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*Educational Technologies*
*Образователни технологии*

# EDUCATIONAL RESOURCES WITH PERCENTAGES FOR THE DEVELOPMENT OF THE VISUAL ESTIMATION

**Toni Chehlarova**
*Institute of Mathematics and Informatics, Bulgarian Academy of Science – Sofia (Bulgaria)*

**Abstract.** Computer models of tasks related to percentage are presented. The files are created with the dynamic software GeoGebra and are provided in the Virtual Mathematics Laboratory, developed by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. The goal is to create conditions for the development of the visual estimation of a percentage, which also supports the understanding of the concept. The computer models contain rectangles and circles. Help and feedback are provided. Options for obtaining a new example and feedback are described. Emphasis is placed on the analogy of the tasks in the four presented topics, each of which contains four tasks. An assessment of the resources is presented, obtained from an anonymous survey with teachers from different subject areas and teaching at different educational levels. The assessment is based on the criteria of easy technical orientation, design, usefulness, entertainment, motivation to solve. The simultaneous development of digital and mathematical competence when working with these resources is commented on, as well as the possibility of their use in STEM centers. Ideas for expanding the resources for checking and developing the percentage calculator in several directions are described.

*Keywords:* computer model; visual estimation; self-assessment; mathematics education; GeoGebra; STEM

## 1. Introduction

Teachers and researchers have assessed the difficulties of many children in learning fractions and more generally, the difficulties in further learning mathematics as a result of this, as well as the difficulties of adults in a number of professions (Lortie-Forgues et al., 2015). Research and the search for ideas to support the formation of competence in students in working with a part of a whole, and in particular the presentation with a percentage, continue (Barbosa Vale, 2021), (Cardoso & Mamede, 2025), (Chehlarova, 2019), (Chehlarova, 2020), (Kiliene, 2023), (Schwarzmeier & Obersteiner, 2025), (Ari et al., 2024) and (Van den Heuvel-Panhuizen, 2003). The study of percentages in Bulgaria is implemented with students at the age of 13 and continues to be a challenge for them, which is evident from the results of both national external assessments and international studies. The problems are related to understanding the concept and solving the basic tasks related to percentages. An essential point in forming the concept of percentage is the ability to evaluate a part of a whole, expressed as a percentage. For this purpose, textbooks and manuals contain single tasks in which a figure is divided into equal parts and the student is asked to determine the percentage of a part of the whole colored in a certain way. For the understanding and development of the ability to determine percentage with some accuracy, it is important to provide conditions for exercises, as well as to emphasize the connection with fractions and decimal fractions.

It is natural to use various technical devices for calculations in everyday life. It is important to form skills for an approximate estimate of an expected result, which will not allow a technical error, for example, when entering data or a technical malfunction of the relevant device, to use the wrong result for the relevant purposes. Both visual assessment and the skill of rough calculation are important.

Here we will present computer models of tasks related to percentage. They are created by analogy with educational resources related to fractions (Chehlarova, 2017). The files were created with the dynamic software GeoGebra (Hohenwarter et al., 2009) and are provided in the Virtual Mathematics Laboratory developed by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

## 2. Computer models with rectangles for the development of the visual estimation of percentage

The Virtual Mathematics Laboratory has developed two topics that provide conditions for the development of the visual estimation of a percentage through models with rectangles. The tasks in the topic at note[1] are accurate to 10%. The text of the first task is "What percent of the rectangle is colored purple?". A rectangle is constructed in the provided file, part of which is colored purple (fig. 1). There are two hide/show checkboxes

– “отговор” (answer) and “помощ” (help). After clicking on the answer checkbox, a record of the searched percentage is displayed/hidden, and after clicking on the help checkbox, a division of the rectangle into 10 equal parts by vertical segments is displayed/hidden.

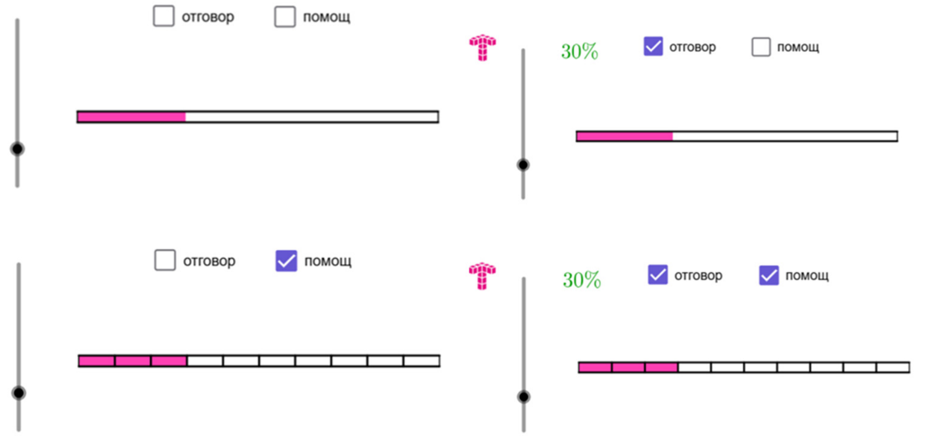


**Figure 1.** A rectangle coloring task to determine a percentage with an accuracy of 10

The provided file also includes a slider parameter with which a new example can be created.

In task 2, creating a new example is done automatically with the “нов пример” (new example) button. As a result, a value is randomly selected, in this case a natural number from 1 to 10 (fig. 2).

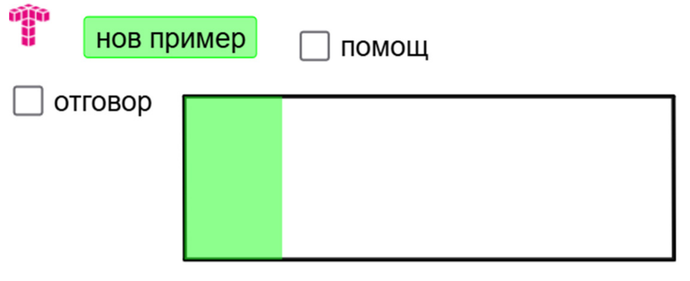


**Figure 2.** Creating a new example randomly with a button

Help is again provided by dividing the rectangle into 10 equal parts with vertical lines, and feedback can be obtained after using the show/hide "отговор" checkbox (fig. 3).

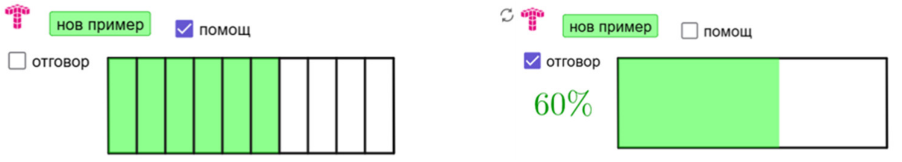


**Figure 3.** Providing help and feedback

There are two reverse tasks – Task 3 and Task 4, in which the percentage is given, and it is necessary to color the part of the rectangle corresponding to this percentage (fig. 4).

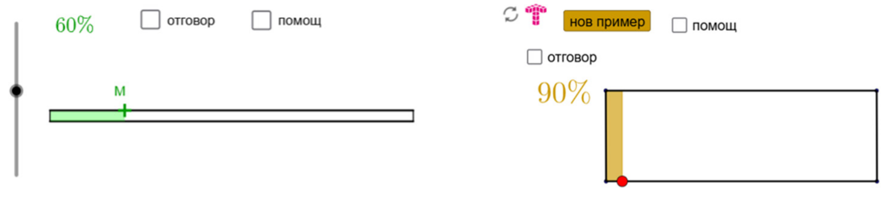


**Figure 4.** Inverse task with coloring a rectangle to within 10

Moving points are used to achieve the coloring. To get help and feedback, the corresponding hide/show checkmarks are used (fig. 5).

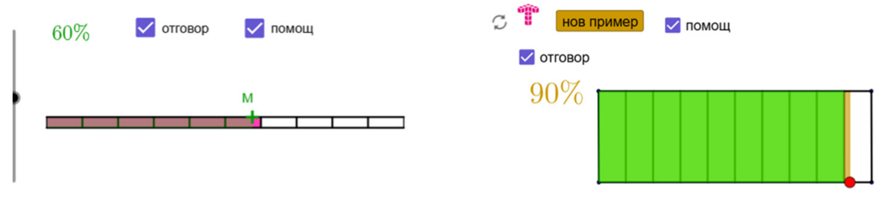


**Figure 5.** Providing help and feedback on the inverse task with rectangle coloring

In Task 3, the next example is created with a slider parameter, and in Task 4 – automatically with a new example button.

This topic uses percentage values with an accuracy of 10.

At note[2], similar tasks follow, but when working with percentages with an accuracy of 1 (fig. 6).

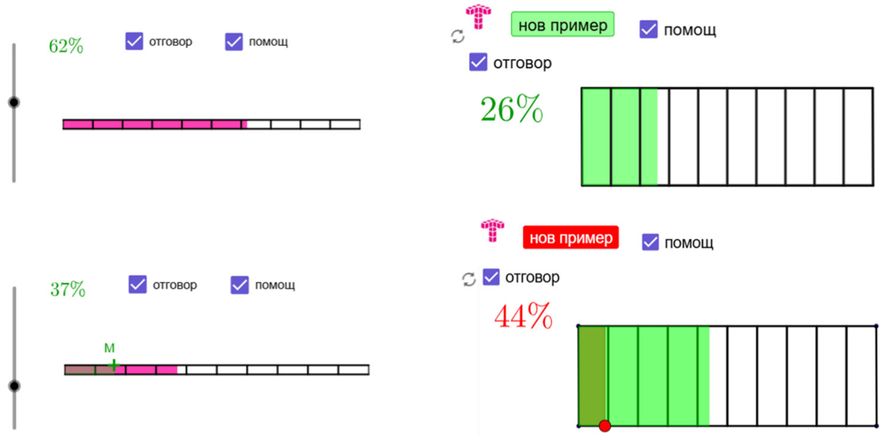


**Figure 6.** Tasks with a rectangle for the visual estimation with an accuracy of 1

This analogy in technical maintenance allows for easy and quick orientation in dynamic files and focusing on the content element.

There is also an analogy in the tasks for the visual estimation of a percentage, in which circles and sectors are used.

## 3. Computer models with circles for the development of the visual estimation of a percentage

The tasks at note[3] use percentages with accuracy up to 10, and at note[4] – with accuracy of 1. We will illustrate with the tasks in the topic with an accuracy of 1. In Task 1, you have to estimate what percentage of the circle is colored green (fig. 7).

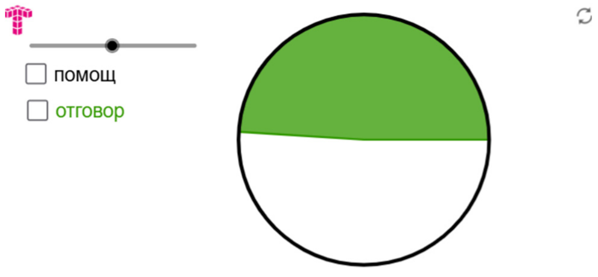


**Figure 7.** Circle coloring task to determine percentage

The help provides dividing the circle into 10 equal sectors (fig. 8). The feedback is by displaying the percentage, after using the show/hide "отговор" checkbox (fig. 10).

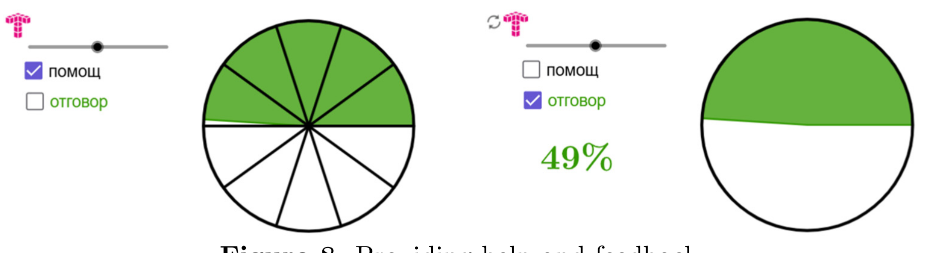


**Figure 8.** Providing help and feedback

To get a new example, the slider-parameter in the file is used. In task 2, a new example is obtained via the new example button (fig. 9).

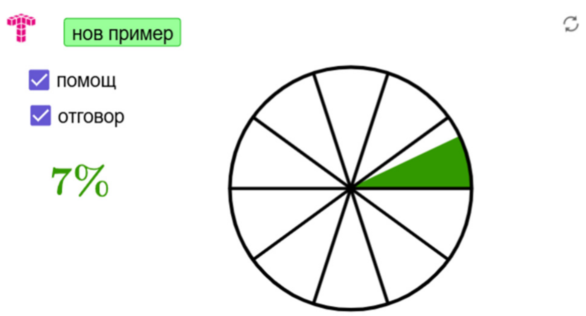


**Figure 9.** Creating a new example randomly with the button

Tasks 3 and 4 are for coloring a part of the circle that corresponds to the percentage specified in the condition. Coloring is realized by moving a point from the corresponding circle. For help and feedback, the hide/show checkboxes are also used here (fig. 10).

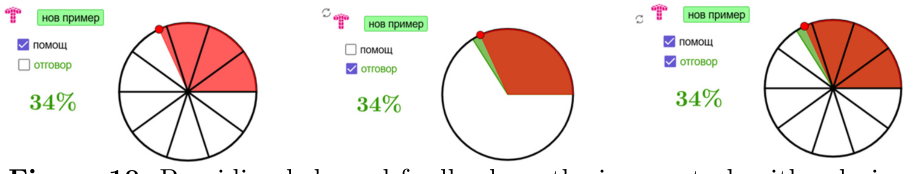


**Figure 10.** Providing help and feedback on the inverse task with coloring a sector in a circle

In Task 3, a new example is obtained with a slider parameter, and in Task 4, after pressing the button for a new example, a number is randomly selected with an accuracy of 1.

The analogy with the tasks where a rectangle is used as a model also provides a reduction in the time for technical orientation here, as well as facilitates the additional use of some of the files, for example, to provide observation and support for understanding and orientation in percentages. For example, for a short time, different colorings can be observed, along with the recording of the percentage corresponding to this coloring.

Table 1 presents data on the tasks in the 4 topics described, which shows the analogy in their composition.

**Table 1.** Tasks data

| | **Rec-tangle** | **Circle** | **Accu-racy 10%** | **Accu-racy 1%** | **For visual estima-tion of percen-tage** | **For coloring of percen-tage from a figure** | **New example with a slider** | **New example with a button** |
|---|---|---|---|---|---|---|---|---|
| **524-1** | + | | + | | + | | + | |
| **524-2** | + | | + | | + | | | + |
| **524-3** | + | | + | | | + | + | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 524-4 | + | | + | | | + | | + |
| 525-1 | + | | | + | + | | + | |
| 525-2 | + | | | + | + | | | + |
| 525-3 | + | | | + | | + | + | |
| 525-4 | + | | | + | | + | | + |
| 526-1 | | + | + | | + | | + | |
| 526-2 | | + | + | | + | | | + |
| 526-3 | | + | + | | | + | + | |
| 526-4 | | + | + | | | + | | + |
| 527-1 | | + | | + | + | | + | |
| 527-2 | | + | | + | + | | | + |
| 527-3 | | + | | + | | + | + | |
| 527-4 | | + | | + | | + | | + |

## 4. Resource assessment

An anonymous survey using GoogleForm was conducted to evaluate the presented resources. Twenty-eight teachers from different subject areas and teaching at different educational levels participated in the survey. Their age characteristics are presented in fig. 11, and the results – in fig. 12.

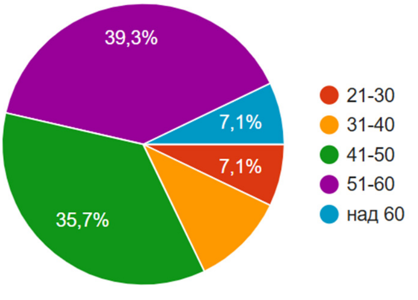


**Figure 11.** Age characteristics of the respondents

The evaluation criteria are: easy technical orientation, design, usefulness, entertainment, motivation to solve. Usefulness, entertainment and motivation to solve were highly rated. Some of the respondents had technical difficulties when working with the resources and the reason is that some of them do not have the skills to work with dynamic resources.

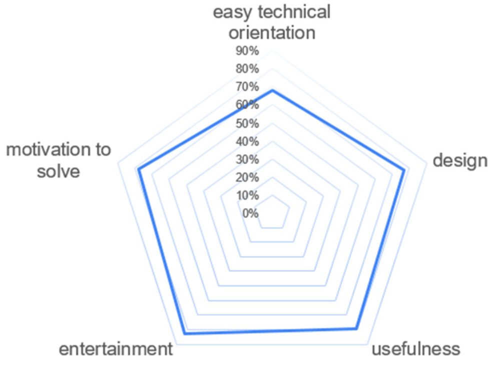


**Figure 12.** Evaluation

Pilot observations show that the desire for self-testing and self-assessment of the visual estimation is high, regardless of age.

## 5. Discussion and conclusion

The possibilities for continuing to create conditions for the visual estimation are in several directions. The inclusion in computer models of: time counting or limiting the time for work; number of examples counting or limiting the number of examples for work; assessing the distance from the correct answer; specifying a final grade will allow to easily organize competitions, which are an additional incentive for self-development. These are also part of the possibilities for further research on the topic. Models with rectangles and circles are included here, with the possibility of easy orientation in dividing them into 10 equal parts. The use of other figures (by analogy with resources for fractions such as Escher-style parquet tiles, Mondrian-style tasks, use of polyominoes, etc.), as well as a different way of coloring a part of the whole are another challenge and, accordingly, the next step in creating educational resources related to the visual estimation when using percentage estimation (Chehlarova, 2013), (Chehlarova, 2017) and (Chehlarova, 2019a).

The use of these resources contributes to the simultaneous development of digital and mathematical competence. They can also be used when working in STEM centers, which are being built in every Bulgarian school and are related to increasing the motivating factors related to self-confidence and the usefulness of mathematics education (Karashtranova, Goldreich & Borisova, 2024). In the context of understanding STEM and STEAM (Chehlarova, 2024), when using the proposed content, STEAM(2) is implemented, and when including other models, for example, parquet tiles in the style of Escher – STEAM(3) , i.e. the subject areas of mathematics, technology and art are included.

### *Acknowledgements*

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**NOTES**

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2. https://cabinet.bg/index.php?contenttype=viewarticle&id=525
3. https://cabinet.bg/index.php?contenttype=viewarticle&id=526
4. https://cabinet.bg/index.php?contenttype=viewarticle&id=527

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✉ **Prof. Toni Chehlarova, Dr.**
ORCID iD: 0000-0001-8472-5217
WoS ResearcherID: AAW-7402-2021
Institute of Mathematics and Informatics,
Bulgarian Academy of Science,
Bl. 8, Acad. G. Bonchev St.
1113 Sofia, Bulgaria
E-mail: toni.chehlarova@math.bas.bg

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# THE IMPACT OF USING GEOGEBRA ON UNDERSTANDING QUADRATIC FUNCTIONS AND EQUATIONS FOR TENTH-GRADE STUDENTS

**Erëmirë R. Aliu[1], Shpëtim Rexhepi[1], and Egzona Iseni[1]**
[1]*Mother Teresa University, Skopje, North Macedonia*

**Abstract.** This study explores the impact of mathematical software, specifically GeoGebra, on tenth-grade students' understanding of quadratic equations and functions. The research was conducted in two classes with similar academic levels: one class (X/3) was taught using traditional methods, while the other (X/4) integrated mathematical software into the learning process. The objective was to analyze how these tools influence students' conceptual understanding, error reduction, and overall engagement in mathematics. The study identifies common difficulties and misconceptions students face while learning quadratic equations and functions. Various examples illustrate the errors encountered and highlight strategies to avoid them. The integration of mathematical software provided students with a more interactive and intuitive learning experience, significantly improving their problem-solving abilities. To assess the impact, a comparative analysis was performed using evaluation tests, questionnaires, and student interviews. The results revealed that students in class X/4 performed better in solving quadratic equations and graphing quadratic functions compared to their peers in class X/3. The Chi-square statistical analysis confirmed that the use of mathematical software positively influenced students' comprehension and accuracy in mathematical problem-solving. These findings emphasize the importance of incorporating technology in mathematics education to enhance conceptual understanding and engagement. The study suggests that educational institutions should integrate mathematical software into their curricula to foster a more effective learning environment. Future research can explore its long-term impact on mathematical proficiency and its application in other areas of mathematics.

*Keywords*: quadratic equations; quadratic functions; GeoGebra; student difficulties

## 1. Introduction and review literature

During the work with the students, we noticed that the motivation for them to learn mathematics in general was not great, and during this work, we tried to approach mathematics differently, especially for the subject of equations and quadratic functions. During the development of the lessons, we tested two classes: in one, we explained equations and quadratic functions using the traditional method, while in the other, we taught the same topic through the use of mathematical software such as GeoGebra.

This paper aims to enhance student's success with equations and quadratic functions by integrating mathematical software tools such as GeoGebra, into classroom instruction. Additionally, we aim to boost students' motivation for learning mathematics, particularly in the context of equations and quadratic functions, through the use of mathematical software.

Mathematics, as a science, has practical applications in solving a wide range of everyday challenges. Equations and quadratic functions are a basic part of mathematics because, during the advancement in learning mathematics, the equations and quadratic functions will be an inseparable part of this process, so a good understanding of equations and quadratic functions is very important for students. After learning linear equations and functions from students, quadratic equations and functions are an advanced topic in mathematics, which falls under the more difficult type of equations and functions. Therefore, for easier understanding and concretization of exercises related to quadratic equations and functions, various mathematical software such as GeoGebra, etc., are used at this time. By using this mathematical software, the learning of this subject becomes very attractive, and the problems are understood very easily. The technology and the time we are living in make this easier because we can also use the resources from the internet for additional lessons. (Kamberi et. al 2022).

Knowing the great importance of quadratic equations and functions, many researchers have conducted studies on learning the quadratic equation and have pointed out that it is an important and interesting research object. There are many scientific works on this topic and each one has come to different conclusions depending on the research type. The primary focus of this study was to elicit a group of high school students' conceptions of quadratic equations with one unknown while considering concept definition and images as theoretical frameworks. The data initially showed that students could not provide a proper definition of quadratic equations with one unknown, and their definitions were not consistent with the formal (standard) definition of quadratic equations.

Students tried to define quadratic equations by stating some properties, which are valid for all equation concepts, instead of stating properties of quadratic equations. (Kabar 2018). The results indicate that most of the students used the factorization technique to solve quadratic equations. This result supports (Bossé & Nandakumar 2005), who claimed that a large percentage of the students preferred to apply the factorization techniques to find the solutions of quadratic equations. Also, in parallel with the results of (Bossé & Nandakumar 2005), the result of this study revealed that factoring the quadratic equations was challenging when they were presented to students in non-standard forms and structures (Didis & Erbas 2015). The results point to the need to create a new item on the research agenda for the international mathematics education research community: if quadratic equations are to remain an important component of lower and upper secondary mathematics curricula, then research is needed to guide teachers on how students think about quadratic equations, and especially on what can be done to help teachers improve students' understanding of variables in this context (Vaiyavutjamai et. al. 2005).

Students get confused when quadratic function concepts are presented in different ways they are not used to. The structure $y = ax^2 + bx + c$ (where $a \neq 0$ and $a$, $b$ and $c$ are constants) is the standard form of a quadratic function form revealing the location of the $y$-intercept $(0, c)$. The vertex form: $y = a(x - p)^2 + p$ distinctly highlights the turning point of the parabola (vertex) represented by $V(p, q)$. Lastly, the factored form: $y = a(x - x_1)(x - x_2)$ indicating the position of the $x$-intercept $(x_1;\ 0)$ and $(x_2\ ;\ 0)$ (Mutambara et. al 2019). Based on the results and discussion of the research, the use of dynamic mathematical software as a learning tool for the topic of quadratic functions proves to be effective.

The media is deemed valid for classroom use based on ratings from media and material experts, as well as feedback from field practitioners and students. The software is superior to conventional teaching methods, as it allows students to effortlessly interpret the graphical representation of quadratic equations, enabling them to formulate broader generalizations. This automatically increases the student's learning achievement and enables math to be a more exciting subject (Barraza Castillo et. al 2014). This discovery provides evidence that the application of GeoGebra can help students grasp the concepts of quadratic functions more effectively. Students of lower classes hardly find themselves when working with GeoGebra, compared to higher classes. (Mollakuqe et. al 2020). This tool offers a dynamic and engaging learning environment, assisting students in establishing connections between algebraic representations and their graphical interpretations. The GeoGebra application offers an intuitive way to present graphs of quadratic functions, enabling students to grasp their concepts more quickly and effectively. (Sumartini & Maryati 2021). The media is considered valid for classroom use based on ratings from media and material experts, as well as feedback from field practitioners and students. The software is superior to conventional teaching methods, as it allows students to easily interpret the graphical representation of quadratic equations and develop broader conceptual understandings (Barraza Castillo et. al 2014).

## 2. Using GeoGebra for understanding second-degree equations and quadratic functions

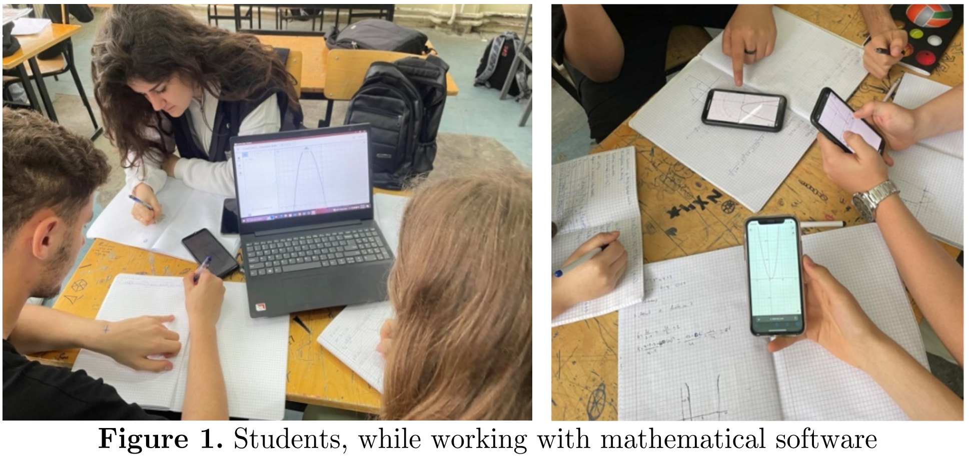

**Figure 1.** Students, while working with mathematical software

To improve and alleviate this difficulty among students, the researchers utilized mathematical software. The use of various math software tools significantly enhanced students' understanding of quadratic equations and functions, while also making mathematics more engaging. This demonstrates that the software is more effective than traditional teaching methods, as it enables students to easily interpret and analyze the graphical representations of quadratic equations and functions (Wijaya et. al 2020). During the research with tenth-grade students, we utilized GeoGebra software. The integration of tools like GeoGebra further enhances the educational aspect, offering an interactive platform for students to visualize and understand exponential functions (Tuda et. al 2024). By enabling students to visualize and manipulate mathematical concepts, GeoGebra fosters deeper comprehension and greater motivation. However, for its successful implementation, adequate teacher training and equitable access to technological resources must be ensured (Aliu et. al 2025).

After we had traditionally explained quadratic equations and functions, the mistakes made by the students while solving the exercises were inevitable. The students mostly learned the formulas and the procedure of solving the exercises mechanically, and could not imagine how the equations of quadratic functions are represented in the graph.

## 3. The impact of using GeoGebra on the understanding of quadratic equations and quadratic functions

We conducted a study with two tenth-grade classes: in one, we used only the traditional method to explain equations and quadratic functions, while in the other, we incorporated mathematical software to teach the same unit. We then administered a test with identical exercises in both classes and obtained the following results. In Table 1 we marked 0 for no exercises solved, 1 for a solved exercise, 2 for two solved exercises, 3 for three solved exercises, 4 for four solved exercises, Class X/3 for students who learned with the classical method and Class X/4 for students who have learned through mathematical software. From the results we conclude that the class that used mathematical software during the learning achieved a better result in the test. Exercise II of the evaluation test was solved by a student in whose class the traditional teaching method was used, a mistake is seen in the exercise to find the peak of the function. Figure 2 shows where the student has mastered the formulas to find the peak of the