to acquire information about the used textbooks and the teachers' opinions about the quality of the textbooks. The questionnaire also provided us with information concerning the teachers' engagement in the detection and correction of the students' knowledge gaps. We used *Tests* prepared by the teachers to detect the gaps in the students' knowledge and skills. Another insight came from the teaching observation of the students of the Faculty of Pedagogy "St. Kliment Ohridski" in Skopje. Since different schools use different textbooks, which cover the syllabus in different ways and through different methodological approaches, we have not used unified tests.

3. Research Results

The mathematics syllabus for fifth grade in the initial education includes four topics from the areas of geometry, data representation and analysis. The testing of the main hypothesis was carried out by applying the statistical method, whereas the testing of the first and second auxiliary hypotheses was conducted by making a comparative analysis, systematization and the use of the analytic-synthetic method. We used the statistical method combined with a comparative analysis, systematization and the analytic-synthetic method to test the third auxiliary hypothesis.

3.1. Testing the Main Hypothesis

The analysis of the tests taken by the students yielded the following results about the number of students who made three or more errors regarding a specific topic:

1) *Data Representation and Analysis* – 98 students,
2) *Measurement* – 102 students,
3) *Space Figures* – 99 students, and
4) *Plane Figures* – 97 students.

In order to examine the main hypothesis at the specific level of significance $\alpha = 0,01$, we test the hypothesis $H_0 : p \leq p_0 = 0,15$ against the alternative hypothesis $H_1 : p > p_0$,

which means that we have allowed a maximum of 15% of the students to make a negative result in each topic. For example, in terms of the topic *Plane Figures* we have $n = 500$ and $s_n = 97$, thus $\frac{s_n - np_0}{\sqrt{np_0q_0}} = 2,75$. Further on, at the level of significance $\alpha = 0,01$, from the table of normal distribution $\mathrm{N}(0,1)$, we get the quantile $z_{1-\alpha}$, to be $P_{H_0}\{\frac{S_n - np_0}{\sqrt{np_0q_0}} \geq z_{1-\alpha}\} = 0,01$, i.e. $z_{1-\alpha} = 2,33$. Since the calculated value $\frac{s_n - np_0}{\sqrt{np_0q_0}} = 2,75$ is greater than the critical value $z_{1-\alpha} = 2,33$, the hypothesis $H_0 : p \leq p_0 = 0,15$ is rejected i.e. the errors made by the students regarding the topic *Plane Figures* are not coincidental. The number s_n is greater than 97 for the other three topics, and all the remaining parameters are the same as for the topic *Plane Figures*, which leads us to the conclusion that also in these cases with a level of significance $\alpha = 0,01$, we should reject the hypothesis $H_0 : p \leq p_0 = 0,15$, i.e. we come to the conclusion that the errors made by the students regarding the other three topics are not coincidental as well. Subsequently, the results indicate that we should reject the main hypothesis and test the auxiliary hypotheses.

3.2. Testing the First Auxiliary Hypothesis

The syllabus for fifth grade (BRO, 2008) has the following structure:

– instructional goals;

– specific goals; for each topic with specified: goals, instructional content, terms that need to be learned, as well as activities and methods recommended for achieving a specific goal;

– didactic recommendations for the realization of the syllabus and

– methods and procedures for evaluating the achievements, i.e. testing the knowledge and skills of the students.

The syllabus is consistent for every topic. The activities and methods recommended for achieving specific goals, i.e. for learning the planned instructional content are completely adapted to the abilities of the students and certain contents are even under the level. The didactic recommendations for teaching certain topics are completely in tune with the general and specific goals of the syllabus, and this is also true for the recommended methods and procedures for evaluating the achievements of the students. Taking into account the previously mentioned, we reject the first auxiliary hypothesis: *The errors are a result of the mathematics syllabus.*

3.3. Testing the Second Auxiliary Hypothesis

In order to test the second auxiliary hypothesis we analyze some errors made by the fifth grade students regarding the topics discussed in Testing the Main Hipothesis.

3.3.1. Errors Regarding the Topic Data Representation and Analysis

Regarding the topic: data representation and analysis we came across a characteristic error in representing the data using a line graph, which is identified in the following problem and in problems of the same type:

Example 1. When solving the problem:

The table contains information about the daily sales of bikes in a department store during 80 consecutive work days:

Number of bikes	1	2	3	4	5	6	7	8
Number of days	1	5	8	6	9	12	5	4

Show the data using the both graphs: bar and line graph.

Asked to represent the information using a bar graph and a line graph, most of the students represented correctly the data given in the above table using a bar graph (Figure 1). However, when they represented the data using line graph, some of them did it as in Figure 2, which was incorrect, and some of them only set the axes in the line graph.

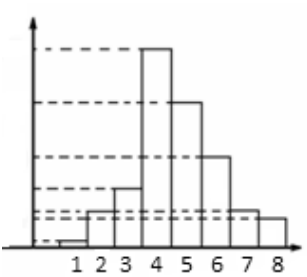

Figure 1

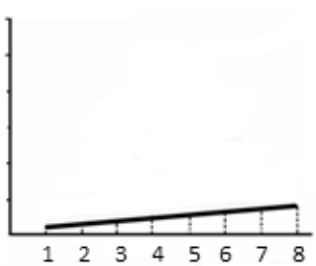

Figure 2

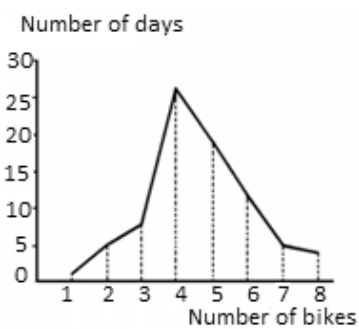

Figure 3

This error is a direct consequence of pattern thinking, which is imposed by the authors of the textbooks in use. Namely, although the third grade students have adopted the term *broken line*, when learning how to represent data by using line graph, all the textbooks (with no exception) in use present a *fixed image* that line graphs are used for only to present data with linear dependence (Stefanovski & Achovski, 2010, p. 154; Dimovski, Krsteska, Jordanovska, Dimitrievska & Paunoska, 2009, p. 143 – 153). The authors have not taken into account that: "...Pattern thinking is also related to the effect of the so-called "functional stability" according to which, the object is used only in the

given form, and a new property, which cannot be seen clearly from the conditions that define the problem, is not required... (Malčeski, 2010, p. 31)."

Naturally, the latter also applies to the learning of terms, especially if the same fixed image is consecutively repeated throughout two or more school years. In this case, it is important to mention that the incorrect adopting of the term line graph, even in cases when it is in a hidden form, will require additional instructional time to correct the fixed image of the students in the upper grades. This will definitely influence the attaining of new knowledge and skills in a negative way. It is necessary that the teachers timely correct this oversight, and give more examples of a line graph with a broken line, together with the examples presented in the textbooks in use, as in this case (Figure 3).

3.3.2. Errors Regarding the Topic Measurement

Regarding this topic, the research shows that the students make several types of errors. One of the most common errors is related to the conversion of the area measurement units, which can be seen from the following example.

Example 1. a) When converting square centimeters, the students frequently wrote:

i) $2dm^2 = 20cm^2$ *ii*) $4dm^2 = 40cm^2$ *iii*) $10dm^2 = 100cm^2$

iv) $2m^2 = 2000cm^2$ *v*) $7m^2 = 7000cm^2$ *vi*) $10m^2 = 10000cm^2$

b) When converting square decimeters, the students frequently wrote:

i) $3m^2 = 30dm^2$ *ii*) $8m^2 = 80dm^2$ *iii*) $6m^2 = 60dm^2$

It is safe to say that the basic reason for the errors in *i*), *ii*) and *iii*) in a) and the examples in b) is the fixed image about the relations between the length measurement units and the incorrect analogy made by the students. However, we cannot find a valid explanation for the errors to be made in *iv*), *v*) and *vi*) in a). The latter leaves us to conclude that the only reason for these errors is the inadequate methodological approach given in the textbooks in use when adopting the area measurement units, and which is non-critically accepted by teachers. Teachers accept the textbooks in a noncritical way. Namely, when expressing the same area in different measurement units not all textbooks offer solved examples, but they immediately present equations of the following type:

i) $2dm^2 = \_\_ cm^2$ *ii*) $4dm^2 = \_\_ cm^2$ *iii*) $10dm^2 = \_\_ cm^2$

iv) $2m^2 = \_\_ cm^2$ *v*) $7m^2 = \_\_ cm^2$ *vi*) $10m^2 = \_\_ cm^2$

vii) $3m^2 = \_\_ dm^2$ *viii*) $8m^2 = \_\_ dm^2$ *xi*) $6m^2 = \_\_ dm^2$

Clearly, this methodological oversight can be corrected if the authors of the textbooks, and the teachers as well, present completely solved examples to the students, such as:

$2dm^2 = 2 \cdot 1dm^2 = 2 \cdot 100cm^2 = 200cm^2$

$3m^2 = 3 \cdot 1m^2 = 3 \cdot 100dm^2 = 300dm^2$

$2m^2 = 2 \cdot 1m^2 = 2 \cdot 100dm^2 = 200dm^2 = 200 \cdot 1dm^2 = 200 \cdot 100cm^2 = 20000cm^2$

At the end of this example, we should mention that one of the reasons for the occurrence of errors of this type, among other things, is the inadequate methodological education of the elementary school teachers. The analysis of the literature used at the faculties of pedagogy in our country, shows that the students learn only general methodology of teaching mathematics, not specialized methodology of teaching mathematics for elementary education. It is also worth mentioning that almost no attention is dedicated to the mathematical problems. All future teachers should know the following about mathematical problems: *the concept of a mathematical problem, types of mathematical problems (classification of the mathematical problems), the functions of the problems, the methodology of solving problems, methods for solving problems (synthetic and analytic method)* and *the role of the counterexamples in mathematics instruction.*

In order to attain the instructional goal: *use of formulae for calculating the area of a rectangle, a square, a cuboid and a cube*, during the teaching observation some of the teachers use tasks like one in the following example.

Example 2. *When one side of the square is multiplied by three and the other one by two, we get a rectangle with an area of* $96cm^2$ *. Calculate how larger is the perimeter of the rectangle when compared to the perimeter of the square?*

When solving this and similar tasks, the students should remember that when the multipliers change, the product changes as well, a principle used indirectly in Task 7, p. 149 in (Stefanovski et al., 2010), or they could also use the already learned algorithm for the value of a rectangle area (Stefanovski et al., 2010, p. 146 – 147). Instead of using this knowledge, some students transforme the area of the rectangle in multipliers, i.e. they write: $96 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, and then try to determine all the possible rectangles with an area of $96cm^2$, and to find the side of the square by trial and error in order to get finally the required answer.

Clearly, this approach is wrong, it requires a lot of instructional time and *does not allow inter-subject integration of the instruction*, which is the primary task of every school subject. In order to correct this error, it is necessary for the mathematics textbooks to offer real integration of the instruction, which the teachers will try to put in practice. It is desirable that the teachers solve a similar example completely,

before instructing the students to do individual work. The solution of the example can be presented in a short version, i.e. the teachers do not have to highlight every stage of the methodology of solving problems. For example, the solution of the task can be presented as:

A rectangle with an area of $96cm^2$ *consists of six squares which are the same as the square given in Figure 4. So the area of the given square is* $96:6=16cm^2$. *Since, the area of the given square is* $P=a\cdot a$, *and furthermore since* $a\cdot a=16\ cm^2$, *we get that* $a = 4cm$. *The perimeter of the rectangle is* $O=10a=40cm$, *and the perimeter of the square is* $L'=4a=16\ cm$. *Hence, the perimeter of the rectangle is* 24 *cm larger than the perimeter of the square.*

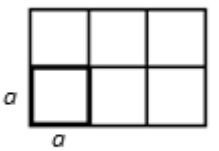

Figure 4

3.3.3. Errors Regarding the Topic Space Figures

In terms of this topic, the basic errors refer to the spatial images, i.e. representations of three-dimensional forms using two-dimensional drawings, as well as "reading" the drawings where three-dimensional forms are shown.

Example 1. When solving the task:

Which of the cubes is the same as the unfolded cube in Figure 5?

less than a third of the students answer that it iss cube a), which is the correct answer, whereas over two thirds of the students give an incorrect answer. One of the basic reasons for this type of errors is due to the fact that most people have problems with images in three-dimensional space,

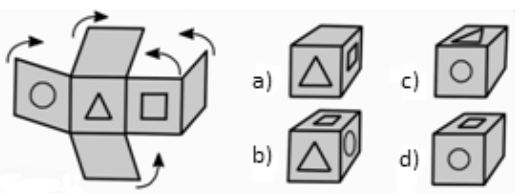

Figure 5


Figure 6


Figure 7


Figure 8

which has been confirmed with a great number of experiments. Hence, the "battle" with this type of errors should be fought by learning a great number of examples. This will enable students to represent three-dimensional forms in a plane, and to use the representations correctly. Several such examples are presented in the following tasks in (Gogovska & Malčeski, 2012).

a) The students are handed worksheets with Figure 6 and are asked the following question: *How many cubes are needed to fill out the cuboid container?* Further on, the students are additionally asked to elaborate the process they have come up to the solution.

We believe that apart from improving the spatial concepts, these and similar tasks can be a good introduction to the term volume of a square. The latter is great reason that tasks of this type should be used in school practice.

b) Similar goals as in the previous example are attained if the students are asked analogous questions regarding Figure 7. However, taking into consideration that in this case the cuboid is not visibly marked as in example a), the practice shows that even the students who have developed advanced spatial concepts, should use Figures 6 and 7, consequently

c) An analogous result, as in the two previous examples, can be attained if the students are asked the same question referring to Figure 8, where instead of a cuboid the question will refer to a cube. In this case, we can also ask how many cubes cannot be seen.

3.3.4. Errors Regarding the Topic Plane Figures

Taking into account that while learning this topic, the students do not attain a great amount of operational knowledge, which is crucial for the testing carried out by the teachers, we did not come across a great amount of errors. Nevertheless, we might say that when it comes to a drawing of an equilateral triangle with sides of 6 *cm*, which needs to be classified according to the angles and according to the sides, a certain number of students answer that it the *triangle isequilateral* in both cases, or *equilateral with respect to the angles, and isosceles with respect to the sides.* In order to correct these and similar mistakes, the students need presenting by drawings of:

a) an acute, obtuse and right scalene triangle,

b) an acute, obtuse and right isosceles triangle and

c) an equilateral triangle, which the authors need emphasizing, is acute.

Naturally, this should be carried out during revision classes, *and the students should be trained to use the scientific method of classification from their earlyt age.*

We have to note that errors of this type occurred while solving a) of the following example.

Example 1. Draw triangles with the given dimensions and determine their type according to the angles and according to the sides:

a) triangle *AOB*: *AO* = 3 *cm, OB* = 3 *cm, BA* = 3 *cm.*

b) triangle *ABC*: *AB* = 5 *cm, BC* = 3 *cm, CA* = 3 *cm.*

c) triangle *CET*: *CE* = 5 *cm, ET* = 7 *cm, TC* = 4 *cm.*

3.3.5. Evaluation of the Currently Used Textbooks by the Teachers

The previous discussion related to the analysis of the errors made by the students indicates that we should accept the second auxiliary hypothesis.

Nevertheless, before deciding whether to accept or reject the second auxiliary hypothesis, we will analyze the results of the survey conducted on a sample of 105 elementary school teachers. The results of the survey bring us to the following conclusions:

a) only 9 teachers, or 8.57% believe that the currently used textbooks cover the syllabus completely, whereas 91.43% or 96 teachers do not share this opinion;

b) only 12 teachers, or 11.43% believe that the currently used textbooks encourage students to learn independently, whereas 93 teachers, or 88.57% believe that the currently used textbooks do not encourage students to learn independently;

c) only 15 teachers, or 14.29% believe that the textbooks are adapted to the psycho-physical abilities of the students, whereas 85.71% believe that they are partially or not adapted at all to the psycho-physical abilities of the students;

d) a significant percentage, i.e. 51.43% of the surveyed teachers believe that the material in the textbook is divided into complete topics, and the language, style and presentation of the material are adapted to the needs of the students, something which 34.28% of the teachers partially agree with, whereas 14.29% of the teachers believe that the textbooks do not meet this criterion as well;

e) 42.86% of the teachers believe that there is a complete correlation of the material to the other school subjects, 39.05% of the teachers believe that there is a partial correlation, and 18.09% of the teachers believe that the textbooks do not correlate to the other school subjects.

f) 34.28% of the teachers think that the illustrations in the textbooks are good, 42.86% are partially satisfied with this component, and only 22.86% of the teachers believe that the textbooks fail to meet the demands in terms of this parameter;

g) the vast majority of the teachers, i.e. 82.86% partially agree that the textbooks have an adequate didactic apparatus (questions, tasks, simple experiments, etc.) which allows guidance and encourages independent learning, evaluation, generalization, revision and practical use of the acquired knowledge and skills.

Taking into consideration the analysis of the errors made by the students, presented in *Errors Regarding the Topic Data Representation and Analysis, Errors Regarding the Topic Measurement, Errors Regarding the Topic Space Figures* and *Errors Regarding the Topic Plane Figures*, the reasons for the occurrence of these errors, as well as the results of the survey, we conclude that we should accept the second auxiliary hypothesis, i.e. we accept that: The errors are the result of the incorrect development and presentation of the syllabus in the textbooks.

3.4. Testing the Third Auxiliary Hypothesis

The third auxiliary hypothesis is tested on the base on the results of the survey, i.e. based on the analysis of the answers of the teachers. Namely, the survey allows us to conclude that:

a) only 24 teachers, or 22.86% correct the errors made by the students when learning new material regularly. The same number of teachers do not correct the errors of their students at all, while 57 teachers, or 54.28% do this occasionally;
b) when using previously acquired knowledge, 40 teachers, or 38.10% correct the errors of the students regularly, 49 teachers, or 46.67% do this occasionally, whereas 15.23% do not correct the errors of the students at all;
c) unbelievably high percentage 72.38% of the surveyed teachers do not systematize the errors of the students and do not engage in activities for their elimination, while only 27.62% do this occasionally.
d) all teachers allow the parents and the students to see the results of the tests. Thus, they actually realize only their formal legal obligation for transparent evaluation;
e) not a single teacher provides the parents and the students with complete feedback about the identified gaps in the knowledge with adequate comments about the errors and directions for their elimination. 49.52% of the teachers have stated that they do this occasionally;
f) only 15.23% of the teachers exchange opinions and experiences about the errors made by the students and develop methods and procedures for their elimination through collaboration.

The discussion above allows us to conclude that we should accept the third auxiliary hypothesis, i.e. we should accept that: The errors are the result of the realization of the syllabus by the teachers.

Taking into consideration the previously stated, we might conclude that the teachers are the only reason why the students make certain errors. However, this conclusion is wrong because, as we have already said, the textbooks are an important factor, as well. We should also look into the training of the students at the faculties of

pedagogy in the Republic of Macedonia. Namely, the analysis of the study programs regarding the methodology of teaching mathematics indicates that the training is incomplete and inadequate. To be exact, the analysis of the textbook (Ачовски, 1998), which is the basic recommended literature for the students at the faculties of pedagogy regarding the subject Methodology of Teaching Mathematics for Grades I-IV (V), indicates that:

a) no attention has been devoted to mathematical thinking and its properties. Hence the future teachers have not been trained to encourage and develop thinking elasticity, depth, rationality, extent, as well as critical thinking. They have also not been trained to encourage the development of positive thinking patterns, i.e. in the process of adopting the technical procedures
b) the scientific methods observation and experiment are almost non-existent, whereas the following scientific methods: comparison, analysis, synthesis, generalization, systematization and abstraction are inadequately presented and there is a tendency to render them subjective, treating them as mental activities;
c) the different types of conclusions, especially analogy, are inadequately and incompletely explored. There is no instance where the authors state that conclusions based on an analogy must be subject to an absolute proof. The same applies to induction as a scientific method. Regarding this matter, no differentiation has been made between the complete and incomplete induction;
d) the mathematical terms and statements are incompletely and inadequately explained. No attention has been devoted to their teaching methodology at all;
e) the didactic principles of mathematical instruction are incompletely elaborated. No attention has been devoted to the principle of learning, which is crucial for the mathematical instruction in the elementary education;
f) when learning mathematical problems, the functions of the problems are not sufficiently elaborated. As far as the methodology and the problem solving methods are concerned and elaborated, only the so-called arithmetic tasks. No counterexamples have been included;
g) in terms of the monitoring, testing and evaluation of the knowledge of the students, very little attention is given to the types and forms of testing. Sufficient attention has not been devoted to the principles, functions and goals of testing. The feedback, which is crucial for detecting the gaps in the knowledge of the students and their elimination, is completely neglected.
h) almost no attention is given to the evaluation and assessment, as well as to the instruments for assessing the achievements of the students. As far as the analysis

of the results of the monitoring and evaluation of the achievements of the students is concerned, we may conclude that the future teachers do not have even the slightest idea that it is necessary for successful monitoring of the development of their students.

4. Conclusion

This paper focuses on the gaps in the knowledge and skills related to geometry, data representation and analysis of the fifth grade students. We listed the reasons behind the identified errors occur as well as suggested procedures to correct and overcome them. As we have seen, there are many objective reasons for the occurrence of the errors which need to be located outside the classroom. The analysis of the position of the educational system at all and the mathematics course within the system shows that:

- although the syllabus for the fifth grade has been reduced in order to "unburden" the students, there are no visible effects of this action. The above is a result of the constant changes of the syllabi, without prior analysis and answer to the questions what, why and how to change the syllabi, as well as how these changes should be implemented in the textbooks,
- the so created syllabi, and the effort to make a universal concept for a textbook, which applies both for Mathematics and Pedagogy, and is also a result of the so-called pedagogization of the educational process, have an extremely negative impact on the mathematics textbooks which are currently used in the Macedonian educational system. Even some of them do not cover important content of the syllabus, and
- there is a tendency to compensate the shortcomings of the textbooks with workbooks, an action that results in decreased use of the textbooks and goes in favor of the wrong popular opinion that mathematics is solving tasks and nothing more.

Consequently, we can conclude that the reasons for the gaps in the knowledge and skills of the students are not just a result of the work of the teachers and the students, which is why it is not fair to hold the teachers responsible for the poor results and their so-called permanent mathematical specialization, which is carried out through various seminars and training. The latter is especially indicative, since in the past two decades there have been numerous seminars for modernizing education (step by step, interactive instruction – active learning, using standards for assessment, etc.), without a comprehensive evaluation of the syllabi and the textbooks, as well as an evaluation whether they are adequate to the syllabus.

Taking into consideration the previously stated, before proceeding to any further changes, it is necessary to conduct a thorough analysis of the syllabi and the textbooks, as well as the segments of the educational system where the future teachers are trained. This means that any future changes need to correspond to the results from the mentioned analyses.

REFERENCES

Ачовски, Д. (1998). *Методика на наставата по математика од I – IV одделение*. Скопје: Педагошки факултет „Св. Климент Охридски". [Achovski, D. (1998). *Methodology of the Mathematical Education I – IV.* Skopje: Faculty of Pedagogy St. Kliment Ohridski.]

Grozdev, S. (2007) *For High Achievements in Mathematics. The Bulgarian Experience (Theory and Practice)*. Sofia: ADE. (ISBN 978-954-92139-1-1)

Димовски, Д., Крстеска, Б., Јордановска, Р., Димитриевска, С. & Пауноска, В. (2009). *Математика за V одделение во деветгодишното основно образование*. Скопје: МОН. [Dimovski, D., Krsteska, B., Jordanovska, R., Dimitrievska, S. & Paunoska, V. (2009). Skopje: *Mathematics for fifth grade in the nine year primary education.* Skopje: MON.]

Князева, Е., Гроздев, С., Георгиева, М. & Гълъбова, Д. (2013). *Синергетичният подход във висшето педагогическо образование (Върху примери от дидактиката на математиката)*. В. Търново: СЛОВО. (ISBN 978-954-439-986-3)

Glavche, M., Malčeski, R. & Anevska, K. (2015). Errors made by students from fifth grade in Macedonia while studying mathematics (pp. 265-274). Varna: Proceedings of the Forty Fourth Spring Conference of the Union of Bulgarian Mathematicians.

Gogovska, V., & Malčeski, R. (2012). The creative teacher, factor for improving mathematics instruction. Busan: Proceedings of the 7-th MCG Conference.

Малчески, Р. (2010). *Методика на наставата по математика*. Скопје: ФОН Универзитет. [Malčeski, R. (2010). *Methodology of the mathematical education*. Skopje: FON University.]

БРО (2008). *Наставна програма за V одделение во деветгодишното основно образование*. Скопје: МОН. [BRO (2008). Mathematics Syllabus for V Grade in the nine year primary education, Skopje: MON.]

Стефановски, Ј. & Ачовски, Д. (2010). Математика за *V одделение во деветгодишното основно образование.* Скопје: МОН. [Stefanovski, J. & Achovski, D. (2010). *Mathematics for fifth grade in the nine year primary education,* Skopje: MON.]

ДОПУСКАНИ ГРЕШКИ ОТ ПЕТОКЛАСНИЦИ В РЕПУБЛИКА МАКЕДОНИЯ ПО МАТЕМАТИЧЕСКИ ТЕМИ ПО ГЕОМЕТРИЯ, ПРЕДСТАВЯНЕ НА ДАННИ И АНАЛИЗ

Резюме. В процеса на учене е естествено да се повявят пропуски, но ако не се идентифицират и отстранят навреме, те се мултиплицират по-нататък в курса на преподаване. Следователно една от задачите на обучението по математика е откриването на допуснатите от учениците грешки и тяхното навременно отстраняване. Допускани грешки от петокласници по аритметика и алгебра са изучени в (Glavche, Malčeski & Anevska, 2015). В тази статия са представени резултатите от научното изследване на някои от най-честите грешки, които допускат петокласниците в Република Македония. В допълнение статията предлага систематичен преглед на идентифицираните грешки и на причините за тяхната поява. Тя предлага също така процедури за елиминиране на идентифицираните грешки.

✉ **Dr. Metodi Glavche, Assoc. Prof**
Faculty of Pedagogy
Ss. Cyril and Methodius University
Skopje, Macedonia

Prof. Risto Malčeski, DSc
Faculty of Informatics
FON University
Skopje, Macedonia
E-mail: risto.malceski@gmail.com

Dr. Katerina Anevska, Assoc. Prof.
Faculty of Informatics
FON University
Skopje, Macedonia
E-mail: anevskak@gmail.com