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Research Insights
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ADVANTAGES OF USING GEOGEBRA SOFTWARE WHEN EXAMINING THE FLOW AND DRAWING A GRAPH OF A FUNCTION

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Abstract. For a long time, the term function has been treated as a mandatory topic in mathematics curricula around the world. It aims to emphasize the importance of this topic of mathematics in higher education, at technical faculties. In addition to the classic way of learning blackboard and chalk, today all over the world a variety of interactive mathematical software is often used in the study of the topic functions. The software helps students to learn how to recognize functions, to examine their properties, and draw them accurately. In this paper, the importance of the mathematical term function in higher education at technical faculties is stated. It also lists the ambiguities that students face when mastering the topic of functions. Then, software for drawing graphs of functions (material that is taught in the subject Mathematics 1 at technical faculties) is listed to help students master that material more easily. Finally, a comparison is made in the results of the final exams in Mathematics 1, before students are recommended to use GeoGebra and after using the software. A conclusion is drawn from the results and the advantages of using the mathematical software are presented.

Keywords: GeoGebra; function; graph; results

1. Introduction

The mathematical notion of function is learned from primary education. However, primary and secondary school students face difficulties in understanding this important mathematical term. In the paper we will mention the ambiguities about the functions that prevail among the students of the technical faculties in higher education. Students often have the task of examining the flow and drawing a graph of a function, the solid students most often know how to examine all the function properties, but at the end they make a mistake when drawing the graph. The weaker ones have problems examining the properties. Quite often they do not know or make mistakes in finding and drawing asymptotes, in examining parity, etc. Confirmation of this are the results obtained of the final exam in the subject

Mathematics 1. At the final exam, students, among other tasks, had the task to examine the flow and draw the graph of the function.

This paper will present the results of the exam in the mentioned subject in the February exam period. We will list the students' mistakes and show some of their graphic drawings. Then the results of the April session, conducted after the consultations in which an exercise was conducted with the GeoGebra software on the mentioned topic, will also be stated.

2. Review of Related Studies

Nowadays, software is presented to us as the most applicable tool for students, because of students' connection with technology. The term Open Source Software (OSS) allows user to download any softwares that are available and suitable for the users. Until August 2010, there are more than 240,000 software projects that have been registered in SourceForge.net which is the world's largest open source software development site[1]. Softwares which are similar to OSS and related to mathematics instructions such as SAGE, FreeMat, GeoNet, JLab, Maxima, Axiom, YACAS, JsMath and others are ready to be downloaded and used in teaching and learning. GeoGebra as a Dynamic Geometry Software can be used as an effective tool in learning by way of visualization to promote learning and enhance understanding. Similarly with the utilizing of Geometer's Sketchpad as a visualization tool in learning mathematical concept found by (Teoh and Fong 2005; Royati et al. 2010) showed that the use of GeoGebra enhanced the students' performance in learning Coordinate Geometry. GeoGebra is a highly used and attractive software for students. Using it, students can construct figures and gain a higher understanding. In the classical method, students construct and practice accuracy themselves. As a result, the chances of giving up mastering the curriculum are high (Mollakuqe et al. 2021) using GeoGebra software gave pedagogical, methodological, and statistical data into teaching circle properties. Praveen and Kwan (2013) discuss in detail the use of GeoGebra software to conduct learning of circles in mathematics. Martinovski (2013; Quinlan 2016; Segal, Stupel, and Oxman 2016[2] promoting GeoGebra as a tool for technology use in mathematics teaching and learning.

Kieran (1992) explores the problems in understanding the term function whether due to insufficient commitment of students or misunderstanding is associated with inappropriate approach to teaching by teachers. Tall (1996) deals with the changing perceptions of functions and calculus in recent years, both in terms of research into cognitive development of concepts and curriculum developments using computer technology. One purpose of the function is to represent how things change. With this meaning it is natural to move on to consider the calculus concepts of the rate of change (differentiation) and cumulative growth (integration).

R. S. Uddin (2011) within his research set out to explore the function of GeoGebra, as a pedagogical tool and mediating artifact in the teaching and

learning of transformation of functions in secondary school mathematics; and whether interaction with these virtual manipulatives enhance the understanding of mathematics concepts. He concludes that the development of mathematical ideas and concepts through computer-based teaching and the role GeoGebra plays in the understanding of and visualization of certain mathematical concepts in high school algebra topics.

Michael, S., Ami, H. and Margaret, S. (2013) in their study describes teacher learning in a teaching experiment consisting of a content-focused methods course involving the mathematical knowledge for teaching function. Prospective and practicing teachers in the course showed growth in their ability to define function, to provide examples of functions and link them to the definition, in the connections they could make between function representations, and to consider the role of definition in mathematics. E. Ruhama (1990) shows how one may approach the question of teachers' knowledge about mathematical topics.

In many countries, the first concepts of calculus (such as functions) are taught by looking at examples, noticing their properties and generalizing from them in some implicit ways. Students have no means to discuss the general truth of a statement, or to examine the validity of a theorem, relative to the mathematical field. This knowledge is nonetheless required by teachers at the university level. The question is, therefore, if it is possible to organize activities for beginning calculus students, which would nevertheless lead them to working on statements and validity of theorems. In his paper B. Immanuel (2003) presented a teaching approach related to the concept of function, which aimed at leading students working within a graphic milieu to producing, discussing, and testing the validity of mathematical statements and theorems.

M. Kocaleva et al. (2016) analysed and compared the results of the electronic testing (e-testing) for the subject Mathematics 2 within University "Goce Delcev" – Stip using statistical data processing. Electronic testing covers the topic "Integral". L. Loku et al. (2019) in their paper research is aimed at data analysis of student's outcomes for the subject mathematics at university level. The results obtained from the overall testing, is processed with statistical data analysis (demographic characteristics, descriptive statistics, frequency distribution). E. Karamazova et al. (2021) analyzed and compared the results of the final exam for the subject Operating Systems for the students from different academic year: one from academic year 2018/2019 when the teaching was performed in a classical way and the second one from academic year 2019/2020 when the teaching was performed online through the platform Microsoft Teams. Both group of students are from Faculty of computer science at University "Goce Delcev" – Stip and they study the subject Operating Systems in the fourth semester. The aim of the paper is using descriptive statistics and other statistical methods to draw a conclusion which way of learning gives better results in student achievement.

R. Trifonov et al. (2018) presented the procedure for determining a conditional extremity of a function with two variables, presented that the point is finding the curve in the space obtained as a cross section on two surfaces and by using free software they have visualization of the conditional extremum. Authors use GeoGebra.

R. Atanasova-Pacemska et al. (2016) analysed the perceptions and attitudes about the use of ICT tools for visualization as a "modern" approach for solving geometry problems in primary schools in Macedonia. The obtained results of the research are processed with the software package SPSS19.

D. Jovanovska et al. (2015) gave a proposal about how to improve the evaluation process in mathematics by using electronic tests created by multimedia software known as Wondershare quiz creator software. Then, comparison is made between students' outcomes when electronic tests were used, and their outcomes gained by old- fashioned testing system pencil/paper. From the research results, they conclude that the method of testing is very helpful for students and teachers.

The education of mathematics and achievement of formal knowledge of mathematics is an important tool for its understanding and application in other sciences. On the other hand, mathematics is not very popular among the students. R. Atanasova-Pacemska et al. (2015) present their research which was conducted among students in the final year of primary education, locally in Stip, Republic of Macedonia, to determine the factors that influence the formation of students' attitudes for mathematics in schools. The results of the survey they were processed with statistical software SPSS 21. The ANOVA was applied to determine whether there is a significant difference between students' attitudes towards mathematics and the factors.

The paper of S. Pacemska et al. (2014) is focused on processes of modernization of teaching mathematics in primary schools by using ICT. K. F. Hollebrands (2007) highlighted that new learning opportunities are provided in technological environments which potentially help students to engage with different mathematical objects and level of understanding. ICT also adds a new dimension to the teaching and learning of Mathematics by helping students to visualize certain mathematics concept[4]. Literatures had shown that the advancement of computer has brought great innovation and thus school teachers need to be competent in using computers so that they would maximize its use in teaching and learning (Kumar, Rose, and D'Silva, 2008).

Teaching and learning with the use of technology has many advantages such as providing greater learning opportunities for students (Roberts 2012) enhancing student engagement (White 2012) and encouraging discovery learning (Bennet 1999). In the teaching and learning of Mathematics, especially geometry, it is important for students to be able to imagine, construct and understand construction of shapes in order to connect them with related facts. Therefore, a computer will assiststudentsin imagining and making observations (Dogan and Icel2010).

3. Objectives of the Study

– To find a solution how to improve the results of the final exam

– To find a solution to make the topic of functions more accessible and more interesting to the students for whom this research is intended

4. Statistical Techniques Used in the Present Study

GeoGebra software and Excel t-test were used for data analyzing and deriving conclusions.

5. Data Analysis and Interpretation

In the main study the focus is on emphasizing the need for new teaching activities in the teaching process for the subject Mathematics 1 in the dispersed studies in the city of Kavadarci at the Faculty of Natural and Technical Sciences at the University Goce Delchev. In Kavadarci is placed one smaller, regional teaching center within the University Goce Delchev – Stip, with smaller number of students compared to the main seat of the Faculty of Natural and Technical sciences in Stip. Mostly local students with different success are enrolled here. There are excellent students but there are also many with poor success. The idea for this research came from the results of the exams in the subject Mathematics 1. Quite often the students had problems with the task of examining the flow and drawing a graph of a function. The teaching was mainly realized with chalk and blackboard. Due to the very extensive teaching content in the mentioned subject of the classes, several tasks were solved using chalk and blackboard. After the completion of the planned material and the realized colloquia, there is a final exam. From the reviewed group of 42 students, 30 received the final exam in the January-February exam period. More detailed information about the group of 42 students can be found in Table 1.

Table 1. Information about students

Variables		N
Group		42
Age	18-21	28
	other	14
Gender	F	27
	M	15

Out of the registered students only 3 had correctly solved the task to examine the flow and to draw the graph of the function. The other ones had an error in examining parity, error in finding asymptotes, error in finding extremes, error in finding crossing points, error in drawing a graph. So, there were 2 students who had exactly examined the property of the function they were given and yet they had not drawn the graph exactly.

The following is an overview of some of the student drawings:

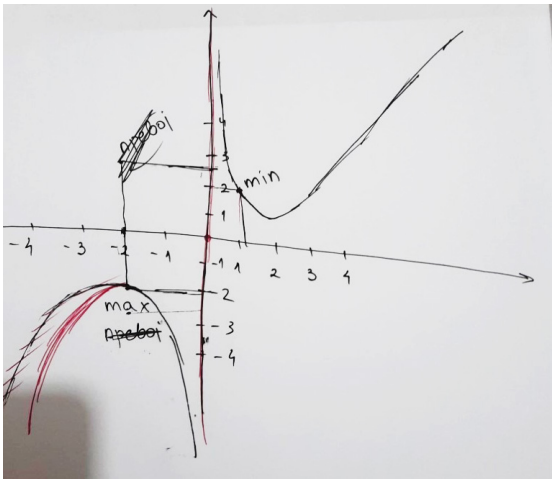

Figure 1. Graph of the function $f(x) = \dfrac{1+x^2}{x}$ drawn by a student at the final exam

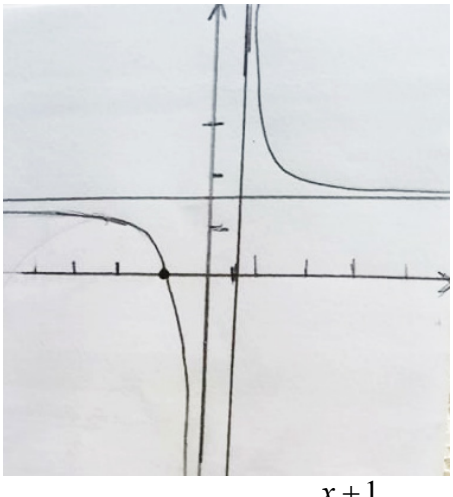

Figure 2. Graph of the function $f(x) = \dfrac{x+1}{2x-3}$ drawn by a student at the final exam

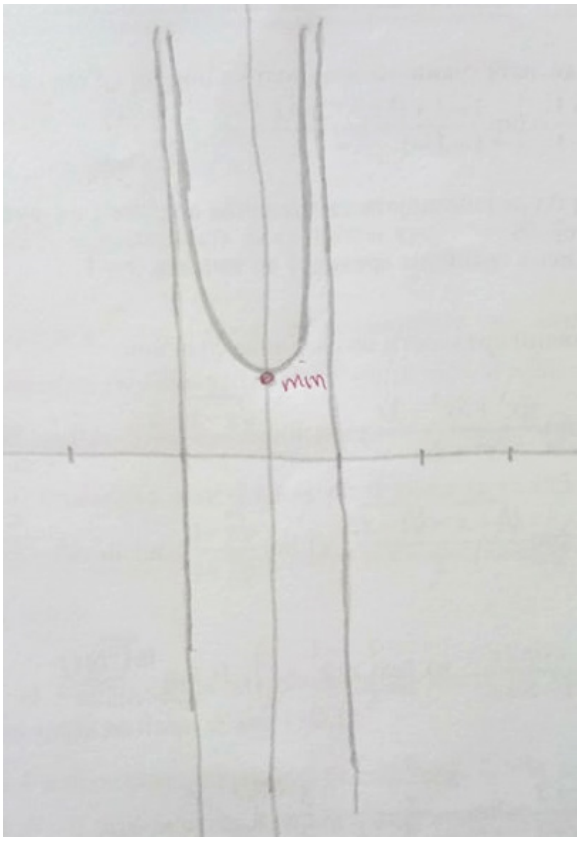

Figure 3. Graph of the function $f(x) = \frac{1}{1-x^2}$ drawn by a student at the final exam

From the few drawings we can see the large number of mistakes and difficulties of the students for this part. The teaching content for the subject Mathematics 1 is extensive and according to the curriculum, as much as it has provided, so much we have in the realization of the teaching dedicated to the part for examining the flow and sketching the graph. From the results we can see that those classes are not enough for the students to master that part of the material. If there was time, of course, mathematical software would be inserted to overcome the ambiguities that students have about the stated content.

After the exam in January-February, the first session after finishing the course, we decided to call the students for consultations and advise them to use mathematical

drawing software to help them overcome the ambiguities and identify mistakes. Without thinking, we recommended it to be a GeoGebra. GeoGebra is a mathematical software used worldwide at all levels of education. We made the decision about the software quite quickly because we knew that we only need a few hours with the students to explain how to install and use it, how to check their results and to see the mistakes. GeoGebra finds the extremities, the transition points if any, the asymptotes, draws a graph and so on. From graph we can see and check that we have examined the parity accurately of the function, to see if there are intersections with the coordinate axes, the intervals of monotony and convexity and concavity.

During consultations we learn GeoGebra software and solving GeoGebra tasks. The functions whose graphs with errors were previously listed were also drawn. From the figures themselves, the students saw their own mistakes and overcame the vague things. The graphs of the previous listed functions in GeoGebra are given in Figure 4, 5 and 6.

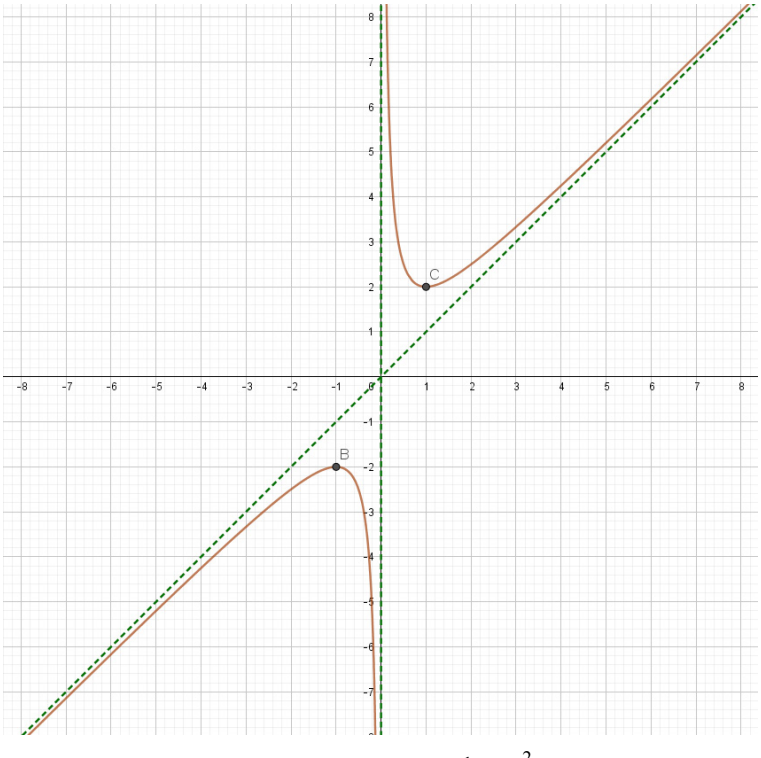

Figure 4. Graph of the function $f(x) = \frac{1+x^2}{x}$ drawn by GeoGebra

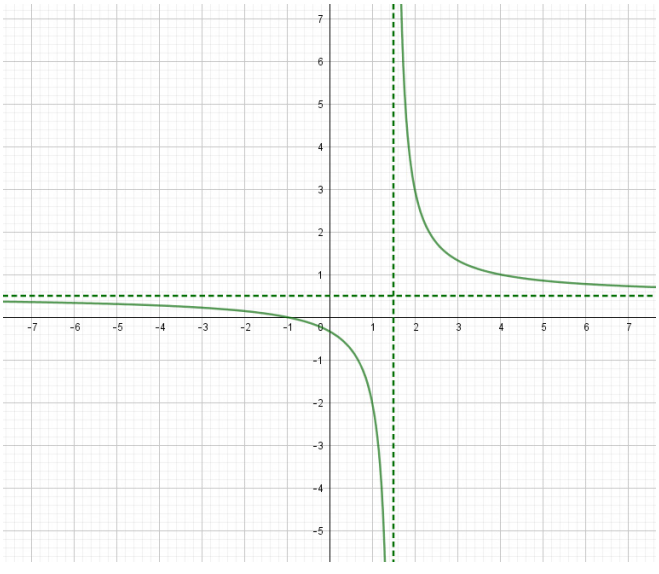

Figure 5. Graph of the function $f(x)=\dfrac{x+1}{2x-3}$ drawn by GeoGebra

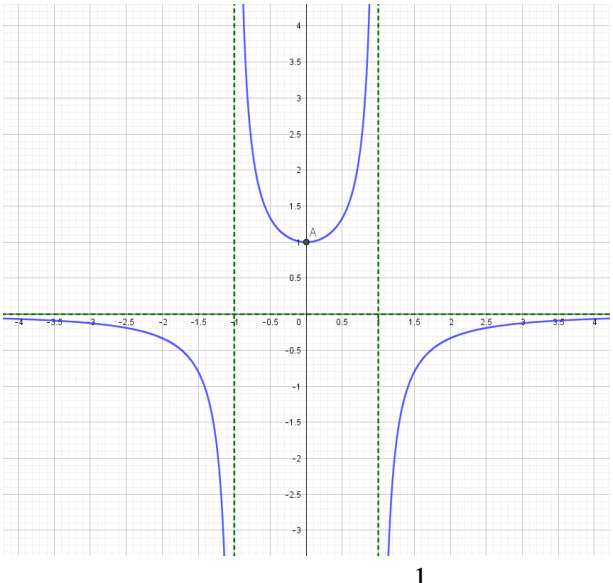

Figure 6. Graph of the function $f(x)=\dfrac{1}{1-x^2}$ drawn by GeoGebra

The students quickly mastered the software i.e., the part of the software that they needed. When practicing tasks at home, they first tried to solve the task and if they encountered a problem, they sought the solution with GeoGebra. Some students solved the task from the beginning to the end and at the end with GeoGebra they just checked the solution and saw the accuracy of the task. Before the exam in the April exam period, the students were invited to another consultation with which we aimed to see if there is an improvement in the students' knowledge. Major improvements were noted.

The test results in February session are shown in the Table 2.

Table 2. Students' achievements in the February session

Number of students	30
Passed	6
did not pass	24
student achievements	Grades: Number of students 10: 2 9: 1 8: 1 7: / 6: 2 5: 24

Statistics is given in Figure 7.

Statistics	
n	30
Mean	5.6333
σ	1.4716
s	1.4967
Σx	169
Σx^2	1017
Min	5
Q1	5
Median	5
Q3	5
Max	10

Figure 7. Statistical data for exam in February session for subject Mathematics 1, Faculty of Natural and Technical Sciences Kavadarci, UGD Stip

From Table 2 we can see that there were 30 students who have taken the exam, but the passing was insignificant in the February session. In the follow there are the results in April session

Table 3. Students' achievements in the April session

Number of students	31
Passed	23
did not pass	8
student achievements	Grades / Number of students: 10 / 1; 9 / 4; 8 / 4; 7 / 6; 6 / 8; 5 / 8

Statistics is shown in Figure 8.

Statistics	
n	31
Mean	6.7097
σ	1.4634
s	1.4876
Σx	208
Σx^2	1462
Min	5
Q1	5
Median	6
Q3	8
Max	10

Figure 8. Statistical data for exam in April session for subject Mathematics 1, Faculty of Natural and Technical Sciences Kavadarci, UGD Stip

From Table 3. it is obvious that there are many passed students with different passing grades. It is proof that the consultations held and the recommended GeoGebra software helped the students to overcome the ambiguities and to pass the exam.

To determine whether the students' achievements exam of subject Mathematics 1 in the February and April sessions depend on learning the topic of Flow Testing

and Sketching a Function Graph without and with the help of GeoGebra software respectively, the following hypotheses were analysed:

– H_0 (Null hypothesis): There is no statistically significant difference between the achievements of students who took Mathematics 1 in the February session without consulting on how to use GeoGebra software when learning to sketch a flow and drawing a function graph and the achievements of students in the April session after the consultation.

– H_1 (Alternative hypothesis): There is a statistically significant difference between the achievements of the students who took the exam in Mathematics 1 in the February exam period without consultations on how to use GeoGebra software in learning to sketch the course and draw a graph of the function and achievements of students in the April session after the consultations.

We performed a t-test to test these hypotheses using two – tailed distribution and paired samples. We get the p-value 0.962803. This means that there is less disagreement between our data and the null hypothesis. In other words, when the p-value is grater then 0.05 is more likely that the group of students using GeoGebra and the group of students who don't use GeoGebra is the same. 5% or 0.05 is the statistical significance. In this case we will accept the null hypothesis and we can say that there is no statistically significant difference between groups of students.

Table 4. Excel T-test

Grades	Number of students	Number of students
10	2	1
9	1	4
8	1	4
7	0	6
6	2	8
5	24	8
p-value	0.962803	

Considering the results of both sessions we can conclude that the consultations are very important and that it is good to advise students in learning mathematics 1 especially on the topic of examining the flow and sketching a graph of a function to use the software GeoGebra, although the t-test showed that there was no significant difference in the results of the students in the two sessions before and after the consultations in which the examples were solved with software.

5. Conclusion

Recently, especially with the outbreak of the Covid 19 pandemic, there have been many changes in the teaching process. Computers have become dominant,

and so have mobile phones. Advanced technology offers many possibilities and all of them should be used to the maximum.

Our research has also shown that it is good to use GeoGebra software to improve knowledge and results on functions in mathematics1, although the test showed that there was no significant difference in students results in the two sessions, but of course not to omit books, printed and electronic, as they are the basis of learning. The benefits and advantages of GeoGebra software are enormous. Critical thinking, understanding and interest are much greater when working with this software compared to not using the software. Then with the use of GeoGebra: more knowledge is gained, tasks are solved quickly and easily, excellent exam results are achieved.

The classic way of learning with chalk and blackboard and notebook and pencil/pen are suitable for teaching and learning, but the probability of errors is very high. Uncertainty, backlogs, and the idea that learning is difficult in math teaching research can be removed using GeoGebra software. With the help of GeoGebra software, more students were motivated to study mathematics, especially the topic of functions, which is also the subject of research. Students gained the courage to solve and check the results and to notice the mistakes.

In the teaching process in mathematics1 next school year we will set aside classes for mastering and using the GeoGebra software.

NOTES

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