*Educational Matters /*
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# COMPARATIVE ANALYSIS REGARDING THE STUDY OF TRANSFORMATIONS IN THE EUCLIDEAN PLANE BY APPLYING COMPLEX NUMBERS

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**Abstract.** Adopting transformations in the Euclidean plane and their application is compulsory for high school students. The practice shows that even advanced students face difficulties when learning such a material, especially the part which refers to group properties. On the other hand, complex numbers are also studied in high school and they are characterized by an analytical apparatus which helps learning the transformations in the Euclidean plane. This paper presents results from a comparative analysis referring to the accomplishments of the students who study transformations in the Euclidean plane and their application by using complex numbers (see (Malcheski et al., 2015)) and in the classical way (see (Mitrović et al., 1998)).

*Keywords*: complex number, transformation, test, assessment

## 1. Introduction

According to the Lisbon Strategy, knowledge and skills are forms of social capital and the society has to take a permanent care of them. Precisely this is one of the reasons for implementing the lifelong education, as well as continuous reevaluation of education and its adaptation to the needs of the society. The strategy of the European Union also supports this. It aims at emphasizing the importance of education, innovations and research for the development of the individuals and their further preparation for successful professional improvement and involvement in social flows. Clearly, this strategy allows a realization of the individual and also results in enriched knowledge and skills of the society in general, which is necessary for overcoming the challenges that the future may bring. The latter imposes a change of the contents and the methods which are used in the realization of the educational process.

Taking into consideration the previously mentioned, the state authorities, the professional associations which deal with education, and many of the teachers invest serious efforts for a successful integration of the Republic of Macedonia in the frames of the European educational sphere. For a long period of time we have been witnessing numerous reforms in the education sector of the Republic of Macedonia, all with the aim to improve the overall educational system, including mathematics education. Nevertheless, the researches carried out by international professional associations have demonstrated that, apart from the numerous projects for modernization of the education, many of which refer to mathematics education, the results are not satisfactory. This especially refers to the preparation of high school students for a successful inclusion at the technical faculties, as well as the applicability of the acquired knowledge. Hence, there is a need to change the mathematics syllabi in certain segments, i.e. to complement them with the goal to:

– improve inter-subject integration of mathematics instruction,

– enable students to acquire permanent and applicable operative and structural knowledge, and

– improve the preparation of the students for successful inclusion in the higher levels of education.

Accomplishing the mentioned goals is a comprehensive and complex task, which requires analysis of the entire material covered in the high school education, and this cannot be accomplished by a single research. So, having in mind that students have continuously displayed poorest results when dealing with transformations in the Euclidean plane and their application, we developed a syllabus which is based on the application of the complex numbers, which can be listed as an elective subject in comprehensive high school, and is the subject matter of the book (Malcheski et al., 2015). Using both, this syllabus and the existing syllabi (see (Mitrović et al., 1998)), we carried out an experiment, which (was) aimed to answer the following question:

When compared to the existing syllabi, does the acquisition of knowledge and skills about the transformations in the Euclidean plane using complex numbers in the high schools enable acquiring advanced operative and structural knowledge?

## 2. Research design

The previously mentioned defines the subject of the research, which is the accomplishment of students, regarding the transformations in the Euclidean plane by following the experimental syllabus, compared to the accomplishments when the existing syllabi are followed.

According to the subject of research we create the following hypothesis:

*The experimental syllabus results in greater knowledge and skills as opposed to the existing syllabi regarding the transformations in the Euclidean plane, their properties and their algebraic structure.*

Since the inability to get a simple random sample, in the period 20 January - 20

May 2014, we carried out an experiment with voluntary participation organized in two groups, a control group and an experimental one, each consisting of 25 students with advanced mathematical knowledge and skills from the high school of science - Module $A$. The experiment included the following stages:

– distribution of the students in a control and an experimental group, the basic criterion being that the students have approximately equal accomplishments in the previous years,

– the students of the control group, using the book (Mitrović et al., 1998) and additional literature, revised the material about the transformations in the Euclidean plane,

– the students of the experimental group, besides using the book (Malcheski et al., 20015), also had lessons which included studying the transformations in the Euclidean plane by using complex numbers,

– after carrying out the lessons, a test was given to the students aiming at assessing the knowledge of the students.

Next in line was analyzing the results, which included:

i) assessment of the validity of the test, i.e. whether the accomplishments of the students from the control and experimental group follow the normal distribution, and

ii) comparison of the accomplishments of the students from the control and the experimental group, carried out by testing the hypotheses referring to the comparison of the mathematical expectations and the distributions of the accomplishments of the students of the control and the experimental group.

## 3. Results of the research

As we have already mentioned, the test was created with the purpose to check the accomplishments of the students from the control and experimental group regarding the transformations in the Euclidean plane, covered by both syllabi. The test contained 8 tasks and the students were given 90 minutes to complete it. The accomplishments of the students were assessed according to a proportional scale, because that is the one that allows application of the Kolmogorov-Smirnov test, that is used to test the hypothesis that the accomplishments of the students have an appropriate normal distribution, i.e. the quality of the test is assessed. Further on, the accomplishments of the students from the control and the experimental group were compared for each task and adequate remarks were given. The test given to the students from the two groups is listed below.

### TEST

**1. (7 points).** Prove that the movement in the plane $E^2$, which has two fixed points, is an identity.

**2. (8 points).** If the indirect symmetry in the plane $E^2$ has at least one fixed point, then it is a line symmetry, whose axis contains the fixed point. Prove it!

**3. (10 points).** The direct isometry in the plane $E^2$ with exactly one fixed point is a rotation. Prove it!

**4. (15 points).** A composition of line and central symmetry, whose center belongs to the line of symmetry, is a line symmetry. Prove it!

**5. (10 points).** The direct isometry in the plane $E^2$, which does not have any fixed points is a translation. Prove it!

**6. (15 points).** The image of a circle under homothety is a circle, and vice versa, every two circles in the plane $E^2$ are homothetic. Prove it!

**7. (20 points).** Construct a circle that passes through the points *A* and *B* and touches the given circle *k*.

**8. (15 points).** Let *O*, *A*, *B* be three non collinear points and *Ik* be an inversion with respect to the circle $k(O,r)$. If $A'$ and $B'$ are images of the points *A* and *B* under the inversion *Ik* , then $\Delta OAB \sim \Delta OB'A'$. Prove it!

Table 1 presents the results from the accomplishments of the students from the control group. Since we do not have information about the arithmetic mean and the standard deviation, and since this information is required for further analysis, they will be calculated by using the data presented in Table 1. The arithmetic mean, i.e. the average number of points scored by the students is $\bar{x}_{25} = 46,32$ , and thus the standard deviationis $\bar{s}_{25} = 14,21$ .

**Table 1.** Accomplishments of the students from the control group

| Student | Points per task | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 7 | 8 | 4 | 0 | 10 | 0 | 0 | 0 |
| | 7 | 7 | 6 | 0 | 10 | 0 | 0 | 0 |
| | 7 | 8 | 6 | 0 | 10 | 0 | 0 | 0 |
| | 0 | 0 | 6 | 15 | 10 | 0 | 0 | 0 |
| | 7 | 0 | 10 | 0 | 0 | 15 | 0 | 0 |
| | 7 | 0 | 10 | 15 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 10 | 15 | 10 | 0 | 0 | 0 |
| | 7 | 8 | 6 | 0 | 10 | 0 | 6 | 0 |
| | 7 | 8 | 10 | 0 | 8 | 0 | 6 | 0 |
| | 7 | 8 | 8 | 0 | 8 | 0 | 8 | 0 |
| | 7 | 8 | 10 | 0 | 8 | 0 | 8 | 0 |
| | 7 | 8 | 10 | 0 | 10 | 0 | 8 | 0 |
| | 7 | 8 | 0 | 15 | 10 | 0 | 0 | 6 |
| | 7 | 8 | 10 | 15 | 0 | 0 | 8 | 0 |
| | 7 | 0 | 0 | 15 | 10 | 0 | 8 | 8 |
| | 7 | 8 | 10 | 15 | 10 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 7 | 0 | 10 | 15 | 10 | 0 | 0 | 8 |
| | 7 | 8 | 10 | 0 | 10 | 15 | 4 | 0 |
| | 7 | 8 | 10 | 0 | 10 | 15 | 6 | 0 |
| | 7 | 8 | 10 | 15 | 10 | 0 | 10 | 0 |
| | 7 | 8 | 10 | 15 | 0 | 0 | 20 | 0 |
| | 7 | 7 | 6 | 15 | 10 | 8 | 0 | 8 |
| | 7 | 8 | 0 | 15 | 10 | 15 | 0 | 8 |
| | 7 | 7 | 6 | 15 | 10 | 15 | 0 | 8 |
| | 7 | 8 | 10 | 0 | 0 | 15 | 20 | 15 |

Furthermore taking into consideration that the test is valid, objective and reliable, i.e. its measure characteristics are correct, then the accomplishments of the students should have normal distribution $\mathrm{N}(46;14^2)$. This is an indicator that firstly we should test the hypothesis $H_0$: the function of distribution $F_X$ of the accomplishments of the students is equal to the normal distribution, i.e. the hypothesis $H_0 : F_X = \mathrm{N}(46;14^2)$. For this purpose, as we have already mentioned, we will apply the Kolmogorov-Smirnov test with a level of significance $\alpha = 0,05$. We apply the same procedures as in the previous analyses, for $z_i = \frac{x_i - 46}{14}$. The calculations are presented in Table 2.

According to the data in Table 2, the maximum value of $|F_n(x) - F(x)|$ is $d_{25} = 0,09146$ and it is achieved for $x = 39$. Since the level of significance is $\alpha = 0,05$ and the number of data is $n = 25$, using the Kolmogorov's criterion table, we find that $d_{25;\,0,05} = 0,2639$. Since

$$d_{25} = 0,09146 < 0,2639 = d_{25;\,0,05}$$

we have no reason to dismiss the assumption that the distribution of the accomplishments of the students regarding the first test is $\mathrm{N}(46;14^2)$.

**Table 2.** Kolmogorov-Smirnov test of the second test of the control group

| $x_i$ | $n_i$ | $F_n(x_i)$ | $z_i = \frac{x_i - 46}{14}$ | $F(x_i)$ | $\|F_n(x) - F(x)\|$ |
|---|---|---|---|---|---|
| 29 | 1 | 0,04 | -1,21 | 0,11314 | 0,07314 |
| 30 | 1 | 0,08 | -1,14 | 0,12714 | 0,04714 |
| 31 | 2 | 0,16 | -1,07 | 0,14231 | 0,01769 |
| 32 | 2 | 0,24 | -1,00 | 0,15866 | 0,08134 |
| 35 | 1 | 0,28 | -0,79 | 0,21476 | 0,06524 |
| 37 | 1 | 0,32 | -0,64 | 0,26109 | 0,05891 |
| 39 | 2 | 0,40 | -0,50 | 0,30854 | **0,09146** |
| 41 | 1 | 0,44 | -0,36 | 0,35942 | 0,08058 |
| 43 | 1 | 0,48 | -0,21 | 0,41683 | 0,06317 |
| 46 | 1 | 0,52 | 0,00 | 0,50000 | 0,02000 |
| 48 | 2 | 0,60 | 0,14 | 0,55567 | 0,04433 |
| 50 | 2 | 0,68 | 0,29 | 0,61226 | 0,06774 |
| 54 | 1 | 0,72 | 0,57 | 0,71566 | 0,00434 |

| | | | | | |
|---|---|---|---|---|---|
| 56 | 1 | 0,76 | 0,71 | 0,76115 | 0,00115 |
| 60 | 2 | 0,84 | 1,00 | 0,84134 | 0,00134 |
| 61 | 1 | 0,88 | 1,07 | 0,85769 | 0,02231 |
| 63 | 1 | 0,92 | 1,21 | 0,88686 | 0,03314 |
| 68 | 1 | 0,96 | 1,57 | 0,94179 | 0,01821 |
| 75 | 1 | 1,00 | 2,07 | 0,98077 | 0,01923 |

The Kolmogorov-Smirnov test will be also used for the experimental group, as well. Table 3 presents the accomplishments in each task by all of the students from the experimental group separately.

**Table 3.** Accomplishments of the students from the experimental group

| Student | Points per task | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 7 | 8 | 9 | 5 | 10 | 0 | 0 | 0 |
| | 7 | 8 | 6 | 5 | 10 | 0 | 5 | 0 |
| | 7 | 8 | 6 | 5 | 10 | 5 | 0 | 0 |
| | 7 | 0 | 6 | 15 | 10 | 0 | 0 | 5 |
| | 7 | 0 | 10 | 5 | 0 | 15 | 6 | 0 |
| | 7 | 8 | 10 | 15 | 0 | 0 | 5 | 0 |
| | 0 | 8 | 10 | 15 | 10 | 5 | 0 | 0 |
| | 7 | 8 | 10 | 0 | 10 | 5 | 4 | 4 |
| | 7 | 8 | 10 | 0 | 10 | 5 | 6 | 5 |
| | 7 | 8 | 8 | 5 | 10 | 0 | 10 | 5 |
| | 7 | 8 | 10 | 0 | 8 | 5 | 5 | 10 |
| | 7 | 8 | 10 | 5 | 10 | 0 | 0 | 15 |
| | 7 | 7 | 0 | 15 | 8 | 5 | 50 | 5 |
| | 7 | 8 | 6 | 15 | 10 | 0 | 8 | 5 |
| | 7 | 0 | 6 | 15 | 10 | 5 | 8 | 8 |
| | 7 | 8 | 10 | 15 | 10 | 0 | 6 | 5 |
| | 7 | 0 | 10 | 15 | 10 | 8 | 5 | 8 |
| | 7 | 8 | 10 | 6 | 10 | 15 | 5 | 6 |
| | 7 | 8 | 10 | 5 | 10 | 15 | 6 | 6 |
| | 7 | 8 | 10 | 6 | 10 | 15 | 9 | 6 |
| | 7 | 8 | 10 | 6 | 10 | 15 | 9 | 6 |
| | 7 | 8 | 10 | 15 | 10 | 8 | 6 | 10 |
| | 7 | 7 | 6 | 15 | 10 | 15 | 8 | 10 |
| | 7 | 8 | 10 | 15 | 10 | 15 | 10 | 8 |
| | 7 | 8 | 10 | 5 | 8 | 15 | 20 | 15 |

Analogously, as in the previous cases, the data in Table 3 shows that the arithmetic mean, i.e. the average number of points scored by the students is, and the standard deviation. According to this, in order to assess the measuring characteristics of the test, with respect to the students of the experimental group, we need to test the hypothesis: the function of distribution of the accomplishments of the students is equal to the appropriate normal distribution, i.e. the hypothesis. For this purpose we will once again apply the Kolmogorov-Smirnov test with a level of significance , for. The calculations are presented in Table 4.

**Table 4.** Kolmogorov-Smirnov test of the second test of the experimental group

| $x_i$ | $n_i$ | $F_n(x_i)$ | $z_i = \frac{x_i - 46}{14}$ | $F(x_i)$ | $\lvert F_n(x) - F(x) \rvert$ |
|---|---|---|---|---|---|
| 39 | 1 | 0,04 | -1,27 | 0,10204 | 0,06204 |
| 41 | 2 | 0,12 | -1,13 | 0,12924 | 0,00924 |
| 43 | 2 | 0,20 | -1,00 | 0,15866 | 0,04134 |
| 45 | 1 | 0,24 | -0,87 | 0,19215 | 0,04785 |
| 48 | 2 | 0,32 | -0,67 | 0,25143 | 0,06857 |
| 51 | 1 | 0,36 | -0,47 | 0,31918 | 0,04082 |
| 53 | 2 | 0,44 | -0,33 | 0,37070 | 0,06930 |
| 55 | 1 | 0,48 | -0,20 | 0,42074 | 0,05926 |
| 57 | 1 | 0,52 | -0,07 | 0,47210 | 0,04790 |
| 59 | 2 | 0,60 | 0,07 | 0,52790 | **0,07210** |
| 61 | 1 | 0,64 | 0,20 | 0,57930 | 0,06070 |
| 63 | 1 | 0,68 | 0,33 | 0,62930 | 0,05070 |
| 67 | 2 | 0,76 | 0,60 | 0,72575 | 0,03425 |
| 71 | 2 | 0,84 | 0,87 | 0,80785 | 0,03215 |
| 74 | 1 | 0,88 | 1,07 | 0,85769 | 0,02231 |
| 78 | 1 | 0,92 | 1,33 | 0,90824 | 0,01176 |
| 83 | 1 | 0,96 | 1,67 | 0,95254 | 0,00746 |
| 88 | 1 | 1,00 | 2 | 0,97725 | 0,02275 |

According to the data, which are given in Table 4, the maximum value of $\lvert F_n(x) - F(x) \rvert$ is $d_{25} = 0,0721$ and it is achieved for $x = 59$. Since the level of significance is $\alpha = 0,05$, and the number of data is $n = 25$, using the Kolmogorov's criterion table, we find that $d_{25;\,0,05} = 0,2639$. Since

$$d_{25} = 0,0721 < 0,2639 = d_{25;\,0,05}$$

we have no reason to dismiss the assumption that the distribution of the accomplishments of the students regarding the first test is $\mathrm{N}(58;15^2)$.

According to what was previously stated, we get the conclusion that the test accomplishments of both groups have normal distribution. The latter allows us to compare them. As we were able to see, the students from the experimental group scored 58.32 points on average, and the students from the control group scored 46.32 points on average. The mean square deviation of the students in experimental group is approximately 15 points, and that of the ones in the control is approximately 14 points. This means that the accomplishments of the students from the experimental group in comparison with the

accomplishments of the students from the control group are higher by about 25,91%., which indicates that studying these contents by applying the experimental syllabus produces significantly better results. Hence, we can conclude that the application of the experimental syllabus, related to the mentioned material, results in improved inter-subject integration of the mathematics instruction in the high schools, as well as better preparation of the students for inclusion in the higher levels of education. We can get more precise confirmation for the above conclusion via the test for the difference of the mathematical expectations when distributions are unknown and large samples. This is possible because in the previous analyses we established that the accomplishments of the students in both groups have the normal distribution. In this case
$\overline{x}_{25}=58,32$ , $\overline{y}_{25}=46,32$ , $n_1=n_2=25$ , $\overline{s}_x=14,21$ and $\overline{s}_y=15,13$ .

We will test the hypothesis $H_0: m_1 \leq m_2$ as opposed to the alternative hypothesis $H_1: m_1 > m_2$ , with the level of significance $\alpha=0,01$

$$\frac{\overline{x}_{n_1}-\overline{y}_{n_2}}{\sqrt{n_2\overline{s}_x^2+n_1\overline{s}_y^2}}\sqrt{n_1 n_2}=\frac{58,32-46,32}{\sqrt{25\cdot 15,13^2+25\cdot 14,21^2}}\sqrt{25\cdot 25}=2,85$$

from the table of normal distribution we find that $z_{1-\alpha}=2,33$ . The final result is

$$\frac{\overline{x}_{n_1}-\overline{y}_{n_2}}{\sqrt{n_2\overline{s}_x^2+n_1\overline{s}_y^2}}\sqrt{n_1 n_2}=2,85>2,33=z_{1-\alpha},$$

This means that we should dismiss the hypothesis $H_0$ , i.e. at a level of significance $\alpha=0,01$ we accept that the mathematical expectation related to the accomplishments of the students from the experimental group is higher than the mathematical expectation related to the accomplishments of the students from the control group. Further on, the mean square deviations $\overline{s}_x=14,21$ and $\overline{s}_y=15,13$ differentiate insignificantly. Nevertheless, before making a final decision about whether to accept or dismiss the set hypothesis, we will compare the distributions of the accomplishments of the two groups. For this purpose, we will apply the test for equality of distributions of two independent normally distributed features, i.e. we will test the hypothesis $H_0: \sigma_1^2=\sigma_2^2$ as opposed to the alternative hypothesis $H_0: \sigma_1^2 \neq \sigma_2^2$ with a level of significance $\alpha=0,10$ . The given conditions indicate that $n_1=n_2=25$ , $\overline{s}_x=14,21$ and $\overline{s}_y=15,13$ , therefore

$$\frac{n_1(n_2-1)\overline{s}_x^2}{n_2(n_1-1)\overline{s}_y^2}=0,88085 .$$

Further on, from the Fisher's distribution table we find that

$$F_{n_1-1,n_2-1;\frac{\alpha}{2}}=F_{24,24;0,05}=2,66 \text{ и } F'_{n_2-1,n_1-1;\frac{\alpha}{2}}=F_{24,24;0,05}=2,66 ,$$

which means that $F_2=2,66$ and $F_1=\frac{1}{2,66}=0,38$ . Therefore,

$$F_1 = 0,38 < \frac{n_1(n_2-1)\overline{s}_x^2}{n_2(n_1-1)\overline{s}_y^2} = 0,88085 < 2,66 = F_2 ,$$

we conclude that there is no reason to dismiss the hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$.

The above stated allows us to conclude that we should *accept the set hypothesis*, which means that *regarding transformations in the Euclidean plane, their properties and their algebraic structure, the experimental syllabus allows gaining advanced knowledge and skills as opposed to the existing syllabi which do not.*

Regarding the accomplishments of the students with regard to the separate tasks, Tables 5 and 6 indicate that regarding all tasks, the students who studied the experimental syllabus accomplished better results than the students who studied the control syllabus. Furthermore, regarding tasks 1-5 there is a little improvement in the results, which is due to the nature of the tasks, i.e. the fact that they are tasks which directly check the level of acquired operative and structural knowledge, i.e. the properties of the transformations in the Euclidean plane are directly applied. Namely, we directly check the assumption that studying the transformations in the Euclidean plane by using the complex numbers increases the inter-subject integration of the algebraic and geometrical contents which are intended to be learned by the high school students. The latter means that students are better prepared for inclusion in the higher levels of education

**Table 5.** Points and average score of points per task

| Task | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Total number of points | 161 | 149 | 188 | 195 | 194 | 98 | 112 | 61 |
| Average by student | 6,44 | 5,96 | 7,52 | 7,80 | 7,76 | 3,92 | 4,48 | 2,44 |

**Table 6.** Points and average score of points per task

| Task | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Total number of points | 168 | 166 | 213 | 223 | 224 | 171 | 191 | 142 |
| Average by student | 6,72 | 6,64 | 8,52 | 8,92 | 8,96 | 6,84 | 7,64 | 5,68 |

Further on, Tables 5 and 6 allow us to see that the students from the experimental group present significantly higher operative and structural knowledge related to the tasks 6-8. Hence, task 6 was adequately and completely solved by 6 and partially by 1 student from the control group, whereas in the experimental group 8 students solved the task completely and 9 nine students provided partial solution of that task. The above results increased the total result of the students of the experimental group for about 75%. There is a similar difference in the accomplishments of the students from the experimental and the control group regarding (the) tasks 7 and 8, where the accomplishments of the experimental group are approximately 70% higher. Further on, regarding the task 8 we can notice that the accomplishments of the students from the experimental group are 2.32 times higher than the accomplishments of the students from the control group. According to analysis of the solutions we can conclude that the differences in the accomplishments of the students from the two groups are mainly caused by the use of the analytical apparatus of the complex numbers for studying the transformations in the Euclidean plane. This is especially true for the study of the group properties and the classification of the similarities and movements. The latter is especially evident when solving the task 8, where the result gained by the students from the experimental group is mainly due to the use of the analytical apparatus when learning inversion.

## 4. Conclusion

One of the goals of mathematics instruction is for the students to acquire comprehensive, applicable and permanent knowledge, which should allow successful inclusion of the students in the higher levels of education. In order to accomplish this goal, a complete integration of the syllabi is necessary, but we must also have in mind that this integration should also allow adequate and appropriate differentiation of instruction. During the previous elaborations we proved that the introduction of the elective subject *Geometry of a complex number*, for which an adequate syllabus was created, and the enabling of the students to elect it along with other mathematical subjects, contribute for accomplishing the set goals. Namely, the research has demonstrated that:

– the experimental syllabus increases the inter-subject integration of the mathematics education, and

– by learning about transformations in the Euclidean plane with the experimental syllabus, students acquire advanced knowledge and skills, which increase their ability for inclusion in the higher levels of education.

Furthermore, this research has been carried out with a goal to figure the answer of the questions *what*, *why* and *how* a certain educational segment should be changed in order for the desired goals to be accomplished. Having in mind the previously mentioned, if any steps are made in the direction to change the curriculum and the syllabi in the education system in the future, it is desirable for the authorities to incorporate the algorithm elaborated in this research.

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