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BINET CONSTRUCTION IN ONLINE STEREO 3D VISUALIZATION

S. Zabunov, [1]M. Gaydarova
[1]*University of Sofia St. Kl. Ohridski*

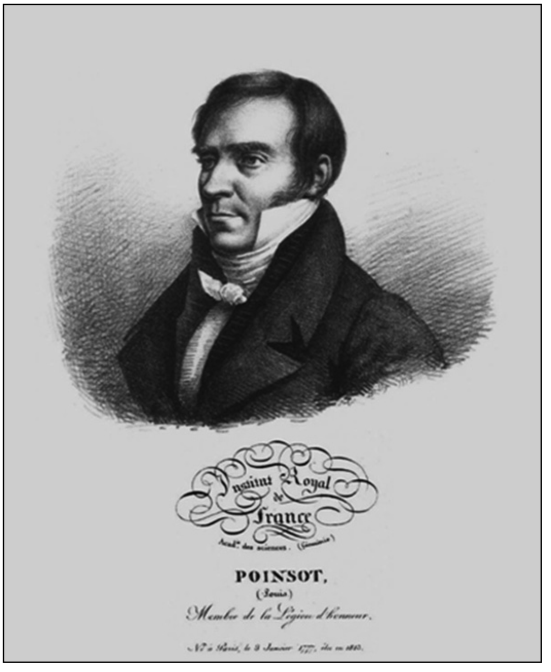
Abstract. The present paper describes an interactive online stereoscopic 3D visualization aimed at e-learning rigid body mechanics. The visualization is realized using a numerical simulation. This learning tool demonstrates free rigid body motion while visualizing the Binet construction. The visualization and the underlying simulation was developed with the idea to help students in their understanding of Newtonian mechanics, mastering its underlying mathematical apparatus and creating a method for observation of phenomena hard to realize in laboratory conditions, such as absence of gravity. The Binet construction is interactively presented in 3D graphics with all of its inherent elements, such as invariant ellipsoid, invariant sphere, angular momentum vector trajectory, etc. The simulation also demonstrates a large number of vector and scalar parameters. This material is directed to university students taking the Analytical (Mathematical) Mechanics courses and Theoretical Physics and General Physics courses. It may be used at any university – the simulation is published with free access on the Internet without restrictions of any kind (**http://ialms.net/sim/**).

Keywords: Binet construction, Poinsot construction, Interactive simulation of free rigid body motion, mechanics in stereo 3D e-learning

Introduction

Poinsot and Binet constructions are celebrated approaches used for many years in teaching rigid body dynamics. Poinsot and Binet constructions are similar in nature, but reveal different properties of free rigid body motion; hence, their separate discussion is essential. Poinsot construction was developed almost 180 years ago by the French mathematician and physicist Louis Poinsot (1851). Nearly at the same time the Binet construction was created by the French mathematician, physicist and astronomer Jacques Philippe Marie Binet. Since then these constructions help students visualize and understand the complex nature of free rigid body motion phenomenon.

This article will try to present to university students an interactive stereoscopic 3D computer visualization of free rigid body motion that is accessible online without charge. The simulation demonstrates the Poinsot and Binet constructions. This way it intends to create a visual and interactive online learning tool used for teaching the Newtonian



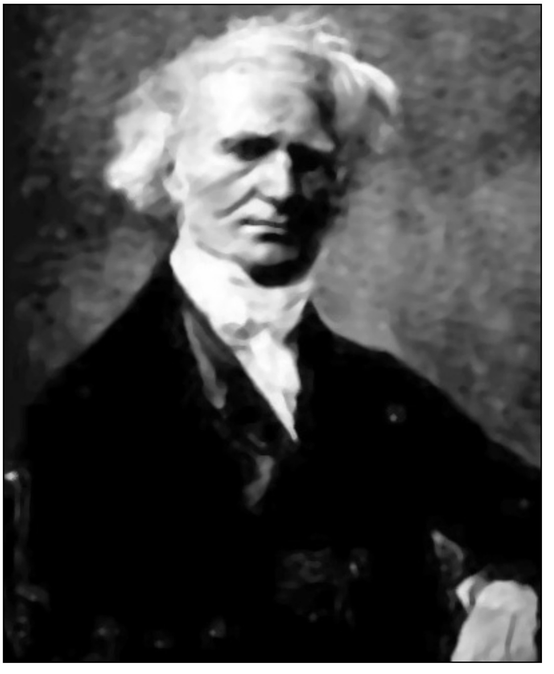
Louis Poinsot (1777-1859)[1)]

mechanics, mastering its underlying mathematical formalism and assuring a method for observation of phenomena hard to realize in laboratory conditions – e.g. the absence of gravity. The Poinsot and Binet constructions are represented in an interactive stereoscopic 3D graphics (Zabunov, 2012). The constructions' inherent elements, such as invariant ellipsoids, invariant sphere, invariant plane, polhode, herpolhode, etc., are shown along with a large number of involved vector and scalar parameters.

The presented e-learning tool targets students taking the Analytical Mechanics courses at the Faculty of Mathematics and Informatics in Sofia University, Bulgaria and the Theoretical Physics and General Physics courses at the Faculty of Physics in the same university. The simulation disclosed herein is not restricted from use in other universities around the world as its access is maintained and shall be maintained free of charge without restrictions.[2)]

Poinsot and Binet constructions physical insight provided by the interactive stereo 3D visualization

Beyond the mathematical formalism that may be better comprehended with the assistance of the visualization, the authors' intention was to bring Poinsot and Binet



Jacques Philippe Marie Binet (1786-1856)[1)]

constructions in motion visualized using 3D graphics based on numerical simulation. These two constructions were for decades a helpful and valuable tool in teaching rigid body motion. Their value has been since proven many times and hence they are taught in theoretical mechanics and analytical mechanics courses in universities throughout the world and are presented in many books (José & Saletan, 1998; Hand & Finch, 2002; Landau & Lifshitz, 1976; Goldstein et al., 2001; Arnold, 1989; Whittaker, 1937).

These constructions have value for the learner, because the student may visualize the constants that govern free rigid body motion. Thus the intended benefit goes beyond assimilation of the mathematical formulas. The real benefit for the student is the visual perception of a real phenomenon and an unreal superimposed construction revealing quantities inherent to this phenomenon. As a result students make conclusions not through recreating the motion as a projection of the mathematical formal definition, but inversely: students observe the visual presentation of the phenomenon and derive conclusions and formalize its behaviour mathematically. The connection between the visual presentation of the phenomenon and the mathematical formalism is the construction that is visually superimposed on the phenomenon. The construction becomes the bridge between the observation of the phenomenon and the taught mathematical formalism. This inverse approach is not novel as the constructions exist for more than 150 years, but till now their application was hampered due the lack of adequate visualization.

The authors note that no artificial visualization may substitute reality – a simulation may only create visualization that resembles reality to a given extent adequate for proper education.

Comparison with modern e-learning visualizations

Authors shall briefly note the reader that simulations of physical phenomena do exist and have a long history. Some modern solutions are: (1) The Massachusetts Institute of Technology (MIT) 3D physics simulations[3)]; this product is online and free. It does not offer rigid body simulation; (2) Software product ThreeDimSim (offering a free test version). The software is a desktop application and offers simulation of mechanical systems of rigid bodies. The simulations are not conservative. No stereoscopic visualization is offered. Poinsot and Binet constructions are not implemented. The product is not presented online nor is it free to use in its full capabilities; (3) Software product NewtonPlayGround v1.53 (offering a free test version).

The herein presented visualization realizes a number of novel approaches by representing free rigid body motion online and in conservative force fields. The stereoscopic visualization offers spatial apprehension of the relations between the observed elements. All that said, we remind the reader that there were no simulations of Poinsot and Binet constructions found online in the thorough personal research we performed at the moment of writing of this manuscript.

The Binet construction

For the sake of simplicity the mathematical details of rigid body motion will be avoided. A brief description of Binet construction follows. If the rigid body is rotating freely (no external torque is present, nor internal friction is observed), the angular momentum vector $\vec{L}$ is constant by value and magnitude. The second constant of motion is the rotational kinetic energy E_{Krot}. Both invariants lead to constants of motion known as constraint ellipsoids ε_ω and ε_L and invariable plane γ. The constraint ellipsoids are named after the two vectors: angular momentum $\vec{L}$ and angular velocity $\vec{\omega}$. The meaning of the constraint ellipsoids is the following. Vector $\vec{L}$ is constraint to point on ellipsoid ε_L while vector $\vec{\omega}$ is constraint to point on ellipsoid ε_ω. These constraints are direct consequence of the two constants of motion mentioned above and the inertial properties of the rigid body. Vector $\vec{L}$ is a constant vector in the space reference frame, but once transferred into the body reference frame, as $\vec{L}'$, it is no longer constant and, generally, continuously rotates (Fig. 1). Nevertheless vector $\vec{L}'$ is still constrained in its magnitude, because rotation does not change magnitude. Vector $\vec{L}'$ has constant length and is restricted to point on the surface of a sphere σ' with radius L. It follows that vector $\vec{L}'$ should point to the intersection of this constraint sphere σ' and its constraint ellipsoid $\varepsilon_{\mathbf{L}}'$ (Goldstein et al., 2001; Arnold, 1989). This intersection is shown on Fig. 1 in the first octant.

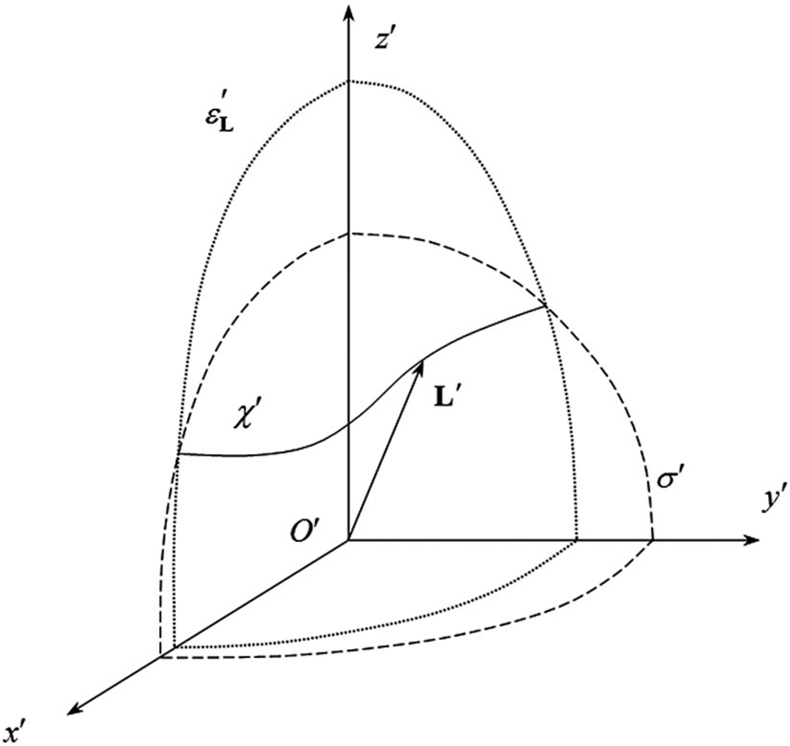


Fig. 1. Binet construction in the body reference frame (first octant)

Thus vector $\vec{L}'$ is constrained to point at curve $\chi' = \varepsilon_{\mathbf{L}}' \wedge \sigma'$, representing the intersection of both $\varepsilon_{\mathbf{L}}'$ and σ'. In the space reference frame the angular momentum vector is $\vec{L}$ and is constant, while the constraint ellipsoid is $\varepsilon_{\mathbf{L}}$ and is rotating. In the space frame the constraint sphere is σ and it is also rotating, but this fact does not present any significant consequence.

The simulation shows constraint ellipsoid $\varepsilon_{\mathbf{L}}$ and sphere σ with contours in the space reference frame, where they are rotating along with the body. The sphere is shown

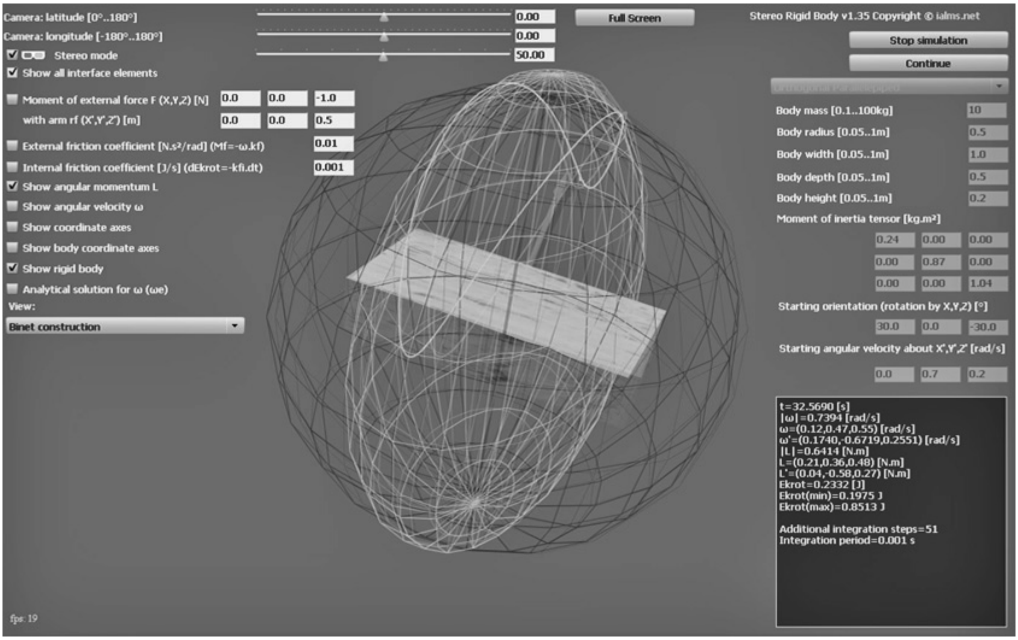


Fig. 2. Binet construction in simulation, stereoscopic visualization: use red-cyan anaglyph stereo-glasses. Angular velocity vector drawing the intersection of ε_L' and σ'. General case – orthogonal parallelepiped[4)]

in dark colour. The intersection is drawn by the tip of vector $\vec{L}$ in light colour (Figs 2 and 3). Students are capable of observing the Binet construction under different initial conditions and examine the motion under the visualized constraints. The intersection is a closed curve which is taco-shaped in the general case. In contrast to Poinsot construction, with the Binet construction the simulation discloses immediately why curve χ, on which vector $\vec{L}$ points, is taco-shaped - it is the intersection of an asymmetric ellipsoid and a sphere. Like in the Poinsot construction, the boundary case of the Binet construction with the body being a symmetric top turns curve χ into a circle as well (Fig. 3). Again, the students may see visually why this is so – the intersection of an axially symmetric ellipsoid and a sphere is a circle.

Conclusion

Although a computer simulation cannot duplicate reality, it may help in studying physical phenomena by visualizing them interactively. Utilizing the Binet construc-

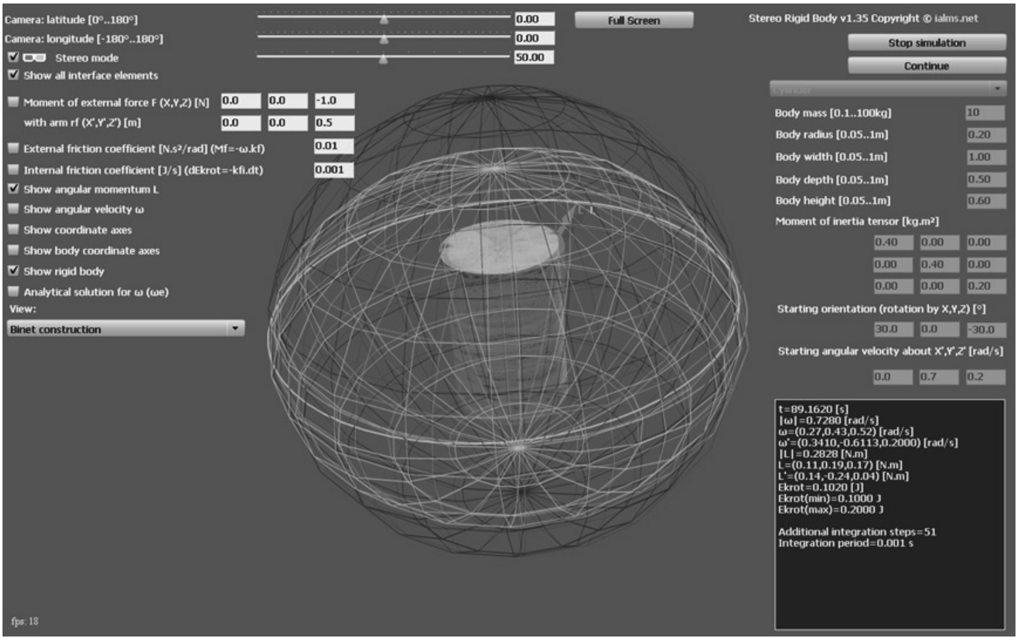


Fig. 3. Binet construction in simulation, stereoscopic visualization: use red-cyan anaglyph stereo-glasses ; symmetric top[4)]

tion simulation provides the student with a visualization of the studied phenomenon superimposed with the visualization of the construction itself, thus creating the correct cognitive path for studying free rigid body motion. Still another benefit of the presented visualization is the interactive capabilities of the simulation. It becomes individualized and lets him or her place the simulation in states, which are most relevant in demonstrating the sought knowledge.

NOTES

1. The portrait is from Wikipedia.
2. The access is from the web address http://ialms.net/sim/ owned solely by the first author. User manual of the simulation is available at http://ialms.net/sim/3d-rigid-body-simulation-tutorial**.**
3. http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/21-Molecules3d/21-Dynamics3d320.html
4. See interactively online http://ialms.net/sim/

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✉ **Dr. S. Zabunov** (corresponding author)
Zh. k. Hipodtuma, 104/A/7/21
1612 Sofia, Bulgaria
E-mail: s14@ialms.net