at least one more swap, and if it is between A and C, the original state is restored, hence $E(3)=4$.

Concerning $E(4)$, four people form 6 unordered pairs, so $E(4)\geq 6$. To see that $E(4)=6$, we proceed as follows, starting with the initial state *Aa, Bb, Cc, Dd*:

– Swap *AB*, then *CD*; the state is now *Ab, Ba, Cd, Dc*.

– Swap *AC*, then *BD*; the state is now *Ad, Bc, Cb, Da*.

– Swap *AD*, then *BC*; the state is now *Aa, Bb, Cc, Dd* and we are done.

Of course, the sequence so far obtained 0, 2, 4, 6, ... will not be followed by 8, since five people form 10 unordered pairs, yielding $E(5)\geq 10$. To see that $E(5)=10$, we proceed as follows, starting with the initial state *Aa, Bb, Cc, Dd, Ee*:

– Swap *AE*, then *AB*, then *BE*; the state is now *Ab, Ba, Cc, Dd, Ee*.

– Swap *CE*, then *CD*, then *DE*; the state is now *Ab, Ba, Cd, Dc, Ee*.

– Swap *AC*, then *BD*; the state is now *Ad, Bc, Cb, Da, Ee*.

– Swap *AD*, then *BC*; the state is now *Aa, Bb, Cc, Dd, Ee* and we are done.

Now let us proceed with the solution of the originally posed general question.

Solution. There are at least $n(n-1)/2$ swaps, as there are that many pairs of people. We will prove that the total number of swaps must be even. Call a pair of Easter eggs "inverted", if the egg with the former letter belongs to the person with the latter one. Let T denote the total number of inverted pairs; at the start $T=0$. A swap performed by adjacent (by letter) people changes T by 1. Any swap is equivalent to an odd number of swaps performed by adjacent people, so it changes the parity of T. Since at the end $T=0$, the total number of swaps must be even. We will show that these two restrictions generate the answer, i.e.,

$$E(n)=n(n-1)/2 \text{ if this number is even and } E(n)=n(n-1)/2+1 \text{ if not.}$$

We construct the required examples for even n by induction, in a way that if $n\equiv 2 \pmod 4$, then the last two swaps are performed by the same pair of people (thus we keep track where the extra swap has gone). We already know that $E(2)=2$ and $E(4)=6$.

Assume first that n is a multiple of 4 and that the people $P_1, P_2, \ldots, P_n$ can perform the task in $n(n-1)/2$ swaps. Take two more people, A and B. For the enlarged set of $n+2$ people we apply the same procedure as for n people with the following modifications:

– If k is odd, each swap of the type P_kP_{k+1} is replaced by the sequence of three swaps AP_k; P_kP_{k+1}; $P_{k+1}A$, which has the same effect (in particular the Easter egg of A is returned to himself).

– If k is even, each swap of the type P_kP_{k+1} is replaced by the sequence of three swaps BP_k; P_kP_{k+1}; $P_{k+1}B$, which has the same effect (the numbering is modulo n).

– Each swap between non-consecutive people in the above numbering is kept unchanged.

At the end the initial state is restored (by the induction hypothesis) and each pair of people has performed exactly one swap, with the exception of AB; so we perform at the end two consecutive swaps between these people and finish the construction.

Assume now that $n=4m+2$ and that the people $P_1, P_2, \ldots, P_{4m-1}, P_{4m}, A, B$ can perform the task in $n(n-1)/2$ swaps, with the last two swaps being both AB. Take two more people, C and D. For the enlarged set of $n+2$ people we apply the same procedure as for n people with the following modifications:

– If k is odd, each swap of the type P_kP_{k+1} is replaced by the sequence of three swaps CP_k; P_kP_{k+1}; $P_kP_{k+1}D$, which has the same effect (in particular the Easter egg of C is returned to himself).

– If k is even, each swap of the type P_kP_{k+1} is replaced by the sequence of three swaps DP_k; P_kP_{k+1}; $P_kP_{k+1}D$, which has the same effect (the numbering is modulo n).

– Instead of performing the two swaps AB, at the end we perform the sequence of 6 swaps for the group of 4 people A, B, C, D as shown while discussing the case E(4)=6.

– Each swap between nonconsecutive people in the above numbering, as well as between one of the people A, B and one of the type P_k, is kept unchanged.

At the end the initial state is restored (by the induction hypothesis) and each pair of people has performed exactly one swap. This finishes the induction step for even n.

Assume now that n is even and the people can perform the task in the minimum number of swaps as described above. The numbering is done in a way that the extra swap (if any) is between non-adjacent persons. Take one more person, A. For the enlarged set of $n+1$ people we apply the same procedure as for n people with the modification:

– If k is odd, each swap of the type P_kP_{k+1} is replaced by the sequence of three swaps AP_k; P_kP_{k+1}; $P_{k+1}A$, which has the same effect (in particular the Easter egg of A is returned to himself).

– Each swap between non-consecutive people in the above numbering is kept unchanged.

At the end each person the initial state is restored (by the induction hypothesis) and each pair of people has performed exactly one swap (except for the one extra swap if $n=4m+2$). Note that for even n, the numbers $n(n-1)/2$ and $n(n+1)/2$ have the same parity. Thus for odd n we also conclude that the least number of swaps is $n(n-1)/2$ if this number is even and $n(n-1)/2+1$ if not.

Note. The induction can be also organized via a uniform induction step from n to $n+4$.

The sequence thus generated was new for the Online Encyclopedia of Integer Sequences; now it is added there as [2]. It is twice the sequence A054925 and is thus linked from there to several seemingly distant combinatorial results, like the number of edges in a median graph, walks in lattices and much more, listed in [1].

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EVOLUTION OF THE CONTENT AND QUALITY OF STANDARDIZED TESTS IN MATHEMATICS IN UKRAINE

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Abstract. The evolution of the content and quality of written state standardized final and entrance testing in mathematics in Ukraine, from the first attempt to implement in 1994 until now, is considered in the article. We describe the features and stages of development of Ukrainian standardized tests in mathematics (external independent assessment and state final attestation), analyze the causes of successes and failures and also prospects for their further improvement. Based on our own statistical experiment, we compare the quality of mathematical preparation of Ukrainian graduates who entered the pedagogical universities of Ukraine in 1994, 2005 and 2020. According to this experiment, we draw conclusions about the existing problems of ensuring the proper quality of mathematics education in Ukraine and give recommendations on possible solutions to these problems.

Keywords: mathematical education; state final attestation; external independent assessment; entrance exams to the university; the quality of mathematical preparation

Introduction

Using of standardized testing in education remains one of the most discussed and debatable topics in modern pedagogical science. There are many scientific publications on this topic – from journal articles to monographs (see Berliner 2011, Kuncel, 2018; Lingard,2013; Miner, 2000, etc.). At the same time, the tone of discussions on the need for such tests, ways how they should be organized and the impact of them on society in general and on the teaching community in particular varies from enthusiastic and positive to critical and negative (see Claiborn, 2009; Garrisson, 2009; Johnson, 2010; Sahlberg, 2015; Wagner & Dintersmith,2015, etc.).

We keep a moderate position and believe that such a complex phenomenon as standardized testing has its own individual characteristics in the educational system of each country, and therefore, its assessment should be balanced and based on a fundamental investigations, including statistical. In this article we consider the evolution of the system of standardized tests in mathematics in Ukraine, as well