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Experiments

DETERMINING THE DEPENDENCE OF THE SPEED OF SOUND IN AIR ON THE TEMPERATURE BASED ON PROPERTIES OF STANDING WAVES

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Abstract. Standing waves are a naturally arising phenomenon in many situations where sound waves propagate in an enclosed medium. Inspired by problem 17 from the International Young Naturalists Tournament (2021), this investigation aims to establish a relationship between the speed of sound in air as a function of air temperature at constant pressure and humidity. This goal is achieved by exploiting certain properties of standing sound waves in a tube, sealed off at one end. By varying the length of the tube and the temperature of the air inside, properties of the air, such as its molar mass, can also be determined.

Keywords: standing waves; speed of sound; temperature; molar mass; experimental investigation; IYNT

1. Introduction

This section focuses on the relevant theoretical models of the study.

1.1. Standing waves

A *standing wave* is a wave which oscillates in time but whose peak amplitude profile does not move in space. In a pipe of length , closed at one end, the air serves as a medium for longitudinal sound waves. These waves vary in terms of distribution of pressure. Let us assume that sound waves first travel to the right. After reflecting off of the closed end of the pipe, the right- and left-moving waves interfere with each other, resulting in a standing wave. The change in pressure in terms of the longitudinal and time coordinates of the right- and left-moving waves are given by the following expressions:

$$\Delta p_{\text{right}}(x,t) = p_{\max}\sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$$

$$\Delta p_{\text{left}}(x,t) = p_{\max}\sin\left(\frac{2\pi x}{\lambda} + \omega t\right)$$

In these equations, denotes the pressure amplitude, – the wavelength, and – the angular frequency.

The resulting standing wave is given by the expression:

$$\Delta p(x,t) = \Delta p_{\text{right}} + \Delta p_{\text{left}} = 2p_{max}\sin\left(\frac{2\pi x}{\lambda}\right)\cos(\omega t),$$

obtained from the identity $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

This wave equation has stationary points, defined as *nodes*, and points that oscillate in time with a maximum amplitude, defined as *anti-nodes*. The resulting wave must have a pressure node at and a pressure anti-node at (known in literature as the “open-closed case”). The corresponding boundary conditions for an anti-node are:

$$\sin\left(\frac{2\pi L}{\lambda}\right) = 1$$

The wavelengths that satisfy this condition are determined by:

$$\frac{2\pi L}{\lambda} = \frac{n\pi}{2} \Rightarrow \lambda = \frac{4L}{n}, \qquad n = 1, 3, 5, \ldots, 2k+1, \qquad k \in \mathbb{Z}$$

If is the speed of sound, then the resonant frequencies are restricted to:

$$v = \lambda f \Rightarrow f = \frac{nv}{4L}, \qquad n = 1, 3, 5, \ldots, 2k+1, \qquad k \in \mathbb{Z}$$

1.2. Speed of sound in air

To derive the formula for the speed of propagation of a sound wave, consider a small enclosed volume of air with cross-section (see Fig. 1):

$$m = \rho V = \rho S x$$

$$\frac{dm}{dt} = \frac{d}{dt}(\rho V) = \frac{d}{dt}(\rho S x) = \rho S\frac{dx}{dt} = \rho S v$$

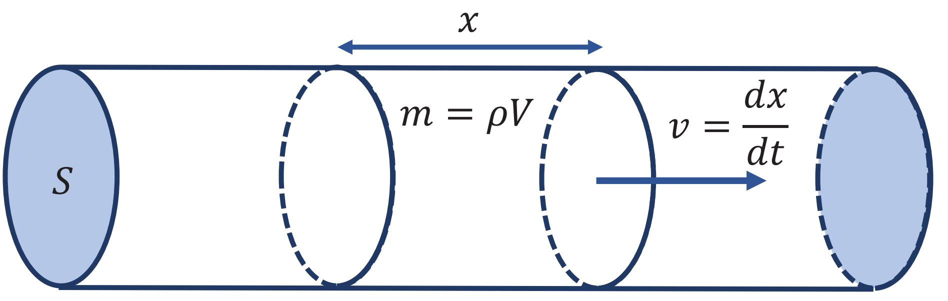


Figure 1. A section of enclosed volume (inside the tube)

The changes in density, temperature, and velocity changes are given by and their changes respectively (see Fig. 2).

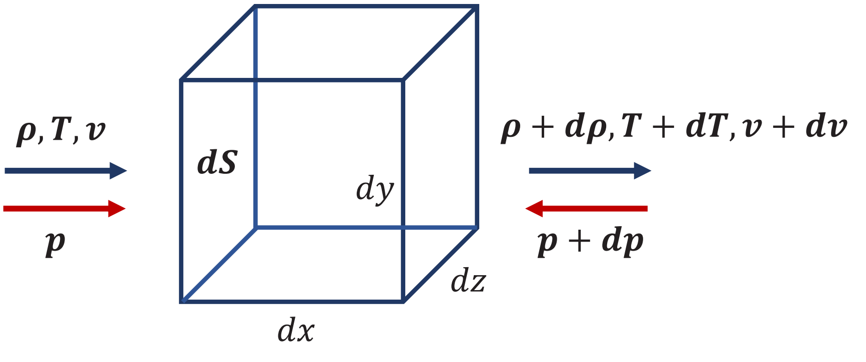


Figure 2. Movement of a sound wave through a small volume of fluid

The mass of the fluid in the volume and the force exerted are given by:

$$dm = \rho\, dV = \rho\, dx\, dy\, dz$$

$$dF = a\, dm = -dp\, dS = -dp\, dy\, dz$$

$$a = \frac{dF}{dm} = -\frac{1}{\rho}\frac{dp}{dx} \Rightarrow \frac{dv}{dt} = -\frac{1}{\rho}\frac{dp}{dx}$$

$$dv = -\frac{1}{\rho}\frac{dp}{dx}dt = -\frac{1}{\rho v}dp$$

$$\boldsymbol{\rho v\, dv = -dp}$$

From the continuity equation , we obtain:

$$\boldsymbol{-v^2\, d\rho = -dp \Rightarrow v = \sqrt{\frac{dp}{d\rho}}}$$

The process is assumed to be adiabatic, therefore: $\boldsymbol{p\left(\frac{1}{\rho}\right)^{\gamma} = const.}$

$$\boldsymbol{\ln p - \gamma \ln \rho = const.}$$

$$\boldsymbol{\frac{d}{d\rho}(\ln p - \gamma \ln \rho) = 0}$$

$$\boldsymbol{\frac{1}{p}\frac{dp}{d\rho} - \frac{\gamma}{\rho} = 0 \Rightarrow \frac{dp}{d\rho} = \frac{\gamma p}{\rho}}$$

From the ideal gas law in differential form . $p = \frac{\rho}{\mu}RT$, , we obtain:

$$\boldsymbol{v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{\mu}}}$$

Where is the molar mass of the gas. In our case, we assume that , since air consists primarily of nitrogen and oxygen – both of which are diatomic molecules.

2. Experimental investigation

For the experimental investigation, a sound with fixed frequency is produced into a tube, whose length can be systematically varied. The modulated sound from the tube is then recorded by a microphone with the software Audacity. When, for a given frequency, a standing wave pattern is achieved, the amplitude of the recorded sound becomes significantly amplified. The outdoor temperature on a sunny day in August is used to achieve the higher temperatures and dry ice is used to achieve the lower temperatures. A thermometer is used to keep track of the temperature.

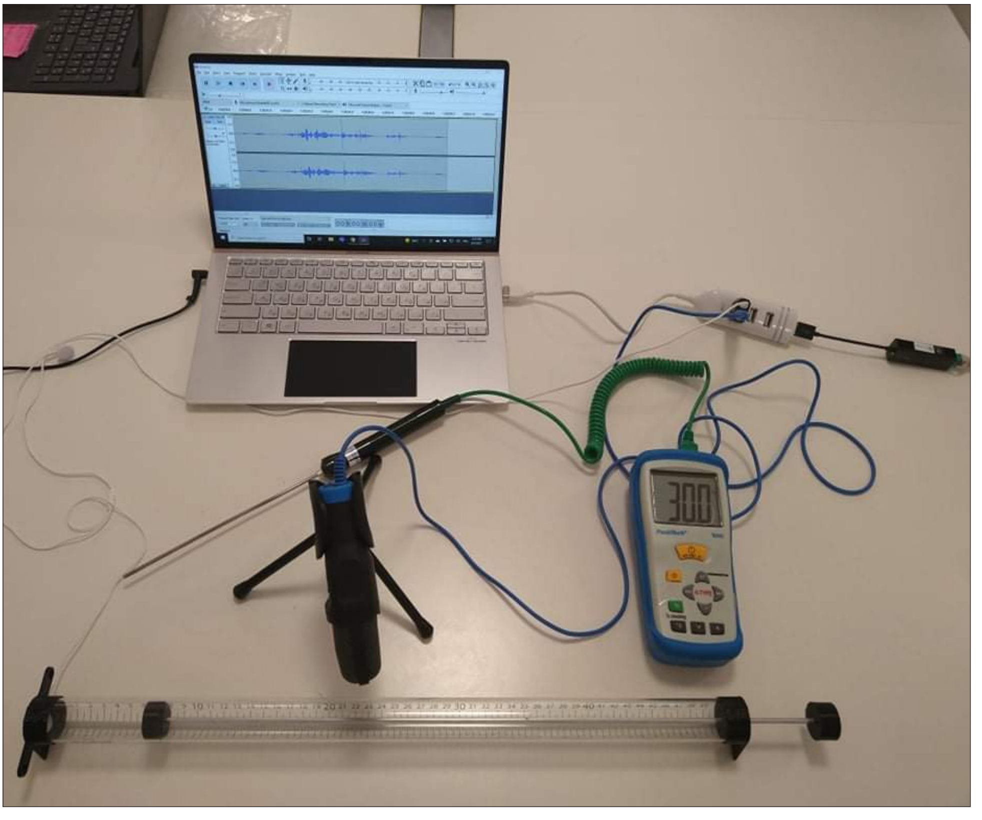
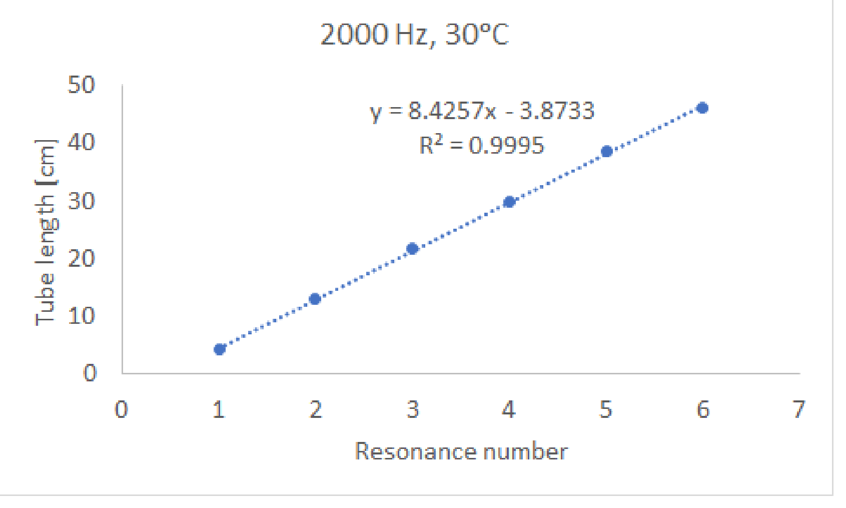


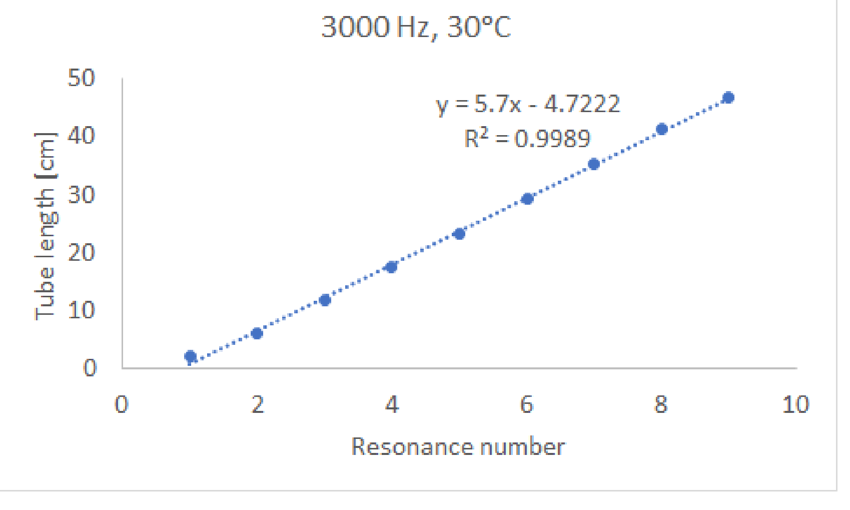
Figure 3. Experimental setup
1. Tube with variable length
2. Earphones
3. Microphone
4. Computer equipped with Audacity
5. Thermometer

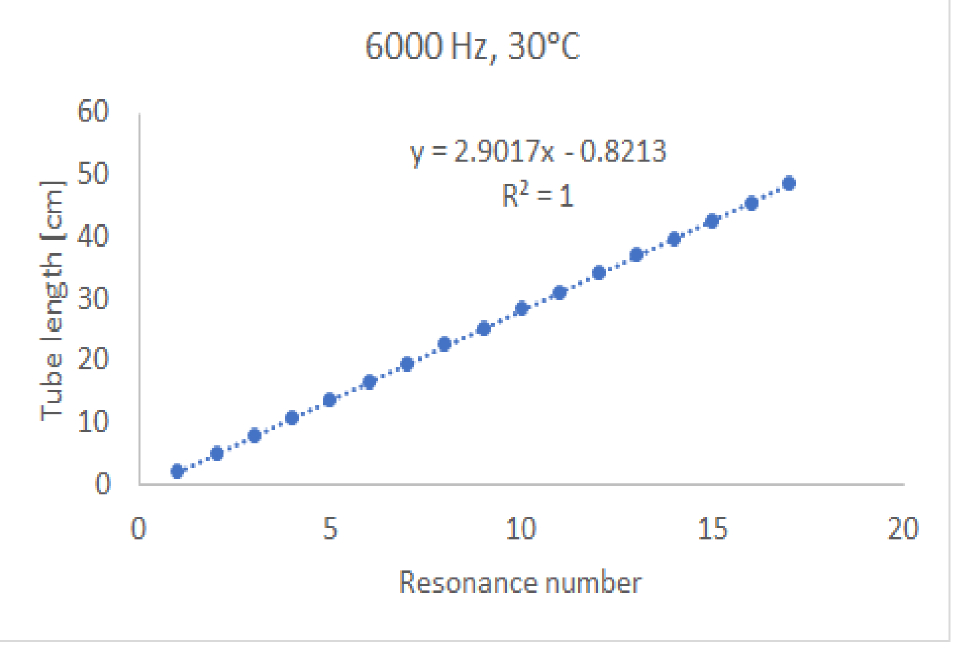
Figure 4. Dry ice used to achieve sufficiently low temperatures

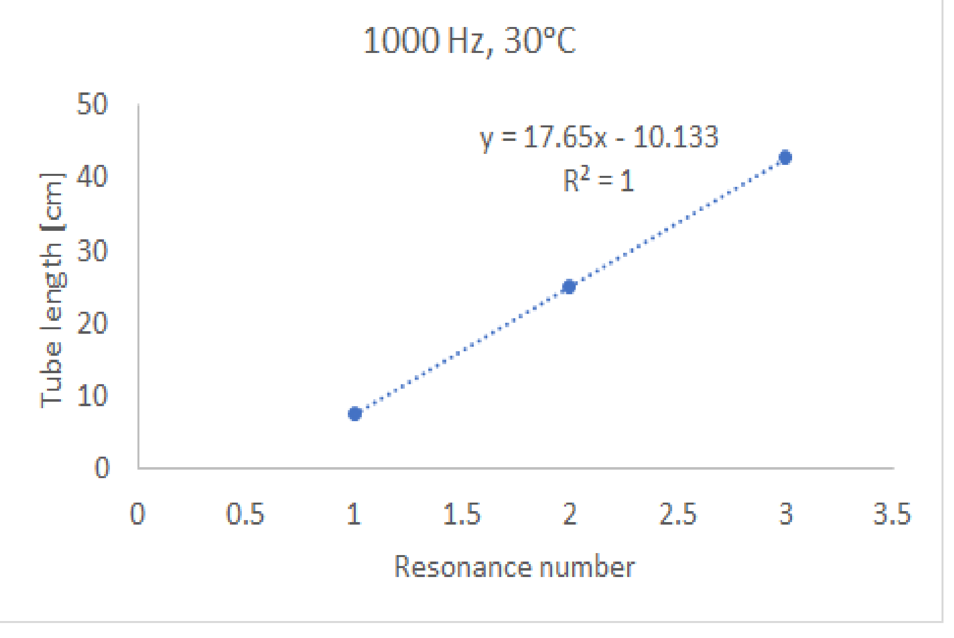
3. Data analysis and results

For each measurement, the resonance number is recorded, as well as the corresponding tube length at that particular resonance. This procedure is repeated for generated frequencies of 1000, 1300, 1600, 2000, 3000, 4000 and 6000 Hz respectively, at temperatures -52, -16, 2, 30 and 49 degrees Celsius for each frequency. A graph of the tube length against the resonance number is plotted. Since each resonance corresponds to an odd value of the integer , the graph is linear with slope . Some of the obtained graphs are shown below:



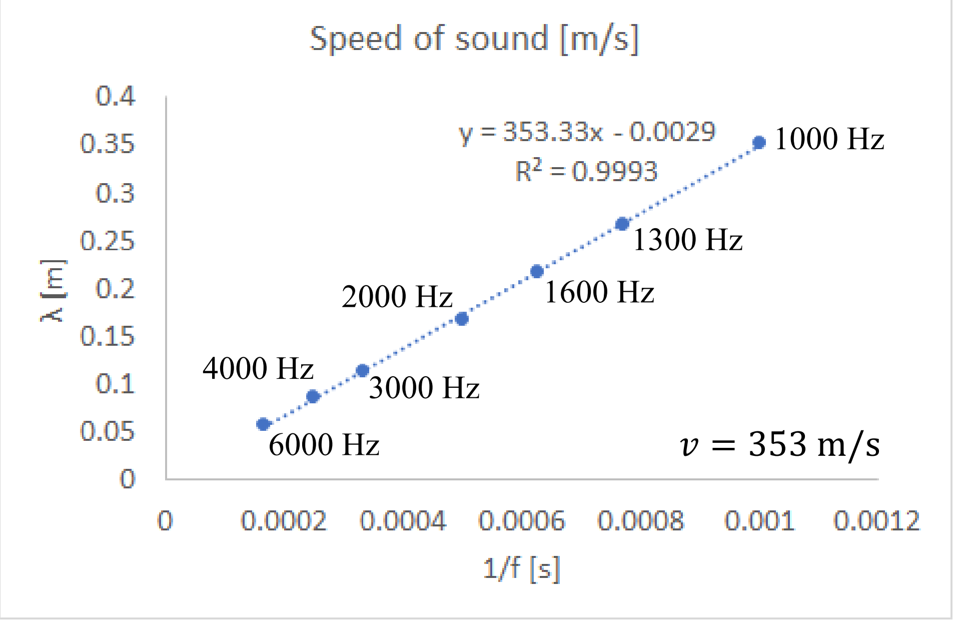



6000 Hz, 30°C
y = 2.9017x - 0.8213
R² = 1
Tube length [cm]
0
10
20
30
40
50
60
0
5
10
15
20
Resonance number


1000 Hz, 30°C
y = 17.65x - 10.133
R² = 1
Tube length [cm]
0
10
20
30
40
50
0
0.5
1
1.5
2
2.5
3
3.5
Resonance number

After we obtain the wavelengths of standing waves from the slopes of the graphs, we plot the resonant wavelengths λ as a function of $1/f$ (the reciprocal value of the frequencies). The slope of this new graph will correspond to the speed of sound in air at the given temperature per the well-known formula $\lambda = v/f$.

An example for temperature 30 degrees Celsius is shown below (all other measurements for other temperatures are analogous):

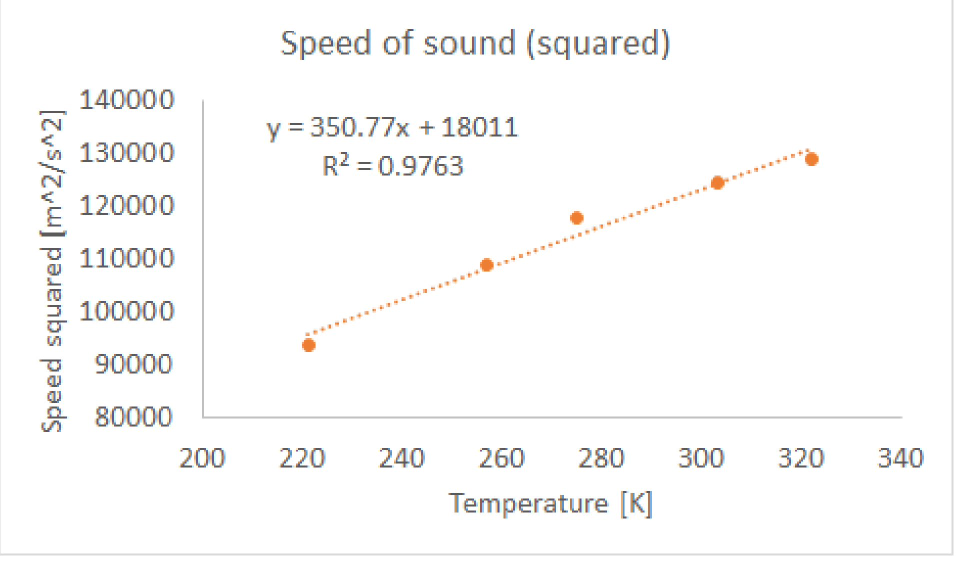


The values for the speed of sound that are obtained by these measurements are displayed in the table:

t [⁰ C]	-52	-16	2	30	49
v [m/s]	306	330	343	353	359

From these values we can construct a linear graph of the speed of sound squared against the absolute temperature (in Kelvins). The slope of the graph k is then used to calculate the molar mass of the air.

$$\boldsymbol{v = \sqrt{\frac{\gamma RT}{\mu}} \Longrightarrow v^2 = \frac{\gamma RT}{\mu} \equiv kT, \qquad \mu = \frac{\gamma R}{k}}$$



Using the obtained experimental results, the obtained value for molar mass of air is **33 g/mol**.

Conclusion

The obtained experimental results coincide with the theoretical predictions for the model of the speed of sound in air with a significant value of the R^2 factor. Thus, the nature of standing waves provides an unusual, however accurate method for deriving the relationship of the speed of sound on air temperature.

NOTES

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