COMPUTER DISCOVERED MATHEMATICS: AN ALTERNATIVE CONSTRUCTION OF MALFATTI SQUARES

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Abstract. In comparison with (Grozdev, Okumura. & Dekov, 2018) an alternative construction of Malfatti squares is presented.

Keywords. Malfatti square; geometric construction; Euclidean geometry; computer discovered mathematics; "Discoverer"

Introduction

In 1985, Hidetosi Fukagawa ((Fukagawa, 1985), Problem 1013) defined the Malfatti squares, named by analogy with Malfatti circles (see Figure 1). Also, Fukagawa suggested the problem of finding a geometric construction of the Malfatti squares.

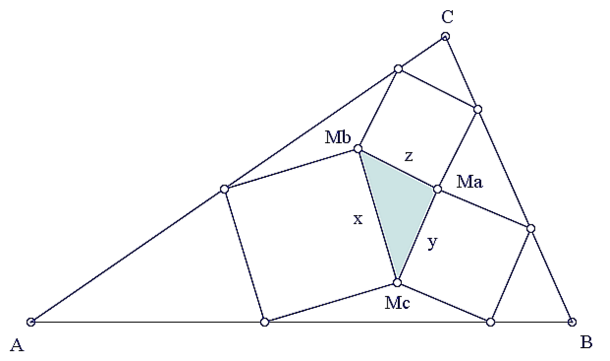

Figure 1

The Fukagawa problem was solved in 1986 by Dan Sokolowsky (Sokolowsky, 1986). In 2007, six new constructions of the Malfatti squares are given by D. Dekov (Dekov, 2007), and in 2008 one additional construction is given by Floor van

Lamoen and Paul Yiu (van Lamoen & Yiu, 2008). In 2018 the authors of the present paper (Grozdev, Okumura & Dekov, 2018) presented 79 new constructions of the Malfatti squares.

In this paper we give a new construction of the Malfatti squares. We use the same method, as in (Grozdev, Okumura & Dekov, 2018). Note that Theorem 1 in this paper is discovered by the computer program "Discoverer", created by the authors.

Recall that the Malfatti-Moses point is point X(3068) in (Kimberling). This point is the centroid of the Malfatti squares triangle. The point was discovered in 2005 by Peter J. C. Moses during his research on the Lucas circles. The Outer Vecten point is point X(485) in (Kimberling). The construction of the Hatzipoalkis triangle of point *P* is given by J.-P. Ehrmann (Ehrmann, 2002).

In (Grozdev, Okumura & Dekov, 2018), in order to find a construction of the Malfatti squares, we have to find the geometric descriptions of two centers of homotheties of the Malfatti squares triangle with two triangles homothetic with it. Theorem 1 gives the possibility to satisfy this requirement.

Theorem 1. Let *P* be a point in the plane of triangle *ABC*, different from the Orthocenter or the Malfatti-Moses point. Denote by L_1 the line through point *P* and Malfatti-Moses point, and by L_2 the line through the Outer Vecten point and the midpoint of points *P* and Orthocenter. The lines L_1 and L_2 intersect in the center of homothety of the Malfatti squares triangle and Hatzipolakis triangle of point *P*.

Proof. We use barycentric coordinates. The barycentric coordinates of the Malfatti squares triangle *MaMbMc* are given in (van Lamoen & Yiu, 2008) and in (Grozdev, Okumura & Dekov, 2018). The barycentric coordinates of the Hatzipolakis triangle *PaPbPc* of point $P=(u,v,w)$ are given in (Ehrmann, 2002) and in (Grozdev, Okumura & Dekov, 2018). These two triangles are homothetic (See (Grozdev, Okumura & Dekov, 2018), Theorem 5). We use formula (3) in (Grozdev & Dekov, 2016) in order to find the barycentric equations of lines *MaPa* and *MbPb*, and then we use formula (5) in (Grozdev & Dekov, 2016) in order to find the intersection of these two lines, that is, the perspector of triangles *MaMbMc* and *PaPbPc*. Denote the point of intersection by X_1.

We use formula (3) in (Grozdev & Dekov, 2016) and we find the equation of the line L_1 defined by point *P* and the Malfatti-Mose point. Then we use the midpoint formula (14) in (Grozdev & Dekov, 2016), in order to find the midpoint *K* of points *P* and the Orthocenter. We use formula (3) in (Grozdev & Dekov, 2016) and we find the equation of the line L_2 defined by point *K* and the Outer Vecten point. Finally, we use formula (5) in (Grozdev & Dekov, 2016) in order to find the intersection point X_2 of lines L_1 and L_2. We see that points X_1 and X_2 coincide. This completes the proof.

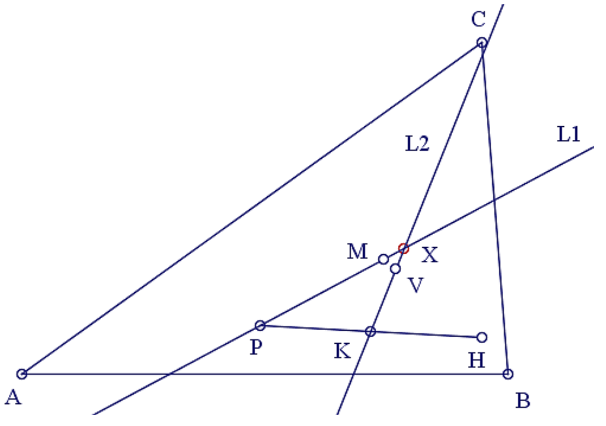

Figure 2

Figure 2 illustrates Theorem 1. In figure 2,

– *P* is an arbitrary point, different from the Orthocenter and the Malfatti-Moses point;

– *M* is the Malfatti-Moses point;

– L_1 is the line defined by points *P* and *M*;

– *H* is the Orthocenter;

– *K* is the midpoint of points *P* and *H*;

– *V* is the Outer Vecten point;

– L_2 is the line defined by points *K* and *V*;

– point *X* is the intersection of lines L_1 and L_2. Also, point *X* is the center of homothety of the Malfatti squares triangle and Hatzipolakis triangle of point *P*.

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МАТЕМАТИКА, ОТКРИТА ОТ КОМПЮТРИ: АЛТЕРНАТИВНА КОНСТРУКЦИЯ НА КВАДРАТИТЕ НА МАЛФАТИ

Резюме. В сравнение с (Grozdev, Okumura. & Dekov, 2018) е представена алтернативна конструкция на квадратите на Малфати.

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