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EDUCATIONAL RESOURCES FOR STUDYING MIDSEGMENTS OF TRIANGLE AND TRAPEZOID

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Abstract. Some educational resources for teaching mathematics in the study of midsegments of triangles and trapezoids are described. Variants of dynamic constructions are considered, in support of characterization studies, as well as for the development of the eye-based proportion estimation. Files for 3D printing models when studying midsegments of triangles and trapezoids are presented. The 3D printed models can be used when working with people with visual impairments.

Keywords: digital competence; mathematical competence; educational computer models; additive manufacturing; 3D models; inquiry-based learning

1. Introduction

Already in preschool education in mathematics, it is appropriate to create conditions for carrying out research in forming an idea of basic geometric figures, for discovering properties and formulating hypotheses, for revealing regularities. Computer models facilitate much of this research, and in some cases are a hard-to-substitute tool when implementing inquiry-based learning. However, the use of computer models for such a purpose is still not a widespread practice, despite the activities of international and national educational projects and programs such as InnoMathEd, Fibonacci, DynaMat, KeyCoMath, MaSciL, STEM PD Net, Scientix, VIVA Cognita, Education with Science (Chehlarova et al. 2021).

In Bulgaria, at the moment, the VIII$^{\text{th}}$ grade is the first high school grade, and the majority of students start studying in newly formed classes. Therefore, it is important to provide, whenever possible, conditions for research work using dynamic software in the teaching of mathematics in VIII grade. This will provide an opportunity for those students who have not worked with similar resources in previous years to get closer to others who already have experience of researching with dynamic constructions.

In the curriculum for general educational preparation in mathematics in the VIII grade on the topic "Triangle and trapezoid", the following competences are formulated as expected learning outcomes: "knows the concept of midsegment in a triangle, its properties and knows how to use them, ..., knows how to apply the properties of an isosceles trapezoid, knows the concept of midsegment in a trapezoid, its properties and knows how to use them, knows how to discover and create situations related to midsegments, knows how to distinguish statements from the topic as necessary and sufficient conditions, knows how to form the negation of statements, substantively related to the topic, is able to analyze the condition of a statement and choose appropriate means of proof"[1]. Several practical activities that can be implemented in the classroom are also indicated, for example using dynamic software to demonstrate the properties of geometric figures, building simple geometric constructions with a ruler and compass or with appropriate software products. And for the establishment of cross-curricular connections with informatics and information technologies, the use of specific dynamic software is recommended - "where a better visualization of the learning process or the formation of certain practical skills is needed"[1].

The existence of a STEM center is a fact in many Bulgarian schools. 3D printers are available in a large part of them, which provides opportunities both for their use by teachers for the preparation of educational resources, and for conducting mathematics classes using a 3D printer for printing models created by students.

Here we are going to present two types of educational resources for VIII grade Mathematics education on the topics of "Midsegment of a triangle" and "Midsegment of a trapezoid".

2. Dynamic computer models

The presented computer models were developed with GeoGebra (Hohenwarter et al. 2008) and are freely accessible in the Virtual Mathematics Laboratory (Chehlarova 2020), developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

Model 2.1. Triangle with midsegment

A triangle and one of its midsegments is constructed. The lengths of the midsegment and the corresponding side of the triangle are shown to the nearest hundredth. The vertices of the triangle are movable points, and by changing their position, different triangles are obtained (fig. 1). This makes it possible to observe multiple triangles in a short amount of time in order to discover properties of a triangle's midsegment.

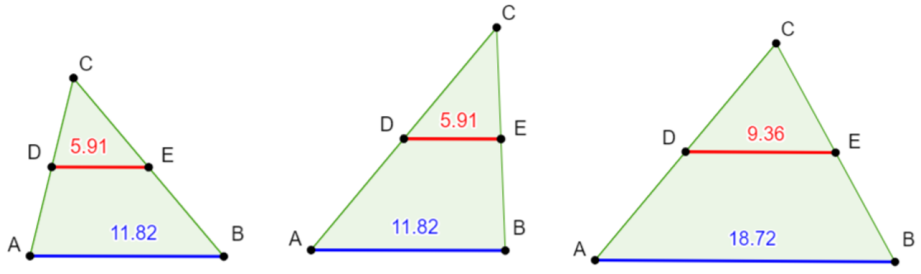

Figure 1. Midsegment study of a triangle

When using computer models, the measurements of the objects are displayed with specified accuracy, in this case – with accuracy to hundredths. Sometimes, due to rounding, an error may occur (with the corresponding precision). For example, in the following fig. 2, the length of the side AB of triangle ABC is 17.35 units to the nearest hundredth, and the length of the corresponding midsegment DE is 8.67 units, i.e. the length of the corresponding midsegment to the nearest hundredth is slightly less than half the length of the corresponding side. Students should know that such results should not prevent the formulation of the hypothesis that the following midsegment of a triangle is twice the corresponding side of the triangle. By changing the precision with which the numbers are displayed, for example to ten thousandths, you can see where the error is coming from.

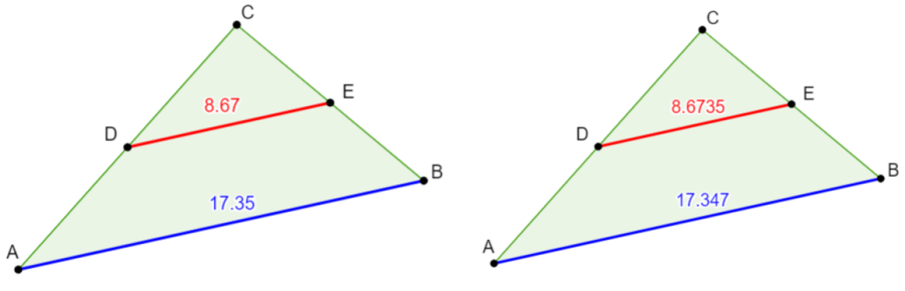

Figure 2. Different display accuracy of numerical values

It is further expected to notice that the midsegment is parallel to the respective side. The degree measures of two angles can be deduced to check the validity of this hypothesis for various specific cases before proceeding to the proof.

When using the *GeoGebra* file, students can observe the step-by-step construction of the dynamic structure used, as well as to observe the construction protocol (fig. 3).

	Име	Икона	Описание	Дефиниция
1	Картинка картинка1			
2	Точка A			
3	Точка B			
4	Точка C			
5	Триъгълник многоъгълник1		Многоъгълник A, B, C	Многоъгълник(A, B, C)
5	Отсечка c		Отсечка [A, B]	Отсечка(A, B, многоъгълник1)
5	Отсечка a		Отсечка [B, C]	Отсечка(B, C, многоъгълник1)
5	Отсечка b		Отсечка [C, A]	Отсечка(C, A, многоъгълник1)
6	Точка D		Среда на b	Среда(b)
7	Точка E		Среда на a	Среда(a)
8	Отсечка f		Отсечка [D, E]	Отсечка(D, E)

Figure 3. Construction protocol

Model 2.2. Triangle with the three midsegments

The lengths of the three sides and the lengths of the midsegments (fig. 4) of triangle ABC are deduced. Additionally, we recommend constructing the lines that divide each of the sides of triangle ABC from its midpoint, for example lines BE and CE, and derive their lengths as well. Thus, a complete perception of the figure is obtained and there is an opportunity to discover other properties related to it.

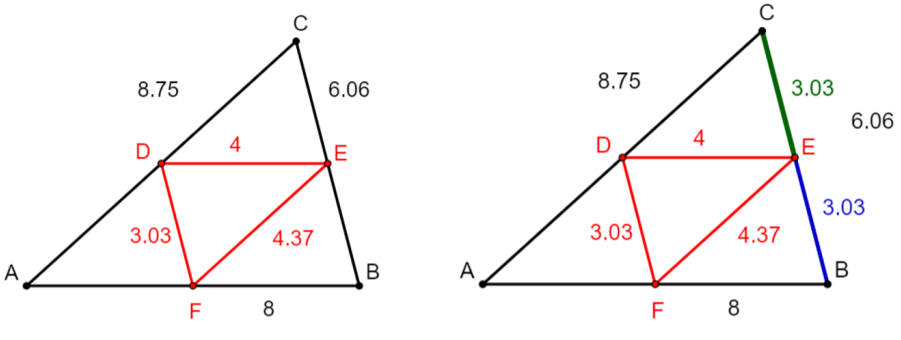

Figure 4. Study with the three midsegments of a triangle

Model 2.3. A quadrilateral and figure whose vertices are the midpoints of its sides

Fig. 5 presents a construction that can be considered both for the study of the figure whose vertices are the midpoints of the sides of a quadrilateral, and as a construction with two triangles with a common side that are in different half-planes about their common side.

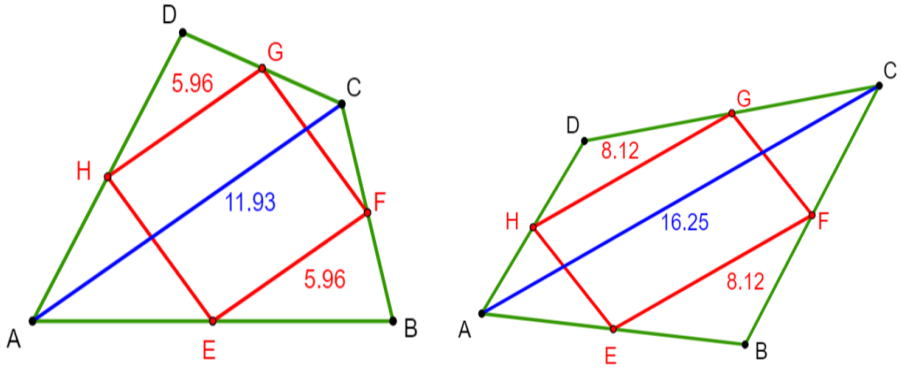

Figure 5. Study of the figure whose vertices are the midpoints of the sides of a quadrilateral

Model 2.4.

Triangle with midsegment and their centrally symmetric images with respect to end of the midsegment.

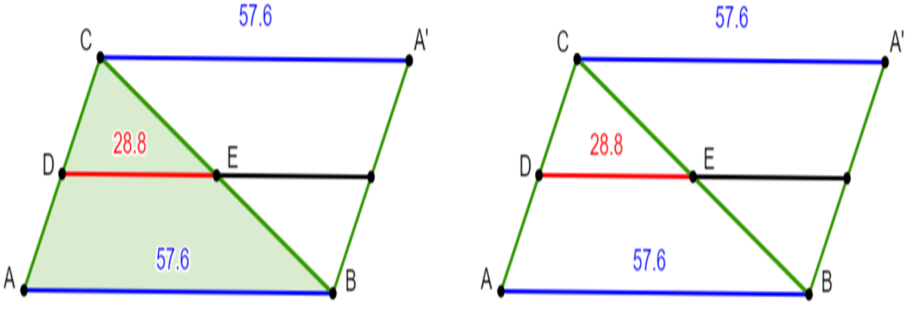

Figure 6. Triangle and midsegment, and their centrally symmetric images

Model 2.5. A model for construction of a midsegment of a triangle

A triangle has been built, a midsegment is expected to be built. This can be implemented as a construction task using elementary and basic constructions, as well as by using available tools from the *GeoGebra* toolbar. In this case, it is appropriate to use the tools for constructing a migsegment and for constructing a section. The construction protocol of fig. 3 can help those who have no experience working with *GeoGebra.*

Model 2.6. Game for the development of the eyesight

One side of a triangle is colored red. Its corresponding midsegment of the triangle should be sketched (fig. 7). For this purpose, an auxiliary section has been built, the ends of which are movable. Self-checking is done by a hide/show check box, in the case of the midsegment.

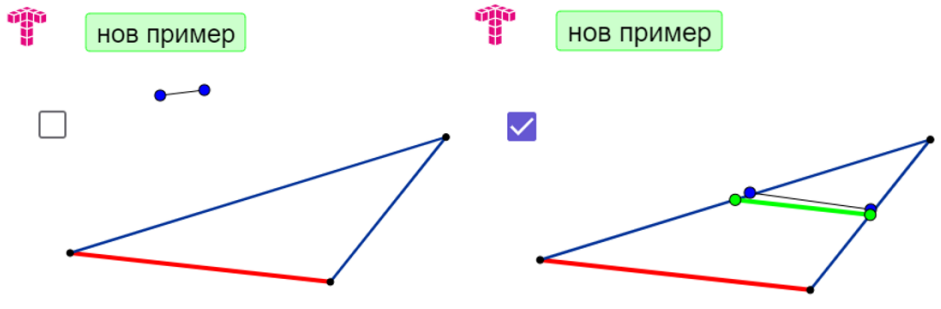

Figure 7. Game for the development of the eyesight

A button is used for a new example. Numbers are randomly selected from specific sets, which are used to determine the coordinates of the vertices of the next triangle. The built-in Refresh button can also be used to get a new example. In a variant of this game to develop the eyesight, a pencil is also used to draw the midsegment.

Models 2.7. Models for the midsegment of a trapezoid

The models for the midsegment of a trapezoid are similar. We believe that analogy is a powerful teaching tool and there are many opportunities for its use in mathematics education that should not be missed, especially when implementing inquiry-based learning. To aid research, it is sometimes appropriate to give additional help or direct students to additional constructions. For example, in the first of the files in fig. 8 shows the sum of the lengths of the trapezoidal bases after using the hide/show checkmark.

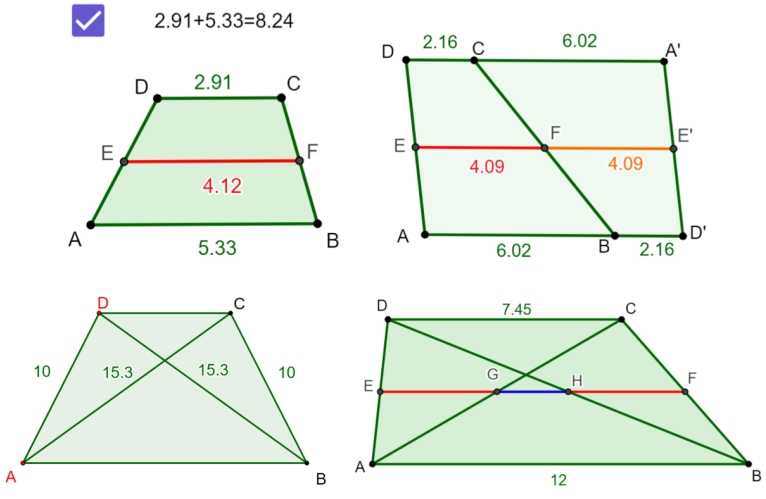

Figure 8. Computer models for trapezoid midsegment studies

3. 3D printed models

Additive technologies can complement learners' knowledge of mathematics; modeling with 3D shape and solid software; variants of filaments used for 3D printers (Kozhuharov 2022), (Kostova et al. 2020), (Ralchev et al. 2021). The preparation of the 3D printer and the printing process will expand their skills to work with a specialized technique (Borgianni et al. 2022), (To et al. 2024). Fig. 9 and fig. 10 present files for 3D printing models of midsegments of triangle and trapezoid models and the corresponding printed models.

Models 3.1

3D printed models connected to a triangle's midsegment

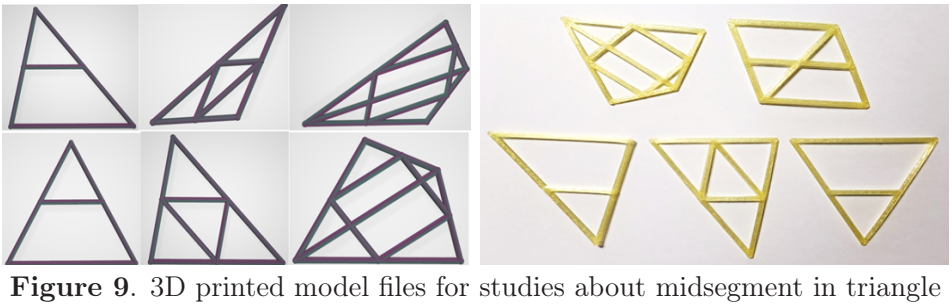

Figure 9. 3D printed model files for studies about midsegment in triangle and corresponding printed models

Models 3.2
3D printed models connected to a trapezoid midsegment

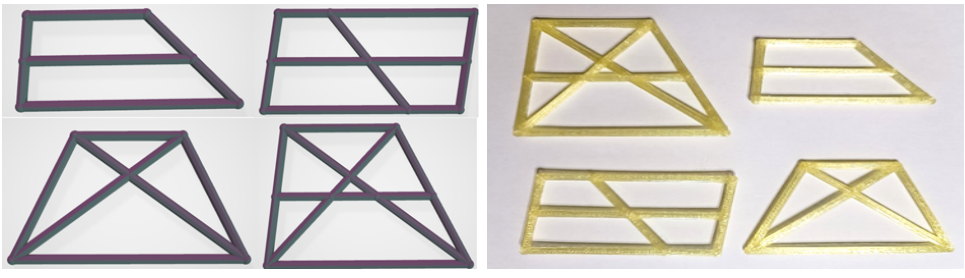

Figure 10. Files for 3D printed models for studies about midsegment in trapezoid and corresponding printed models

They can be used in class, as well as when working with people with visual impairments (Chehlarova 2024).

4. Discussion

Students who have even a little experience working with dynamic structures like the ones presented in the article can quickly handle with making research and formulating hypotheses. For the rest of the VIII grade students, doing research with the suggested files is a suitable step towards the formation of research skills with mathematical objects.

In the Virtual Mathematics Laboratory, there is a large number of auxiliary materials with dynamic constructions, including some with applied problems, with contest problems, etc., but to provide conditions for conducting research and rediscovering properties of mathematical objects by a large number of VIII grade students, including those without a focus on mathematics (Chehlarova & Valkov 2024), it is appropriate to work with tasks of little complexity and related to the curriculum for general education.

When learning about the midsegment of a triangle and the midsegmant of a trapezoid, the 3D printed models make it easier to work with students with visual impairments, but can also help to understand and remember other students, who struggle with math learning.

Didactic games have proven to be a powerful educational tool for teaching mathematics as well (Nikolova & Tuparova 2018), (Pavlova & Toncheva 2020), (Chehlarova 2021), (Gachev 2024). And in this case, a short game for the development of the eyesight with midsegment has its educational effect.

One of the goals of the online competition "VIVA Mathematics with a computer", organized by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences and distributed with the help of the Union of Mathematicians in Bulgaria, is precisely the distribution of computer tools for

mathematical research and solving mathematical and applied tasks with sufficient accuracy (Kenderov 2022), (Kenderov et al. 2021). Resources from past competitions can be stimulating for VIII graders to expand their competence in using computer models to solve a variety of tasks.

For all students, working with ready-made 3D printed models can be a motivation to independently create 3D printed models. The study of mathematics in the context of STEAM through such resources and the creation of new ones in the available STEM centers in Bulgaria can be defined as STEAM$^{(5)}$ (Chehlarova 2024a), denoting the number of subject areas it covers.

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NOTES

1. Mathematics curriculum for Grade VIII (general education) from the academic year 2024/2025.
2. https://cabinet.bg/index.php?contenttype=viewarticle&id=521
3. https://cabinet.bg/index.php?contenttype=viewarticle&id=523

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