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Образователни технологии

EDUCATIONAL RESOURCES WITH PERCENTAGES FOR THE DEVELOPMENT OF THE VISUAL ESTIMATION

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Abstract. Computer models of tasks related to percentage are presented. The files are created with the dynamic software GeoGebra and are provided in the Virtual Mathematics Laboratory, developed by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. The goal is to create conditions for the development of the visual estimation of a percentage, which also supports the understanding of the concept. The computer models contain rectangles and circles. Help and feedback are provided. Options for obtaining a new example and feedback are described. Emphasis is placed on the analogy of the tasks in the four presented topics, each of which contains four tasks. An assessment of the resources is presented, obtained from an anonymous survey with teachers from different subject areas and teaching at different educational levels. The assessment is based on the criteria of easy technical orientation, design, usefulness, entertainment, motivation to solve. The simultaneous development of digital and mathematical competence when working with these resources is commented on, as well as the possibility of their use in STEM centers. Ideas for expanding the resources for checking and developing the percentage calculator in several directions are described.

Keywords: computer model; visual estimation; self-assessment; mathematics education; GeoGebra; STEM

1. Introduction

Teachers and researchers have assessed the difficulties of many children in learning fractions and more generally, the difficulties in further learning mathematics as a result of this, as well as the difficulties of adults in a number of professions (Lortie-Forgues et al., 2015). Research and the search for ideas to support the formation of competence in students in working with a part of a whole, and in particular the presentation with a percentage, continue (Barbosa Vale, 2021), (Cardoso & Mamede, 2025), (Chehlarova,

2019), (Chehlarova, 2020), (Kiliene, 2023), (Schwarzmeier & Obersteiner, 2025), (Ari et al., 2024) and (Van den Heuvel-Panhuizen, 2003). The study of percentages in Bulgaria is implemented with students at the age of 13 and continues to be a challenge for them, which is evident from the results of both national external assessments and international studies. The problems are related to understanding the concept and solving the basic tasks related to percentages. An essential point in forming the concept of percentage is the ability to evaluate a part of a whole, expressed as a percentage. For this purpose, textbooks and manuals contain single tasks in which a figure is divided into equal parts and the student is asked to determine the percentage of a part of the whole colored in a certain way. For the understanding and development of the ability to determine percentage with some accuracy, it is important to provide conditions for exercises, as well as to emphasize the connection with fractions and decimal fractions.

It is natural to use various technical devices for calculations in everyday life. It is important to form skills for an approximate estimate of an expected result, which will not allow a technical error, for example, when entering data or a technical malfunction of the relevant device, to use the wrong result for the relevant purposes. Both visual assessment and the skill of rough calculation are important.

Here we will present computer models of tasks related to percentage. They are created by analogy with educational resources related to fractions (Chehlarova, 2017). The files were created with the dynamic software GeoGebra (Hohenwarter et al., 2009) and are provided in the Virtual Mathematics Laboratory developed by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

2. Computer models with rectangles for the development of the visual estimation of percentage

The Virtual Mathematics Laboratory has developed two topics that provide conditions for the development of the visual estimation of a percentage through models with rectangles. The tasks in the topic at note[1] are accurate to 10%. The text of the first task is "What percent of the rectangle is colored purple?". A rectangle is constructed in the provided file, part of which is colored purple (fig. 1). There are two hide/show checkboxes

– “отговор” (answer) and “помощ” (help). After clicking on the answer checkbox, a record of the searched percentage is displayed/hidden, and after clicking on the help checkbox, a division of the rectangle into 10 equal parts by vertical segments is displayed/hidden.

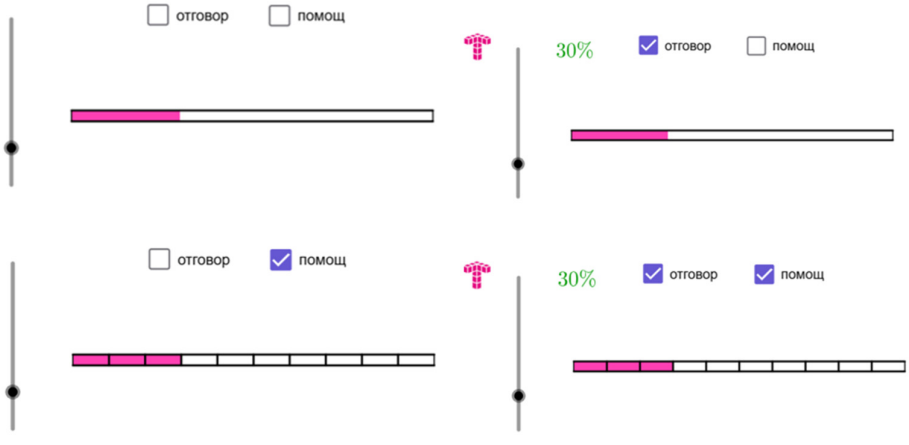

Figure 1. A rectangle coloring task to determine a percentage with an accuracy of 10

The provided file also includes a slider parameter with which a new example can be created.

In task 2, creating a new example is done automatically with the “нов пример” (new example) button. As a result, a value is randomly selected, in this case a natural number from 1 to 10 (fig. 2).

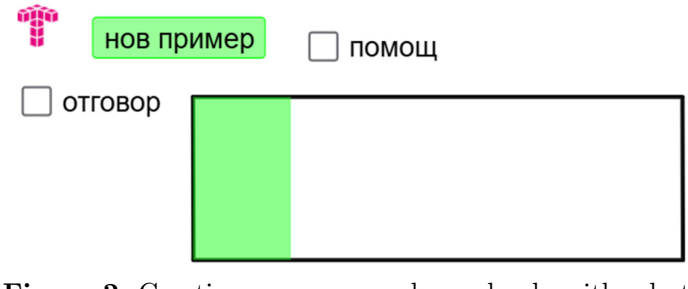

Figure 2. Creating a new example randomly with a button

Help is again provided by dividing the rectangle into 10 equal parts with vertical lines, and feedback can be obtained after using the show/hide "отговор" checkbox (fig. 3).

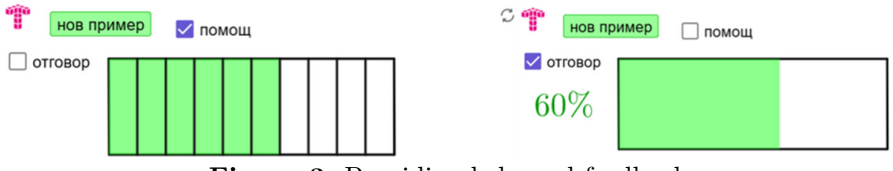

Figure 3. Providing help and feedback

There are two reverse tasks – Task 3 and Task 4, in which the percentage is given, and it is necessary to color the part of the rectangle corresponding to this percentage (fig. 4).

Figure 4. Inverse task with coloring a rectangle to within 10

Moving points are used to achieve the coloring. To get help and feedback, the corresponding hide/show checkmarks are used (fig. 5).

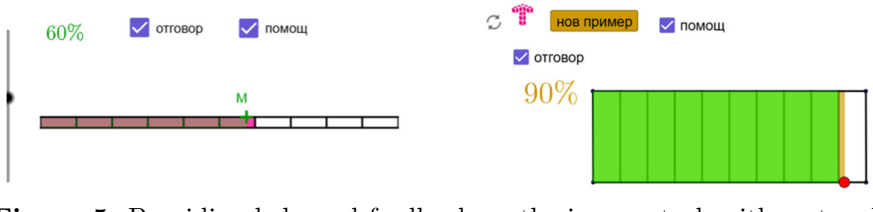

Figure 5. Providing help and feedback on the inverse task with rectangle coloring

In Task 3, the next example is created with a slider parameter, and in Task 4 – automatically with a new example button.

This topic uses percentage values with an accuracy of 10.

At note[2], similar tasks follow, but when working with percentages with an accuracy of 1 (fig. 6).

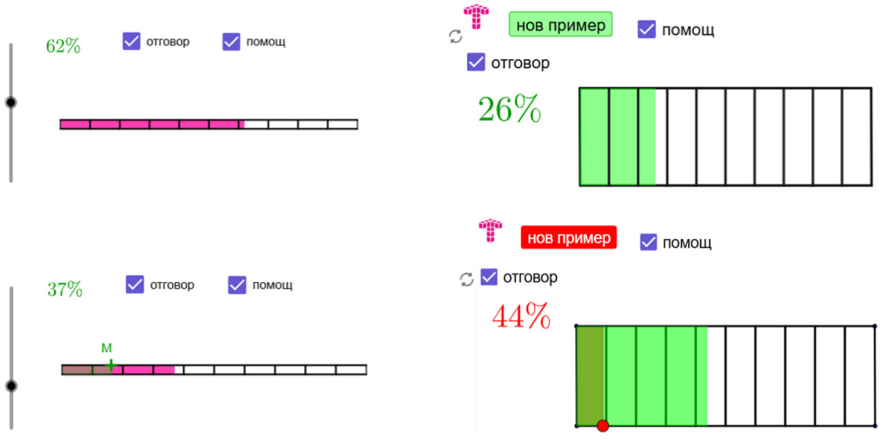

Figure 6. Tasks with a rectangle for the visual estimation with an accuracy of 1

This analogy in technical maintenance allows for easy and quick orientation in dynamic files and focusing on the content element.

There is also an analogy in the tasks for the visual estimation of a percentage, in which circles and sectors are used.

3. Computer models with circles for the development of the visual estimation of a percentage

The tasks at note[3] use percentages with accuracy up to 10, and at note[4] – with accuracy of 1. We will illustrate with the tasks in the topic with an accuracy of 1. In Task 1, you have to estimate what percentage of the circle is colored green (fig. 7).

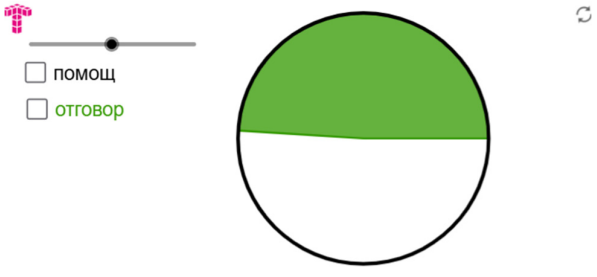

Figure 7. Circle coloring task to determine percentage

The help provides dividing the circle into 10 equal sectors (fig. 8). The feedback is by displaying the percentage, after using the show/hide "отговор" checkbox (fig. 10).

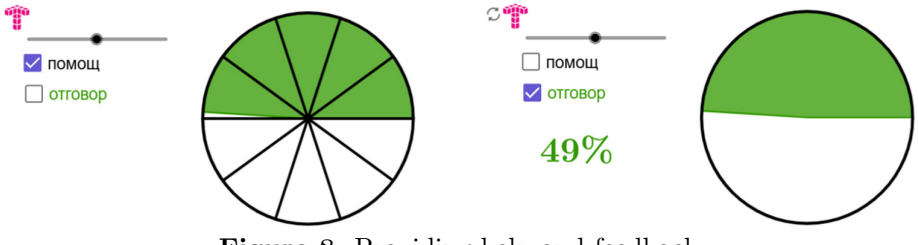

Figure 8. Providing help and feedback

To get a new example, the slider-parameter in the file is used. In task 2, a new example is obtained via the new example button (fig. 9).

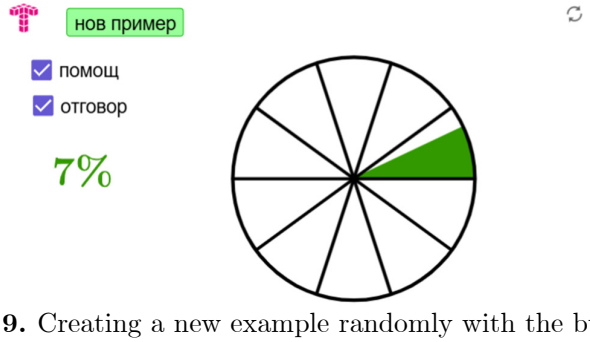

Figure 9. Creating a new example randomly with the button

Tasks 3 and 4 are for coloring a part of the circle that corresponds to the percentage specified in the condition. Coloring is realized by moving a point from the corresponding circle. For help and feedback, the hide/show checkboxes are also used here (fig. 10).

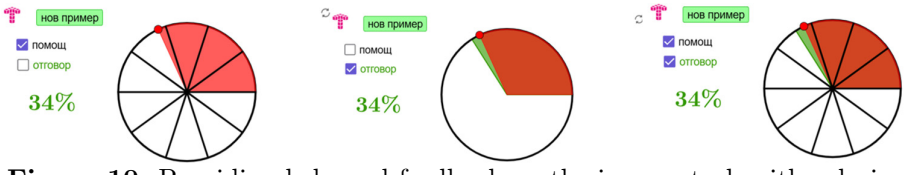

Figure 10. Providing help and feedback on the inverse task with coloring a sector in a circle

In Task 3, a new example is obtained with a slider parameter, and in Task 4, after pressing the button for a new example, a number is randomly selected with an accuracy of 1.

The analogy with the tasks where a rectangle is used as a model also provides a reduction in the time for technical orientation here, as well as facilitates the additional use of some of the files, for example, to provide observation and support for understanding and orientation in percentages. For example, for a short time, different colorings can be observed, along with the recording of the percentage corresponding to this coloring.

Table 1 presents data on the tasks in the 4 topics described, which shows the analogy in their composition.

Table 1. Tasks data

	Rec-tangle	Circle	Accu-racy 10%	Accu-racy 1%	For visual estima-tion of percen-tage	For coloring of percen-tage from a figure	New example with a slider	New example with a button
524-1	+		+		+		+	
524-2	+		+		+			+
524-3	+		+			+	+	

524-4	+		+			+		+
525-1	+			+	+		+	
525-2	+			+	+			+
525-3	+			+		+	+	
525-4	+			+		+		+
526-1		+	+		+		+	
526-2		+	+		+			+
526-3		+	+			+	+	
526-4		+	+			+		+
527-1		+		+	+		+	
527-2		+		+	+			+
527-3		+		+		+	+	
527-4		+		+		+		+

4. Resource assessment

An anonymous survey using GoogleForm was conducted to evaluate the presented resources. Twenty-eight teachers from different subject areas and teaching at different educational levels participated in the survey. Their age characteristics are presented in fig. 11, and the results – in fig. 12.

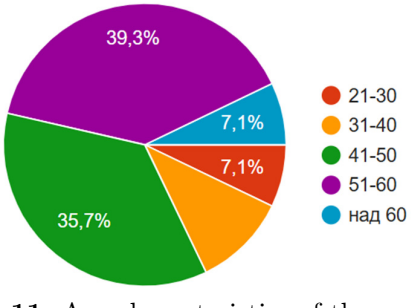

Figure 11. Age characteristics of the respondents

The evaluation criteria are: easy technical orientation, design, usefulness, entertainment, motivation to solve. Usefulness, entertainment and

motivation to solve were highly rated. Some of the respondents had technical difficulties when working with the resources and the reason is that some of them do not have the skills to work with dynamic resources.

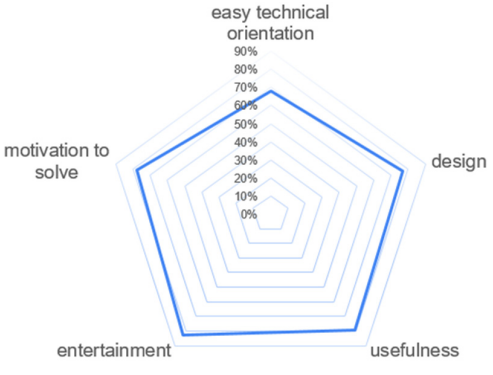

Figure 12. Evaluation

Pilot observations show that the desire for self-testing and self-assessment of the visual estimation is high, regardless of age.

5. Discussion and conclusion

The possibilities for continuing to create conditions for the visual estimation are in several directions. The inclusion in computer models of: time counting or limiting the time for work; number of examples counting or limiting the number of examples for work; assessing the distance from the correct answer; specifying a final grade will allow to easily organize competitions, which are an additional incentive for self-development. These are also part of the possibilities for further research on the topic. Models with rectangles and circles are included here, with the possibility of easy orientation in dividing them into 10 equal parts. The use of other figures (by analogy with resources for fractions such as Escher-style parquet tiles, Mondrian-style tasks, use of polyominoes, etc.), as well as a different way of coloring a part of the whole are another challenge and, accordingly, the next step in creating educational resources related to the visual estimation when

using percentage estimation (Chehlarova, 2013), (Chehlarova, 2017) and (Chehlarova, 2019a).

The use of these resources contributes to the simultaneous development of digital and mathematical competence. They can also be used when working in STEM centers, which are being built in every Bulgarian school and are related to increasing the motivating factors related to self-confidence and the usefulness of mathematics education (Karashtranova, Goldreich & Borisova, 2024). In the context of understanding STEM and STEAM (Chehlarova, 2024), when using the proposed content, STEAM(2) is implemented, and when including other models, for example, parquet tiles in the style of Escher – STEAM(3) , i.e. the subject areas of mathematics, technology and art are included.

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NOTES

1. https://cabinet.bg/index.php?contenttype=viewarticle&id=524
2. https://cabinet.bg/index.php?contenttype=viewarticle&id=525
3. https://cabinet.bg/index.php?contenttype=viewarticle&id=526
4. https://cabinet.bg/index.php?contenttype=viewarticle&id=527

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