functions, while also making mathematics more engaging. This demonstrates that the software is more effective than traditional teaching methods, as it enables students to easily interpret and analyze the graphical representations of quadratic equations and functions (Wijaya et. al 2020). During the research with tenth-grade students, we utilized GeoGebra software. The integration of tools like GeoGebra further enhances the educational aspect, offering an interactive platform for students to visualize and understand exponential functions (Tuda et. al 2024). By enabling students to visualize and manipulate mathematical concepts, GeoGebra fosters deeper comprehension and greater motivation. However, for its successful implementation, adequate teacher training and equitable access to technological resources must be ensured (Aliu et. al 2025).

After we had traditionally explained quadratic equations and functions, the mistakes made by the students while solving the exercises were inevitable. The students mostly learned the formulas and the procedure of solving the exercises mechanically, and could not imagine how the equations of quadratic functions are represented in the graph.

3. The impact of using GeoGebra on the understanding of quadratic equations and quadratic functions

We conducted a study with two tenth-grade classes: in one, we used only the traditional method to explain equations and quadratic functions, while in the other, we incorporated mathematical software to teach the same unit. We then administered a test with identical exercises in both classes and obtained the following results. In Table 1 we marked 0 for no exercises solved, 1 for a solved exercise, 2 for two solved exercises, 3 for three solved exercises, 4 for four solved exercises, Class X/3 for students who learned with the classical method and Class X/4 for students who have learned through mathematical software. From the results we conclude that the class that used mathematical software during the learning achieved a better result in the test. Exercise II of the evaluation test was solved by a student in whose class the traditional teaching method was used, a mistake is seen in the exercise to find the peak of the function. Figure 2 shows where the student has mastered the formulas to find the peak of the function. However, his confusion is in the graphical representation of the vertex by setting the vertex points as zeros of the function.

Table 1. The number of exercises solved during the assessment test

Number of test exercises	**The number of students who have solved the exercises from class X/3**	**The number of students who have solved the exercises from class X/4**
0	10	0
1	13	10
2	7	11
3	7	9
4	3	10

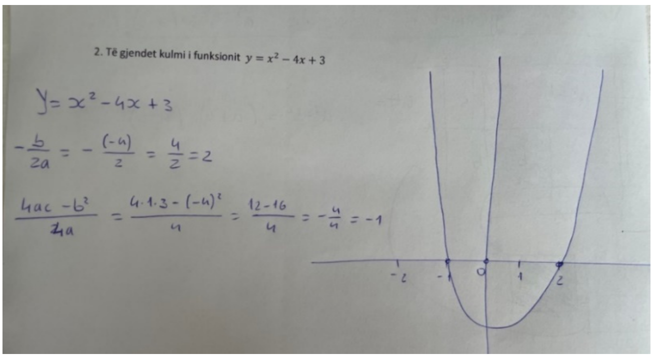

Figure 2. Exercise II of the test solved by the student from X/3

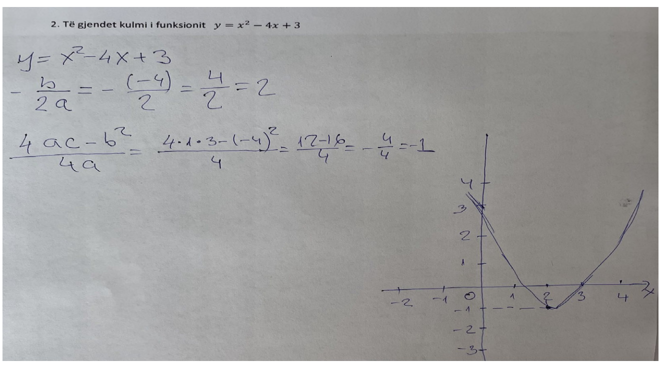

Figure 3. Exercise II of the test solved by the student from X/4

In Figure 3, the exercise was solved by an average student, but in whose class the modern method was used during the explanation of the quadratic equations and functions unit. It can be seen that the student has no problem at all with solving the exercise and presenting the function graphically.

In exercise III, the student was asked to choose a biquadratic equation. In Figure 4, the exercise was solved by a good student but in whose class the traditional method of explanation was used, and the student encountered difficulties in remembering the process of solving the exercise. The biquadratic equation is well replaced with $x^2 = t$, but then the student encountered difficulties while solving the created quadratic equation and did not come to the result.

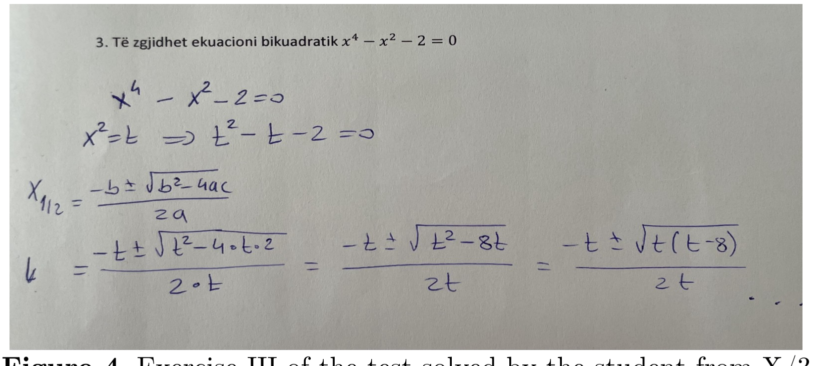

Figure 4. Exercise III of the test solved by the student from X/3

In Figure 5, the exercise was solved by a good student in whose class the modern method of explanation was used, namely for this problem the GeoGebra software was used, which shows a step-by-step solution to the exercise, and once the student has thoroughly mastered this type, they can successfully solve related exercises.

Now we consider exercise IV of the test. In Figure 6, we have the exercise solved by the student, a good student of the class where the traditional explanation method was used in that class, at first glance it can be seen that the peak of the function, the zeros of the function have been found exactly but their presentation on the graph is done completely wrongly. The student has confused the zeros of the function with its peak by placing them incorrectly on the graph and has not been able to present the final form of the function since the points on the graph are irregularly placed.

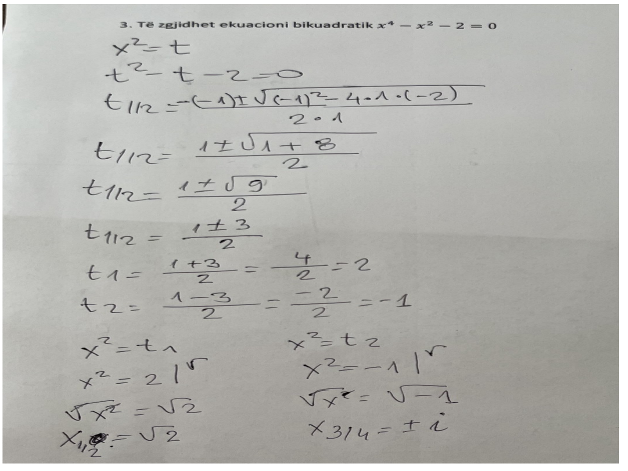

Figure 5. Exercise III of the test solved by the student of X/4

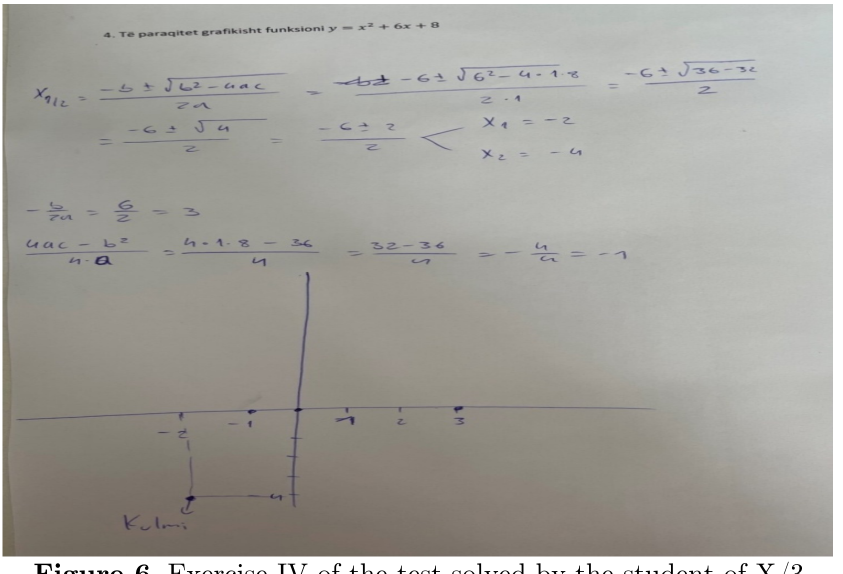

Figure 6. Exercise IV of the test solved by the student of X/3

In Figure 7, the exercise was solved by another good student in the class where the modern explanation method was used, that is, mathematical software was used and the student solved the exercise correctly and without mistakes. After the test results, we interviewed the student with the best success and the student with the weakest success.

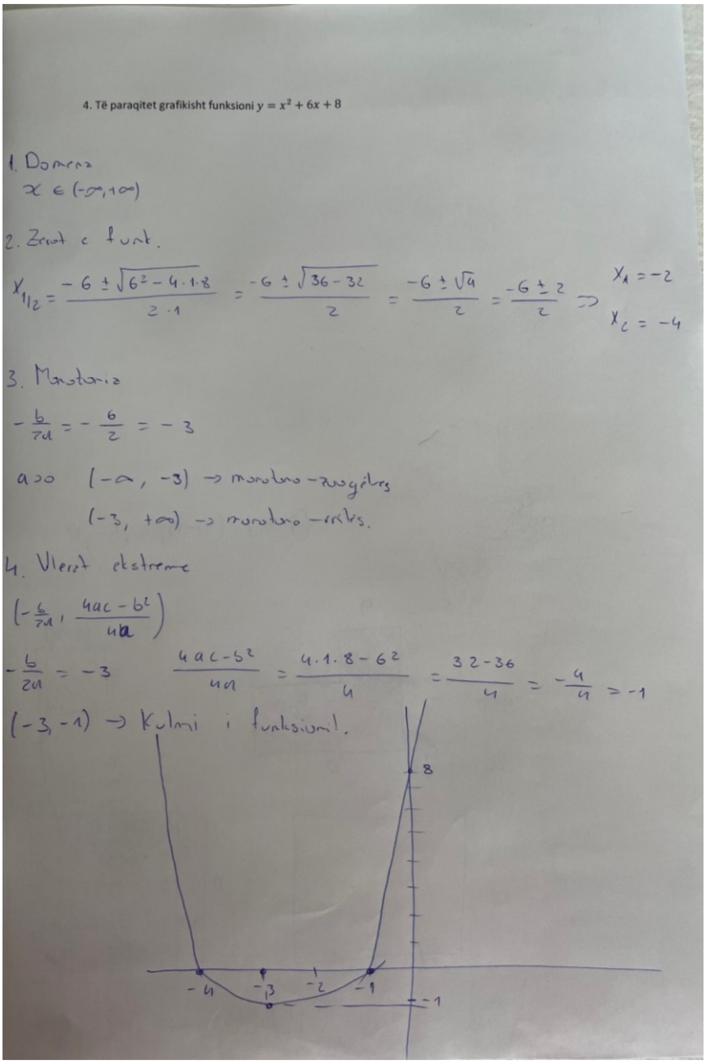

Figure 7. Exercise IV of the test solved by the student of X/4

Interview with the student with the best success

Interviewer: What was the problem you encountered while solving exercises with equations and quadratic functions?

Student: Acquiring formulas for solving quadratic equations, for finding the peak of a function, Viet's formulas, and other formulas.

Interviewer: Which math software do you use the most?

Student: GeoGebra

Interviewer: What math software has helped you the most when understanding quadratic equations and functions?

Student: They helped me concretize the exercise, especially on quadratic functions, that is, where the zeros of the function, the coordinates of the vertex and the shape of the graph of the function are placed.

Interview with the least successful student

Interviewer: What was the problem you encountered while solving exercises with equations and quadratic functions?

Student: I encountered difficulties in the algebraic part, i.e. in replacing numbers in formulas and then calculating them.

Interviewer: Which math software do you use the most?

Student: I don't use any of the software, as I'm not good at technology.

Interviewer: What math software has helped you the most when understanding quadratic equations and functions?

Student: I liked GeoGebra the most when you used it during the explanation in class because with it I saw the total selection of the exercise, i.e. step by step the entire solution process. But since I haven't practiced it myself to solve the exercises, I haven't been able to understand them well.

Students, in whose class the modern method of explanation is used, i.e., through mathematical thinking, think that the application of this method during the explanation of mathematics, especially the equation and quadratic functions unit, is the best way for them to 'basically understand the problems and exercises in this learning unit. Through this approach, students believe that their focus on learning mathematical topics has significantly improved, as they are now more engaged with technology and its application in explaining mathematical concepts, particularly equations and quadratic functions. This, in turn, enhances classroom engagement and simplifies the learning process. Students, in whose class the modern method of explanation is used, i.e. through mathematical software such as GeoGebra, think that the application of this approach during the explanation of mathematics, especially the equation and quadratic functions unit, is the best way for them to 'basically understand the problems and exercises in this learning unit. Through this approach, they believe that the focus on learning mathematical topics has significantly increased, as they are now more passionate about technology and its application in explaining mathematical units, particularly equations and quadratic functions. This, in turn, enhances the classroom experience and simplifies the learning process. High school students in X classes find it easier to work with GeoGebra compared to students in lower grades (Aliu et. al 2021).

The difference between the students who used the modern method and the students who used the traditional method of learning is clearly visible, not only in the test results but also during engagement in class or even solving homework, since the students who used the modern method of learning, that is, with the application of the software, their engagement in class and the choice of homework without mistakes was significantly greater than among students where only the traditional method was used.

After both classes that participated in the research were introduced to the two methods of explanation, the traditional and the modern method, we created a questionnaire about which of the methods the students understood the learning unit more.

The questionnaire

1. What was the method by which you best understood quadratic equations and functions?

I. Modern method II. Traditional method

2. Do you have difficulty using mathematical software during individual work?

I. YES II. NO

Table 2. Observed values

	Answers to the first question			
Answers to the second question	X\Y	YES	NO	Total
	YES	31	2	32
	NO	3	4	8
	Total	34	6	40

Table 3. Expected values

	Answers to the first question			
Answers to the second question	X\Y	YES	NO	Total
	YES	28	4	32
	NO	6	2	8
	Total	34	6	40

Table 4. Chi-square value calculation

O-observed values	E-expected values	$O-E$	$(O-E)^2$	$(O-E)^2/E$
31	28	3	9	0.32
2	4	-2	4	1
3	6	-3	9	1.5
4	2	2	4	2
				4.82

Common variables are given with the values X and Y given with the values {Yes, No}. After receiving the students' answers, we collected the data and created two tables for the values we received and the values we expected. Then we calculated the chi-square value using Table 4.

Next, we will calculate the value Chi-square:

$$X^2 = \frac{(31-28)^2}{28} + \frac{(2-4)^2}{4} + \frac{(3-6)^2}{6} + \frac{(4-2)^2}{2} = 4.82$$

With significance level $\alpha = 0.05$, we put $X^2_{1,0.05} = 3.841$, so the critical domain is $C = (3.841, \infty)$ and since $4.82 \in C$, we conclude that X and Y are dependent.

4. Discussion

This study aimed to explore the impact of using mathematical software, specifically GeoGebra, on tenth-grade students' understanding of quadratic equations and functions. The research was conducted in two classes with similar academic performance levels, where one class (X/3) was taught using traditional methods, while the other (X/4) integrated mathematical software into the learning process. The findings of this study indicate that the use of mathematical software significantly improved students' comprehension and problem-solving abilities related to quadratic equations and functions.

One of the key observations from this research was that students in class X/3, who followed traditional teaching methods, faced difficulties in visualizing quadratic functions and understanding the relationship between algebraic and graphical representations. Errors related to misidentifying coefficients, incorrect application of formulas, and challenges in solving quadratic equations were prevalent. Conversely, students in class X/4, who utilized mathematical software, demonstrated a higher level of accuracy in solving problems and a better conceptual understanding of quadratic equations and functions. The integration of technology enabled them to engage with interactive visualizations, making abstract concepts more tangible.

A crucial aspect of the research was the comparative analysis of test results between the two classes. The statistical evaluation revealed that students in class X/4 performed significantly better in solving quadratic equations and graphing quadratic functions. The use of Chi-square analysis confirmed that the success rate was higher among students who used mathematical software, highlighting its effectiveness in reducing common mistakes. This suggests that technological tools play an essential role in improving mathematical comprehension by allowing students to explore and interact with mathematical concepts dynamically.

Additionally, the study examined the motivational impact of mathematical software on students. Interviews with students from both classes indicated that those in X/4 found the learning process more engaging and intuitive. The ability to visualize problems and receive instant feedback through software applications contributed to a deeper understanding and greater interest in mathematics. In contrast, students in X/3 often relied on memorization without fully grasping the underlying concepts, leading to confusion in problem-solving.

These findings align with existing literature that supports the integration of technology in mathematics education. Previous studies have shown that digital tools enhance students' ability to conceptualize and apply mathematical principles effectively. The results of this study reinforce the argument that incorporating technology into the curriculum can facilitate a more effective learning environment, particularly in topics that require graphical interpretation and algebraic manipulation.

Based on these findings, it is recommended that mathematics educators integrate software tools into teaching methodologies to enhance students' engagement and understanding. Schools should consider investing in technological resources and training teachers to use digital platforms effectively. Future research can explore the long-term impact of mathematical software on students' performance and its applicability to other mathematical topics.

In conclusion, this study highlights the transformative potential of mathematical software in learning quadratic equations and functions. By bridging the gap between algebraic expressions and their graphical representations, digital tools provide students with a more interactive and engaging learning experience. The improved results of students in class X/4 serve as strong evidence of the benefits of integrating technology in mathematics education, paving the way for more effective teaching strategies in the future.

5. Conclusion

This study demonstrated that the integration of mathematical software significantly enhances students' understanding of quadratic equations and functions. Through a comparative analysis of two tenth-grade classes, it was observed that students who used software tools such as GeoGebra achieved higher results than those who followed traditional teaching methods. The findings confirmed that digital tools provide a more interactive, visual, and engaging approach to learning, leading to improved problem-solving skills and a deeper conceptual understanding of mathematical principles.

The results from the assessment test and Chi-square statistical analysis showed that students in class X/4 achieved higher accuracy rates and demonstrated fewer misconceptions compared to their peers in class X/3. The ability to visualize and manipulate equations graphically enabled students to correct errors more efficiently and understand abstract mathematical concepts with greater clarity. Furthermore, interviews with students indicated that the use of mathematical software increased motivation and interest in mathematics, making the learning experience more enjoyable and dynamic.

These findings emphasize the necessity of integrating technology into mathematics education. As digital tools become more accessible and user-friendly, educators should consider incorporating them into their teaching methodologies to support students' learning processes. This approach not only strengthens conceptual understanding but also prepares students for an increasingly technology-driven academic and professional landscape.

Given these findings, it is strongly recommended that educational institutions implement the use of mathematical software in their curricula. Future studies could investigate its impact on other mathematical topics and the long-term retention of mathematical concepts. Ultimately, embracing technology in education can transform the way mathematics is taught, making it more intuitive, effective, and engaging for students.

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Educational Technologies
Образователни технологии

BRIDGING THE GAP: A PEDAGOGICAL TOOL FOR TEACHING MATHEMATICAL MODELING WITH SPREADSHEETS

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Abstract. The widespread use of information and communication technologies (ICT) offers new opportunities in many topics of mathematics education. As science and technology are constantly evolving, information technology is becoming increasingly intertwined with education. Modeling, simulation and visualization are already proven methods in teaching subjects such as physics, chemistry or engineering. These methods can help students see connections more clearly and develop their creative thinking. This paper aims to further explore this direction in the field of mathematics education, with focus on differential equations. We chose spreadsheets as our tool to calculate and visualize the processes described by differential equations. We demonstrate a wide range of applications of differential equations through real-life examples, such as in modeling physical, biological, and economic processes. This method provides students a better understanding of the practical usefulness and applicability of these equations. The study thus shows how integrating ICT into mathematics education can help students gain a deeper understanding of the underlying mathematical concepts and improve their mathematical thinking and problem-solving skills. ICT tools enable teachers to use interactive and engaging teaching methods, resulting in an exciting and practical education for students. This paper outlines the potential of ICT in mathematics education, with a focus on the use of spreadsheets for modelling and visualization. It highlights the benefits of integrating technology into the classroom to enhance student learning and engagement.

Keywords: mathematics education; spreadsheets; differential equations; interdisciplinary education; advanced uses of ICT; real-world problems

1. Introduction

This study is an extended version of the previous conference paper which evaluates the usability of spreadsheets in education. In this paper we narrow our focus to differential equations aimed to support mathematics education by using ICT technologies to solve otherwise complex problems addressed by the field of modeling (Paksi et al., 2022b). The prepared teaching tool is available online[1].

This study is primarily aimed at undergraduates, especially those who are open to new methods and technologies. Integrating ICT into mathematics education offers significant benefits: it provides modern, interactive and illustrative teaching methods that help students understand theory and apply it in practice. In university-level education, traditional methods are often unable to arouse students' interest or deepen their knowledge. New teachers can particularly benefit from incorporating these innovative tools into their workflow, as they can convey complex mathematical concepts in a more efficient and motivating way. Such approaches not only develop students' problem-solving skills, but also introduce real-life applications of mathematical modelling.

The development of science and technology brought changes to the whole world. Information Technology (IT) is no exception either. Over the past few years the repertoire of available digital tools expanded and they also caught up in education (Oleksandr et al., 2023). At the same time the mentioned advancement raised the stakes: the required digital skills to master for general computer literacy are diverse.

To train people who successfully overcome modern challenges we must pay attention to the content of education and its presentation (Talhofer, 2017).

In the field of education, IT has gradually become indispensable in the last decade (Rodrigez-Jim´enez, 2023). The shift towards remote learning and the need for digital solutions further increased its importance. With the aid of IT, education can go on without significant interruptions in the digital space. It became essential to devise methods for instructing specialized subjects like physics, electronics, and chemistry. Fortunately, a solution was already in existence before the demand arose, as the field of modeling and simulation had been a researched discipline for quite some time.

There are many tools available for teaching mathematical models, but most of them require the usage of high-level programming language. The usage of Excel for teaching natural sciences is crucial because provides an opportunity for students to develop skills in data analysis, modeling, and scientific thinking (Pohoriliak et al., 2023).

Spreadsheet applications have a relatively long history and their usage often constitutes part of computer (and digital) literacy. Computational thinking (CT) surfaced as part of digital literacy and is mostly described as a problem-solving approach imported from computer science. CT got attention over the years because the skillset it promotes is universally usable (Borkulo et al., 2023). Hence, the task at hand was for educators to develop and refine the necessary tools, experiments, and teaching materials, making them available for educational purposes (Svitek et al., 2022).

A common feature of similar educational tools is that they all require a strong mathematical background to be effectively applied and to support deeper understanding of students (Serra & Godoy, 2011).

Today, crafting educational tools remains a challenging endeavor. It may seem logical that everyone can access the same content since most people have a computer at home. However, the reality is more intricate. To ensure access to the same content, individuals must be able to access the relevant digital environment. This may entail installing specific software, while in other cases, merely having a web browser suffices. When it comes to the former situation, the software can either be paid or free. If it demands high performance, users may encounter factors that disturb the user experience or even prevent access to the content. This is particularly relevant in the context of modeling and simulation, and from an educator's perspective, such barriers are unacceptable.

Public education systems typically prefer solutions that are freely accessible to all (Paksi et al., 2022a). Several companies have longstanding commitment to supporting education. Spreadsheet applications are part of the curriculum and widely used for various tasks in the labor market. In Slovakia, the Microsoft 365 software suite, including the aforementioned member of the software family, is available free of charge for educational purposes. In recent years, the emergence of Google Suite in educational institutions has diversified the range of tools used and made it more difficult to develop out of the box, platform-independent learning materials. Thus, the primary challenge remaining for educators in higher educational institutions (HEIs) is to adapt the teaching material to effectively facilitate the teaching of modeling simulation and visualization (MSV).

2. Pedagogical approach

We define methodological approach as a set of ideas, principles related to the nature of learning along which the educational process is implemented. The term pedagogical methodological approach covers the ideas about teaching and learning. The goal of pedagogical methodological approaches is to maximize the success of the educational process. It is not unique that different educational institutes and teachers combine multiple approaches during teaching sessions (Harizanov, 2023). This is necessary to meet the needs of the specific blend of students and the curriculum at the same time. Nowadays teachers can choose from multiple approaches, for this study we used and combined the problem-based learning, deep learning and interdisciplinary learning approaches.

Problem-based learning (PBL) is a pedagogical approach that has emphasis on theoretical foundations. It is an instructional method, where students are presented with an open-ended question or a real-world problem. The primary objective is to systematically gather information, develop a viable solution, present their results and express their own insights related to the topic. The chosen problem must be carefully selected to invite the students on a journey to carry out their own research, organize and evaluate the collected information. In PBL, the educational roles have undergone changes: the teacher assumes a mentoring role to guide students during their research, while students gain a prominent role in the problem-solving process. The PBL relates to a particular context and situation by engaging participants in the processing of authentic scenarios as opposed to abstract theoretical constructs (Csóka & Czakóová, 2021; Tempelmeier, 2016).

Deep learning utilizes understanding and thinking as the method's main pillars. It is characterized by thorough understanding of the fundamentals of the subject matter accompanied by critical thinking and the ever-promoted problem-solving of IT. The mentioned core competencies are complemented by collaborative work, communication, and the autonomy of one's own learning. Such learning can help cultivate positive beliefs and attitudes about oneself which provides motivation for continuous learning (Marton & Saljö, 1976a; Marton & Saljö, 1976b; Csoka et al., 2022).

Interdisciplinary learning is a pedagogical approach that draws a wide variety of perspectives from diverse academic disciplines. It not only introduces these viewpoints in the learning environment but also requires that collaborative tasks actively share, discuss, and integrate them. Interdisciplinarity is essential to solve complex, real-world challenges that draw from multiple science fields and also demand expertise. By directing students' attention to a particular problem or topic while exploring it through the perspective of multiple disciplines helps them to organize their knowledge their own way and supports their comprehension of their own intellectual maturation. Moreover, exposure to interdisciplinary learning can foster critical thinking and metacognitive skills (Zhu & Burrow, 2022).

3. Implementing MSV at a glance

The making of mathematical models is the process of encoding and decoding reality, in which a natural phenomenon is reduced to a formal numerical expression. There is an essential difference between the mathematical model and the laws of physics. The first is a representation of a particular system in mathematical terms, while the second is a general statement based on a physical theory. Modeling, simulation and visualization (MSV) are all tools that help the user better understand, predict, test and optimize real-world systems and processes without having to work directly with the real system. The first step in the process is to create the mathematical model. We set the parameters required for the model according to our best knowledge, then run the simulation to imitate and reproduce the behavior of the real system (Meng et al., 2020). After that, we evaluate the results. If they meet the defined expectations, we have reached the end of the process, otherwise we examine whether the mathematical model was correct, or if the parameters need to be adjusted.

3.1. Role of MSV in education

MSV plays a useful role in teaching mathematics and science subjects (Bilbokaite, 2016), as it allows students to gain a deeper understanding of abstract concepts and phenomena (Niazi & Temkin, 2017). These pedagogical tools help students not only to interpret knowledge as passive recipients, but also to become active participants in the learning process. One of the main advantages of MSV is that they provide concrete and tangible examples of mathematical and other scientific principles. Borba also states that many processes and concepts can be tied to visual representations, which can be built to help the understanding of the hidden mathematical structure (Borba, 2005). In this way, mathematics and scientific knowledge do not remain at the level of abstract theories but can also introduce their application in real life.

Creating models requires students to analyze phenomena, form associations, derive algorithms, test and reformulate hypotheses. Modeling is particularly useful in constructivist learning environments where students explore, experiment, create, collaborate and reflect on real-world problems. MSV also helps students to connect theoretical knowledge with practical application. For example, when a student calculates the velocity (including air resistance) of a free-falling body, or simulates a chemical reaction, they apply their theoretical knowledge to a real problem. In this way, one will experience how the concepts and formulas learned can be used in real life, resulting in motivation and deeper understanding. In addition, modeling and visualization help students develop their critical thinking and problem-solving skills (Smirnov & Bogun, 2007). The visualization provides an opportunity for students to experience the process. For example, in the case of this study, differential equations are given a form that is easy to interpret even for laics, thus promoting a deeper understanding of the topic (Csoka2021a; Svitek et al., 2022). When a student analyzes a complex model, they employ a set of skills related to computational thinking: logical- and algorithmic thinking, information processing, recognition and solving of subtasks. These skills are useful not only in mathematical and scientific fields, but also in other areas of life.

3.2. The information and communication technologies in mathematics education

Information and Communication Technologies (ICT) are processes, methods and laws related to the recording, analytical-synthetical processing, storage, retrieval and dissemination of scientific information (Mikhailov et al., 1966; Wellisch, 1972). The goal of ICT is to make the mentioned processes more effective, safe and easy to use. The use of ICT plays an increasingly important role in the teaching of various educational subjects. Such development cannot be attributed alone to the overwhelming use of digital tools, but one must acknowledge that ICT is capable of transforming and enriching education in ways that match with modern pedagogical goals. ICT has the potential to spark students' scientific interest and engage them (DeWitte & Rogge, 2014). The use of these technologies increases the interactivity and enjoyment of learning (Bowers & Berland, 2013) and can be a tool for bridging the ever-growing gap between mathematics and other subjects (Jehliˇcka & Rejsek, 2018).

Interactive learning environments, simulations and online labs allow students to experiment and practice without exposing themselves to real world risks. This way aids students in developing a deeper understanding of natural sciences and encourages curiosity and discovery. In addition, the digital space makes it possible to model and observe events that would not be possible in real life due to lack of time, financial background, resources, too large-, small scale or are simply too dangerous. Furthermore, ICT helps learners to connect what they have learned (theory) with lifelike problems, situations, events (practice). For example, in mathematics and physics they can use software and computer modeling to understand abstract concepts and apply them to solving real-world problems (Oliveira & Nápoles, 2017). In this way, students experience the practical application of the knowledge acquired during learning, which should be the goal of the whole teaching process.

3.3. Spreadsheets in education

Spreadsheets represent a group of application packages used for tabular calculations. Spreadsheets are inexpensive, can be run on machines with low-end specifications, and are widely used by companies, institutes, and all levels of education. In addition, introduction to spreadsheets is part of the general IT curriculum (Arganbright, 1993).

In general, students can support their ideas with numbers and graphs, or keep a record of daily activities using spreadsheets (Abramovich et al., 2010). However, not many people use spreadsheets in the classroom to draw new conclusions about a topic, since spreadsheets are usually not developed with the intention of provoking new ideas or creating an environment of debate. Depending on the topic, spreadsheets like Microsoft Excel can be involved in high school education on a basic level, and in HEIs for more advanced tasks and topics.

Spreadsheets are present virtually everywhere in today's engineering field: from elementary numerical analysis in the general engineering field to software quality control, cache-based parallel processing systems in the electrical industry. Spreadsheet simulation models can be used as a platform to understand the mechanisms behind a discrete event, as well as for system dynamics approaches. Their advantages include gaining software knowledge in a short period of time, wide availability and smooth usability (Skafa et al., 2022).

3.4. The interdisciplinary role

The use of mathematics in interdisciplinary education (IE) can provide various scientific and pedagogical benefits, states (Jehlička & Rejek 2018). This approach connects mathematical thinking with other disciplines and real-world problem-solving situations, which expands students' cognitive and intellectual skills. Interdisciplinary education allows the integration of mathematics with the help of computer science into other disciplines such as computer science, physics, biology, engineering, i.e. STEM areas (Doig & Jobling, 2019). As a result, students gain a broader perspective and can see the relevance of mathematical principles and methods to real life and other disciplines. This improves general knowledge and scientific literacy.

An interdisciplinary approach fosters problem-solving ability and critical thinking. Students encounter complex problems that are not limited to just one area or subject but involve knowledge from several diverse disciplines of different educational levels (Lucas et al., 2019). As a result, students must use different approaches to solve problems. It is expected that this process helps to develop their creativity and analytical skills. In interdisciplinary education, the application of mathematical principles and methods to real problems creates real value. Students learn how to use mathematics via real-world examples, such as cooling a tea, the spreading of cancer cells, financial decision-making, data analysis, environmental protection, and more. This practical applicability helps to transfer knowledge to real life.

4. Process of implementation

Differential equations are mathematical tools that can be used to model changes and processes in specific systems. Differential equations are useful in many scientific fields, such as physics, modeling chemical reactions, analyzing biological systems, describing economic processes, and many other applications. One of their most important features is to describe the relationships that govern changes over time in a system. So, if we are interested in how the parameters of a system change over time or how the properties of a particular system change in a particular area, we can model these processes using differential equations. Differential equations also play an important role in scientific research and engineering practice. With their help, the behavior of different systems can be predicted, analyzed and optimized. In addition, they can solve complex problems that would be difficult to deal with other methods.

4.1. Continuous models

By continuous models we mean continuous state and continuous-time differential equations, which can be classified as members of the Differential Equation System Specifications (DESS) group (Zeigler et al., 2018).

Differential equations are a common means of describing natural, technical, and economic processes, i.e., continuous mathematical models are often possible to describe with their help. The theory of Ordinary Differential Equations (ODE) deals with the study of such models among others. These studies focus primarily on the solving of different types of tasks, primarily examining the conditions under which the task will be correctly set. It is rarely possible to produce a solution in a closed form (i.e., to specify it using formulas that contain known and easy-to-evaluate functions). Therefore, from a practical point of view, an approach in which we seek the solution in an approximate form with the help of some numerical method is unavoidable. These methods allow us to produce a numerical solution with high accuracy and reliability (Atkinson et al., 2009). For differential equation models, a derivative function is used to specify the speed of change in the status variables. At any given time of the time axis, for a given state and input value, only the speed state changes are known. From this information it is necessary to calculate the state that will occur at any time in the future.

When we want to express this in the form of an equation, we need a variable that represents the current state. In our case, it is represented by the $z(t)$ state variable. The current input $u(t)$ indicates the speed at which the actual content is changing, which is expressed by the equation

$$\frac{dz(t)}{dt} = u(t)$$

where output $y(t)$ is equal to the current $z(t)$ state. By further, shaping the equation, we can get the classic ODE state equation representation:

$$z^{\cdot}(t) = f(z(t), u(t))$$

Most continuous-time models are actually described (or converted) in the form of an equation that does not give an explicit value to the state $z(t)$ after a certain period of time. Thus, in order to get the state trajectories, the ODE must be resolved. The problem is that obtaining a solution to the equation is not only exceedingly difficult, but in some cases, impossible. Very few ODEs have analytical solutions in the form of known functions and expressions. This is the reason why ODEs are usually solved with numerical integration algorithms that provide approximate solutions (Zeigler et al., 2018).

4.2. Explicit Euler method

Let $[a, b]$ be the interval, where the initial value problem $\dot{y} = f(t, y)$ solution must be found, where $y(a) = y_0$. Instead of searching for the solution of a differentiable function, which satisfies the initial problem $\{(t_i, y_i)\}$ set of points are generated and these points are used for approximation, where $y(t_i) \approx y_i$. To determine the set of points that approximately satisfy the differential equation, we first select the abscissas of the points. For simplicity, we divide the $[a, b]$ interval into M equal parts and select the mesh points (Mathews & Fink, 1999): $t_i = a + ih$ and $i = 0,1, \dots, N$, where $h = \frac{b}{N}$.

The value h is the step length. Next, an approximation can be given for the differential equation.

$y' = f(t, y)$ $[t_0, t_N]$ where $y(t_0) = y_0$.

The standard Euler-method can be written as follows:

$$t_{i+1} = t_i + h \qquad y_{i+1} = y_i + \mathrm{h}f(t_i, y_i), \qquad i = 0,1, \dots, N-1$$

The above described can be easily translated into an algorithm (see Listing 1), that determines the substitution values of the chosen differential equation at an arbitrary interval in fixed steps.

It is important to mention that there are many other ways to prepare the previous algorithm. The first feature of the algorithm we created is that it will not determine the values on the interval set by the user, instead extending to a maximum 1 step larger from the right limit.

Listing 1: Pseudocode for Explicit Euler method

```
Input: a, b, y0, h, f
  a - Initial time
  b - Final time
  y0 - Initial value
  h - Step size
  f - Function
begin
     y(1) = y0
     t(1) = a
     i = 1
     while t(length(t)) < b
          y(i+1) = y(i) + h * f(t(i),y(i))
          t(i+1) = t(i) + h
          i = i+1
     end
end

Output: t, y
  t - Time array
  y - Value array
```

4.3. The examined platforms

The User Interface (UI, see fig. 1) consists of two main parts: on the side down the user can set the initial values (I) the specific values (II) related to the exact problem to be modeled and the equation (III) itself, while the other part contains a XY Scatter chart showing the results. The initial values are the following: model simulation start time, model simulation end time, initial value and the step size expresses the detailedness. The problem specific values are model dependent and vary by each model, therefore they must be specified manually. These values have further usage because the model-specific equation resides in (III). The results of the actual model are shown