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CHALLENGING THE STUDENTS‘ SENSE OF MATHEMATICS VIA DECODING PROBLEMS IN CHERNORIZEC HRABAR TOURNAMENT

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Abstract. Under consideration is a thematic branch from the Chernorizec Hrabar (ChH) math tournament, which aims to check and develop the student's sense of mathematics. The ChH competition papers include several types of test items related to codes. These test items require just basic math knowledge but a high level of nonstandard reasoning and a kind of synthetic mathematical abilities. The authors' standing point is to present by examples how the idea of (de)coding evolves in competition papers for students in different ages.

Keywords: mathematic competitions; sense of mathematics; (de)coding problems; synthetic competence; gifted education

1. Introduction

The Chernorizec Hrabar math tournament (ChH) is a multiple-choice competition (Lazarov et al. 2004). It started in 1992. In the beginning, there was just one competition paper for all participants who were 9th – 11th grade advanced students in mathematics. The competition format changes during the years and today's version includes divisions for students of different grades, starting with 3rd-4th and finishing with 11th – 12th graders. The competition papers for the senior groups consists of 30 test-items (TIs) with an answer and four distractors. The scoring system is: 0 points for a wrong choice; 3 points for blank; 5, 7, 9 (depending on the stated a priory difficulty) points for the correct choice. Let us note that such scoring does not tolerate gambling. The general idea for choosing the answer is not to solve the math problem but to have grounded reasoning for this (Lazarov 2019).

The thematic range of ChH competition papers is quite large and as a rule, only complex knowledge and skills allow the participants to succeed. But, even these do not guarantee a highest score because of some TIs, which requires some sense of mathematics for finding the correct answer in a very limited time (Lazarov 2011). One of this kind TIs are the decoding problems. In the following, we are going to discuss the role of decoding problems in developing student's synthetic competence. Our method is based on the main points about gifted education given in (House 1991).

2. About coding-decoding

Coding-decoding processes are very important in the modern live. Let us point just two milestones in the mankind history:

– the deciphering of Egyptian hieroglyphs by Jean-François Champollion that opened the ancient Egypt for the world;

– the battle for Atlantic during the World War II when Britain defeated Germany mainly because Alan Turing braked the Enigma code.

The above examples indicate that the humans who possess decoding abilities are rather rare phenomenon and the development of their talent requires special training. Nowadays, such people are welcome in communication technology as top-level experts in cryptography. However, the Bulgarian secondary school curriculum does not provide enough space to highlight the students who have aptitude to deal with (de)coding. Here comes the ChH math competition to fill the gap.

3. Two simple examples of decoding problems in ChH

Decoding problems of type ***puzzle*** are common for early math education. Puzzles appear regularly in ChH competition papers for the youngsters. We are going to start this section with the first puzzle in ChH, which is intended for the secondary school students (it is not a proper decoding problem).

(2000, 9 – 11)[1] Two two-digit numbers are multiplied as shown on the right. The last digit of the product is:

(A) 2 (B) 1 (C) 4 (D) 6

$$\begin{array}{cccc} & & * & * \\ \times & & * & * \\ \hline + & & * & * \\ & * & 1 & \\ \hline * & * & * & * \end{array}$$

(E) cannot be determined

Reasoning. The first digit of the product is 1. The only possible value for the second addend is 91. Since we want to obtain as a final result a four-digit number, then the tens-digit of the first addend should also be 9. Since both addends are divisible by a two-digit number, then they are equal, i.e. the sum is $91 + 910 = 1001$. Examples: the product can be either 91×11 or 13×77.

Answer (B).

Comment. In fact, the most important attribute of the coding is missing in the TI: the one-to-one correspondence between the objects (decimal digits) and their codes (asterisks). We included the TI here mainly because it is the first puzzle in ChH.

Below are two puzzles satisfying the one-to-one correspondence under some conditions.

(2014, 5 – 6) Let ТЕК be the least odd solution of the puzzle

ТЕК+ТЕК=ЧИФТ.

Find the product Ч·И·Ф·Т.

(A) 0 (B) 48 (C) 72 (D) 96 (E) none of these

The correspondence Ч↔1 is obvious. Since 2·ТЕК ≧1000 then Т≧5. Considering the last digits, we conclude that Т is an even number. To satisfy the minimality we take Т=6, К=3, И=2, Е=4. Thus ЧИФТ=1246.

Answer (B).

Comment. This easy TI gives an idea about what kind of didactical challenges we state by the decoding problems:

– finding some exact matches (Ч↔1) from the equation;

– using the additional requirements (minimal and odd) to determine the alternatives.

The math knowledge for elaborating the TI is basic. Here the student's sense of mathematics plays crucial role.

Note. ТЕК means ODD, ЧИФТ means EVEN.

(**2016, 7 – 8)** In the number puzzle Ч·Е·Р·Н·О = Р·И·З·Е·Ц the different letters represent different non-zero digits, and the same letters represent identical digits. Which number is represented by the letter "И" in the solution where the four-digit number РОЗИ is as large as possible?

(A) 2 (B) 3 (C) 4 (D) 6 (E) none of these

Reasoning. There are eight different letters in the puzzle, so exactly one of the numbers 1 to 9 is not involved. If the numbers 5 and 7 are present, they must be on both sides, i.e. to be in the place of Е or Р. After cancelling out the identical letters, we get Ч·Н·О=И·З·Ц. Among the other digits, the prime factors "2" are 1+2+1+3, which is an odd number, so not all even numbers can be present, i.e. 5, 7 and one even number are already missing. Then the number 9 must be present; if it is on one side, then 3 and 6 must be on the other. We get the options 9·8·1=6·3·4 and 9·4·1=6·3·2. The largest value of РОЗИ is attained in the second case for Р=8, О=9, З=6, И=3.

Answer (B).

Comment. This puzzle challenges slight knowledge in arithmetic but mostly the student's sense of mathematics. Indeed, the distribution of primes on the both sides of the equation should be felt, the rest is routine.

Note. ЧЕРНОРИЗЕЦ ХРАБЪР is the transcription of the tournament name ChH in Bulgarian, which gives the TI a genuine native beauty. In the puzzle, З is the letter from the Bulgarian alphabet; there is no reason to believe it is the digit 3.

4. Letters substututions

The coding by letters shift is known at least since Julius Caesar. We adopted the idea of cyclic replacement of the letters to challenge the students' computational thinking in early grades. Often the computational thinking disguises the sense of mathematics deficit.

(2018, 5 – 6) Each of the letters in the word ЧЕРНОРИЗЕЦ is replaced by the preceding letter in the alphabet and each of the letters in the word ХРАБЪР is replaced by the next letter in the alphabet. What is the correct coding?

(A) ЦДПМНПЗЖДХ ЦСБВЬС
(B) ЦДПМНПЗЖЕХ ЦСБВЬС
(C) ЦДПМНПЗЖДХ ЦСАВЬС
(D) ЦДПМНПЗЖДЧ ЦСБВЬС
(E) none of these

Answer (A).

Comment. The students are expected to fill the table below with a shift of the letters in the (first row second column) to the left according to the alphabetic order and a shift to the right in the (first row third column). We accommodated the answer at the first position (A) to save efforts and time to those who elaborated the algorithm accurately.

Plain	Ч Е Р Н О Р И З Е Ц	Х Р А Б Ъ Р
Cipher	Ц Д П М Н П З Ж Д Х	Ц С Б В Ь С

Note. The Bulgarian alphabet is

АБВГДЕЖЗИЙКЛМНОПРСТУФХЦЧШЩЪЬЮЯ

(2018, 7 – 8) The following coded message was received today:

ШЖСОПСЙИЖЧ ЦСБВЬС.

The answer must be the password

НАРОДНИ БУДИТЕЛИ

encoded with the same code. Which message should be sent?

(A) ОБСПЕОЙ ВФЕЙУЖМЙ
(B) ОВСПЕОЗ ВФЕЙУЖМЙ
(C) ОБСПЕОЙ ВФГЙУЖМЙ
(D) ОБТПЕОЙ ВФЕЙУЕМЙ
(E) ОБСПЕОЙ ВФЖЙУЖМЙ

Reasoning. The received message is obtained from ЧЕРНОРИЗЕЦ ХРАБЪР by replacing each letter in the Bulgarian alphabet by the next one.

Answer (A).

Comment. In this hard TI, in addition to the above one, the students should recognize the encryption method (the Caesar's code) and after that to apply it. Here the sense of mathematics is crucial: at first students should connect the information about the Bulgarian holiday Ден на НАРОДНИте БУДИТЕЛИ with the tournament ЧЕРНОРИЗЕЦ ХРАБЪР ciphered as ШЖСОПСЙИЖЧ ЦСБВЬС. Here each word matters, e.g. ***today*** contains the information about the Bulgarian holiday when the ChH tournament takes place.

5. Decoding problems about phrases

The first proper decoding problem in ChH was suggested by Tsvetan Pavlov. It follows the Champollion's idea to recognize something familiar in a text – in the Rosetta stone this was the name of King Ptolemy V (known from the Greek text) in the hieroglyphic script.

(**2002, 9 – 12)** Professor Solomonovsky gave Dr. Pythagorov an encrypted sentence of a famous mathematician, written in Bulgarian. The sentence contained the name of the mathematician. After having found that in encrypted text

тхра щтесх чн итйнунат т оьуьийта нюс смт сеанзняс щсцтщ хн битхурхрщ р хнцт хн хсюнцтщ зт оятхуншюр есчр щтесх арт емт бсоерфатщ жтяен ор яншпарж

the following Bulgarian pairs of letters (cipher↔plain) have been swapped:

т↔е, р↔и, х↔д, с↔о, ю↔к, н↔а, щ↔м

Dr. Pythagorov easily deciphered the sentence and found that the name of the mathematician is:

(A) Архимед (Archimedes)

(B) Лайбниц (Leibniz)

(C) Нютон (Newton)

(D) Кантор (Cantor)

(E) Дирихле (Dirichlet)

Reasoning. The practice is ***the quotation ends with the author's name***.[2)] If so, the options are only (A), (B) and (D), because the ciphered word is of 7 letters. It follows from the correspondence *н↔а* that the desired name is Лайбниц.

Answer (A).

Comment. The clumsy wording is typical for the PISA problems and it is considering as a bad style in the Bulgarian math education. For our excuse, there was no precedent to include TIs of this kind in math competition papers until that moment (as far as we know). Nevertheless, the TI contains very important feature of the decoding: informal approach to the text. The clue that a quotation ends with the name of the author is not mathematical at all. It is a pure challenge for the student's sense of mathematics.

Challenge. Try to decipher the whole sentence.

The informal approach evolves in the next TI.

(**2015, 7 – 8)** Като използвате ключа за кодиране на фразата[3)]

"Мачо ирлопргаче мпюта ра моцизабе ба хзарача",

determine what the encoding of Черноризец Храбър is.

(A) Кезнозирек Фзатъз

(B) Тезбозиред Фзанъз

(C) Тезносирег Фзабъз

(D) Кезнозирек Фратъз

(E) Тезнозирек Фзатъз

Reasoning. Here the second line is the ciphering of the first one. The vowels are preserved and the consonants are grouped in pairs, which are interchanged in the original and the coding, as follows:

к↔м, т↔ч, з↔р, п↔л, в↔г, ф↔х, н↔б, д↔ц.

Answer (B).

Comment. The formulation in the TI is self-reference. It includes also formatting: the initial sentence is coded just below itself (tacit information) and the coding is two-fold: the vowels are preserved and the consonants are grouped in pairs. Here partial decoding does not work – the answer and the distractors required full scale decoding. Since no math knowledge and skills are necessary, the only instrument for elaborating the TI remains the student's sense of mathematics. The statistics is: 81 respondents; 5 correct (2 in the bottom quartile, which means *gambling*; the other 3 are in the top quartile), 34 blank, 42 wrong. The large number of blanks indicate that many students did not felt the idea. We consider this as underdeveloped sense of mathematics for decoding problems in a large fraction of the advanced students.

6. Living-languages puzzles

A feature of the ChH competition papers[4)] for senior division is inclusion of math puzzles with linguistics elements. In these TIs, a math carcass[5)] is dressed in an exotic language (foreign for the Bulgarian students)[6)]. Some numbers are written in this language following the genuine grammar rules. We are going to illustrate what kind of challenge are these TIs for the student's sense of mathematics.

(2019, 9 – 12) The following equations between one-digit numbers are written in Estonian:

$$neli : kaks = kaks;$$
$$kaheksa : neli = kaks;$$
$$neli + kaks = kuus.$$

Then the product $neli \cdot kaks \cdot kaheksaneli \cdot kaks \cdot kaheksa$ equals:

(A) $nelikümmend\ kaks$

(B) $kakskümmend\ kaheks$

(C) $kuuskümmend\ kaks$

(D) $kuuskümmend\ neli$

(E) $kuuskümmend\ kaheksa$

Reasoning. Let us start with the math carcass rewriting the equations in math-mode. Denote $X1 = neli$, $X2 = kaks$, $X3 = kaheksa$, $X4 = kuus$. Then

$X1: X2 = X2,\ \ X3: X1 = X2,\ \ X1 + X2 = X4.$

Now

$X3 = X1 \cdot X2 = X2 \cdot X2 \cdot X2.$

which is possible only for $X2 = 2, X1 = 4, X3 = 8, X4 = 6$. Thus

$X2 \cdot X1 \cdot X3 = 64$.

Returning to Estonian: the given answer options are of the type

$X k\ddot{u}mmend\ Y$,

where X and Y stand for the names of digits mentioned above. **We can assume** they mean „$10X + Y$" or „$10Y + X$". Hence, 64 in Estonian **must contain** $kuus$ and $neli$ – there is only one option among the given ones.

Answer (D).

Comment. The math part is not a challenge at all for the target group. The tricky moments are **the assumption** and the **conclusion about the structure of the answer**. Here the sense of mathematics comes to do the job. The statistics is: 81 respondents; 43 correct (all with high scores in the Tournament), 18 blank, 20 wrong. Our conclusion: the students' sense of mathematics performed quite good.

(2015, 9 – 12) The following equations between one-digit numbers are written in Tagalog[7)]:

$dalawa + tatlo = lima$;
$tatlo + lima = walo$;
$dalawa + lima = pito$.

Just one among the five numbers mentioned above is not prime. Which of the following numbers written in Tagalog equals the sum pito+walo ?" (Tagalog uses the decimal system.)

(A) $labindalawa$
(B) $labintatlo$
(C) $labinlima$
(D) $labimpito$
(E) $labinsiyam$

Reasoning. The prime one-digit numbers are 2, 3, 5 and 7, so they should all appear here. The number 1 is not present among the numbers in equations, as this could only happen in $1 + 2 = 3$, while each summand appears in two equalities. Hence, 2 and 3 are not sums, so one of the equalities is $2 + 3 = 5$. From the first equality $lima$ is greater than the two summands and since it is less than $walo$ and $pito$, we must have $5 = lima$. Hence, 2 and 3 are $dalawa$ and $tatlo$ in some order. Then $pito$ and $walo$ are 7 and 8 in some order and their sum is 15. All answer options begin with $labin$ (or $labim$ before a bilabial consonant), followed by a word declared as a one-digit number X, so this should be the two-digit number 10+X. As $7 + 8 = 15$ and $5 = lima$, the answer is (C).

Comment. This TI is composed of a sophisticated math part and requires some linguistic analysis of the answer options. Having in mind quite limited time for dealing with the TI, we may assume that only a high-level synthetic competence allows the student taking it.

Note. Beyond the solution, the mentioned numbers are:

$2 = \mathit{dalawa}$; $3 = \mathit{tatlo}$; $5 = \mathit{lima}$; $7 = \mathit{pito}$; $8 = \mathit{walo}$; $12 = \mathit{labindalawa}$; $13 = \mathit{labintatlo}$; $15 = \mathit{labinlima}$; $17 = \mathit{labimpito}$; $19 = \mathit{labinsiyam}$.

7. Note

Bulgarian experience in extracurricular math activities related to linguistics went to the early 1981 (as far as we know). In this period, there were a club for gifted students associated to the Joint Center for Mathematics and Mechanics (a unique scientific center that incorporated the Institute of Mathematics of the Bulgarian Academy of Sciences with the Faculty of Mathematics and Mechanics of the Sofia University). In the framework of this club, several booklets were published and two of them were ***Mathematics and Linguistics*** by Radoslav Pavlov and Galya Angelova and ***The Names of the Numbers*** by Ruslan Mitkov. However, there were no problems of this type included in the math competitions. Nowadays, the math-linguistic competitions have their own place in the extracurricular activities. ChH is the only tournament that formally keeps alive the genetic ties of math linguistics with mathematics in secondary school.

8. Conclusions

We presented here how far the evolution of the didactical challenges about (de) coding went in the ChH competition papers examining students' sense of mathematics. In the junior level there are decoding TIs of recreational type. The intermediate level contains decoding TIs of higher difficulty intensifying the students' sense of mathematics. Elaborating complex coding TIs at the senior level requires the student's sense of mathematics to be amalgamated with the math knowledge and skills in a synthetic competence.

The paper presents a branch of the ChH didactical strategy, related to coding-decoding processes. Parallel to it, we are developing another branch in senior-level competition papers – decoding pseudo-cods of algorithms. Combining these ideas with the traditional math branches as arithmetic, geometry, probability etc., we hope that ChH becomes a platform for manifestation the students' talent in mathematics and subjects, which are closely related to it. As an Appendix, we submit some more TIs for the readers who enjoy this thematic.

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Borislav Lazarov proposed the conceptual framework of the study, the paper design and the didactical analysis. Ivailo Kortezov did the proof reading.

NOTES

1. The notation supposes to be decoded as "TI for 9 – 12 grade from the 2000 year competition paper". In the following, the TI code will have analogous meaning.
2. Similar practice allowed breaking the cipher used by the Japanese military attachés in 1942. Viewed: https://www.nsa.gov/Portals/70/documents/about/cryptologic-heritage/historical-figures publications/publications/misc/tiltman.pdf.
3. Translation: „Use the key to encode the phrase".
4. The competition papers are created by Borislav Lazarov.
5. Boyanka Savova submits the math parts of this type TIs of.
6. Ivailo Kortezov does the linguistic parts of the TIs.
7. Austronesian language.

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APPENDIX

(2019, 7 – 8) The coding is as follows: any vowel is replaced by the next letter in the Bulgarian alphabet and any consonant is replaced by the anterior letter. Which is the coding of ЧЕРНОРИЗЕЦ ХРАБЪР?

(A) ЦЖПМППЙЖЖХ ФСБАЬП
(B) ЦЖПМПСЙЖЖХ ФПБАЬП
(C) ЦЖПМППЙЖЖХ ФПБАЬС
(D) ЦЖПМППЙЖЖХ ФПБАЬП
(E) none of these

Hint. The Bulgarian alphabet is

АБВГДЕЖЗИЙКЛМНОПРСТУФХЦЧШЩЪЬЮЯ.

The letter C does not participate in the coding, which rejects the options (A) – (C).
Answer (D).

(2013, 7 – 8) In the following equations written in the artifitial language Sol-resol, the words mean one-digit natural numbers:

$$\boldsymbol{redodo + remimi + remimi = relala}$$

$$\boldsymbol{redodo = remimi - redodo.}$$

Which number corresponds to $relala$?

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

Reasoning. Since $remimi$ is a one-digit number and a sum of two $redodo$, it should be 2, 4, 6 or 8. If the $remimi$ is at least 4, the $redodo$ is at least 2 and the $relala$ exceeds 9, which is a contradiction. It remains $remimi$ to be 2, $redodo$ to be 1, so the $relala$ is 5.
Answer (E).

(2016, 9 – 12) If n is a number, let @(n) be equal to the remainder of 2^n when divided by 9. For example, @(4)=7, because the remainder of 2^4=16 when divided by 9 is 7. All words below denote non-zero digits in Maltese:

@(disgħa) = tmienja
@(tmienja) =erbgħa
@(erbgħa) = sebgħa
@(sebgħa) = tnejn
@(tnejn) = erbgħa
@(tlieta) = tmienja
@(ħamsa) = ħamsa
@(sitta) = wieħed
@(wieħed) = ?

(A) erbgħa (B) sebgħa (C) tmienja (D) disgħa (E) tnejn

Reasoning. As in the example above, we get @(1)=2, @(2)=4, @(3)=8, @(5)=5, @(6)=1, @(7)=2, @(8)=4, @(9)=8. Denote *D=disgħa, M=tmienja, E=erbgħa, S=sebgħa, N=tnejn, L=tlieta, H=ħamsa, I=sitta, W=wieħed*. Now the given identities read @(*D*)=*M,* @(*M*)=*E,* @(*E*)=*S,* @(*S*)=*N,* @(*N*)=*E,* @(*L*)=*M,* @(*H*)=*H,* @(*I*)=*W*. We need @(*W*). As we see, application of @ leads to $E \to S \to N \to E$, which according to the initial calculations is valid only for the triple 2, 4, 7 (we just need to find which is which). The only two different non-zero digits that yield the same digit other than the above are @(3)=8 и @(9)=8. We infer $M=8$, while 3 and 9 are L and D in some order (this is actually the right order, however that is irrelevant for the solution). Now @(8)=4 yields $4=E$, so $7=S$, $2=N$. The equality $@(I)=W$ can be only @(6)=1, that is, $I=6$, $W=1$. Finally, $@(1)=2=N=$*tnejn*.

Answer (E).

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