Problems
Задачи

A NEW FORMAL GEOMETRICAL METHOD FOR BALANCING CONTINUUM CLASSES OF CHEMICAL REACTIONS

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Abstract. In this article a new formal geometrical method is developed for balancing continuum class chemical reactions. Here are treated new continuum classes of aliphatic hydrocarbon chemical reactions which possess atoms with integer oxidation numbers. All considered continuum reactions are reduced to a set of hyperplanes, which intersection is a hyperline that contains the required coefficients of reactions. Also, particular reactions derived from the general continuum classes are balanced in such a way that they do not lose their continuum properties. To this method is given an advantage, because the so-called chemical ways for balancing chemical reactions are inconsistent. Actually, here offered geometrical method is the first scientific method, which treats chemical reactions as *n*-dimensional geometric entities. By this method, the author proved again that balancing chemical reactions does not have anything to do with chemistry, because it is a pure mathematical subject.
Keywords: geometrical method, chemical reactions, balancing.

Introduction

The best method of balancing chemical reactions would be one which could be applied to all oxidation-reduction reactions. Presently, there are such methods in chemistry and mathematics and they are created by virtue of algebraic principles.

The aim in the balancing of an oxidation-reduction reaction should be to secure a stoichiometrically correct final reaction and the method applied should emphasize the fundamental phenomena of certain class of reactions and take into account whatever other factors may be involved in a particular case which may modify the course of the reaction.

Generally speaking, balancing chemical reactions is an excellent topic for students who have chemistry as a major subject of study (Risteski, 1990). Mass balance of chemical reactions is one of the most highly studied subjects in chemical education. In fact, balancing chemical reactions provides a tremendous demonstrative and pedagogical example of interconnection between chemistry and linear algebra. In chemistry there are lots of so-called *methods* for balancing chemical reactions, but all of them have limited usage, because they hold only for some elementary chemical reactions. Actually, they are not methods, just particular procedures founded by virtue of experience, but without any formal criteria. A survey of the references which treat problem of balancing chemical reactions through the prism of chemistry is given in the previous author's research works (Risteski, 2007a; 2007b; 2008a; 2008b; 2009).

Most current chemistry textbooks generally support the *ion–electron procedure* as the general balancing tool that best makes use of *chemical principles*. Since, the author of this article was astonished by the given advantage of that particular procedure, he posed the following question: *why do they do that*? This question does not have a philosophical disposition, just an intention to mention to chemists that it is a big fallacy; in the last decade, it is very well-known that only the mathematical methods are consistent methods for balancing chemical reactions. So-called *chemical methods* for balancing chemical reactions are inconsistent, because they consider chemical reactions in an informal way, which produces only paradoxes (Risteski, 2010; 2011).

Now, logically this question arises: *what are chemical principles*? According to Risteski (2010) the best short answer to this question is: *'chemical principles' are not defined entities in chemistry, and so this term does not have any meaning. They represent only a main generator for paradoxes. Actually, 'chemical principles' are a remnant of an old traditional approach in chemistry.*

In order for readers to have a better picture about the balancing chemical reactions, let's make a small digression. Really, until the second half of the 20th century there was no mathematical method for balancing chemical reactions in chemistry, other than the algebraic method. Then, chemists on an inertial way balanced just simple particular chemical reactions using only change in oxidation number procedure, partial reactions procedure and other slightly different modifications derived from them. So-called *chemical principles* were an assumption of traditional chemists, who thought that the solution of the general problem of balancing chemical reactions is possible by use of chemical procedures. But, practice showed that the solution of the century old problem is possible only by using a contemporary mathematical method (Risteski, 2007a).

Also, in (Risteski, 2010) the author emphasized very clearly, that *balancing chemical reactions is not chemistry; it is just linear algebra*. From a scientific view point*, a chemical reaction can be balanced if and only if it generates a vector space*. That is a necessary and sufficient condition for balancing a chemical reaction. This shows that chemical reaction must be considered as a formal whole, in a right sense of the word, if we like it to be balanced in a correct way. In the opposite case, as it was done by the *chemical methods*, one obtains only the absurd (Risteski, 2011).

Here, considered aliphatic hydrocarbon chemical reactions belong to the class of two generator chemical reactions with non-unique coefficients. These reactions are *continuum reactions*, because the problem of their coefficients determination reduces to the *generalized continuum problem* (Risteski, 2012).

A new geometrical method

In this section we shall develop a new geometrical method for balancing continuum chemical reactions. For that purpose, we shall introduce a whole set of auxiliary definitions from n-dimensional geometry (Kendall, 2004) and real finite-dimensional vector spaces (Halmos, 1987) to make the chemistry work consistently. The more abstract the theory is, the stronger the cognitive power is.

Let $\mathscr{X}$ be a finite set of molecules.

Definition 1. *A chemical reaction on $\mathscr{X}$ is a formal linear combinations of elements of $\mathscr{X}$, such that*

$$\rho : \sum_{i=1}^{m} a_{ij}x_j \rightarrow 0, \ (1 \leq j \leq n). \tag{1}$$

The coefficients x_j, $(1 \leq j \leq n)$ satisfy three basic principles (corresponding to a closed input-output static model): (i) the law of conservation of atoms; (ii) the law of conservation of mass, and (iii) the reaction time-independence.

Proposition 2. *Any chemical reaction can be reduced to a set of hyperplanes of its atoms*.

Proof. Since every chemical reaction can be presented in a matrix form $\boldsymbol{Ax} = \mathbf{0}$, then it corresponds with (1). In fact, the expression (1) is a set of hyperplanes. Opposite, if (1) holds, then exists $\boldsymbol{Ax} = \mathbf{0}$.

Let us now consider an arbitrary subset $\mathscr{A} \subseteq \mathscr{X}$.

Definition 3. *A chemical reaction* ρ *may take place in a reaction combination composed of the molecules in $\mathscr{A}$ if and only if Dom*ρ $\subseteq \mathscr{A}$.

Definition 4. *The collection of all possible reactions in the stoichiometrical space* ($\mathscr{X}$, $\mathscr{R}$), *that can start from $\mathscr{A}$ is given by*

$$\mathscr{R}_{\mathscr{A}} = \{\rho \in \mathscr{R} \mid Dom\rho \subseteq \mathscr{A}\}. \tag{2}$$

Theorem 5. *Let* $U = span\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *in a vector space V of the chemical reaction* (1) *over the field* $\mathbb{R}$. *Then,*

*U is a subspace of V containing each o*f $\boldsymbol{v}_i$, $(1 \leq i \leq n)$, (3)

U is the smallest subspace containing these vectors in the sense that any subspace of V that contains each of $\boldsymbol{v}_i$, $(1 \leq i \leq n)$*, must contain U.* (4)

Proof. First we shall prove (3). Clearly $\mathbf{0} = 0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n$ belongs to U. If $\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$ and $\boldsymbol{w} = b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + \cdots + b_n\boldsymbol{v}_n$ are two members of U and $a \in U$, then

$$\boldsymbol{v} + \boldsymbol{w} = (a_1 + b_1)\boldsymbol{v}_1 + (a_2 + b_2)\boldsymbol{v}_2 + \cdots + (a_n + b_n)\boldsymbol{v}_n,$$

$$a\boldsymbol{v} = (aa_1)\boldsymbol{v}_1 + (aa_2)\boldsymbol{v}_2 + \cdots + (aa_n)\boldsymbol{v}_n,$$

so both $\boldsymbol{v} + \boldsymbol{w}$ and $a\boldsymbol{v}$ lie in U. Hence U is a subspace of V. It contains each of $\boldsymbol{v}_i$, $(1 \leq i \leq n)$. For instance, $\boldsymbol{v}_2 = 0\boldsymbol{v}_1 + 1\boldsymbol{v}_2 + 0\boldsymbol{v}_3 + \cdots + 0\boldsymbol{v}_n$. This proves (3).

Now, we shall prove (4). Let W be subspace of V that contains each of $\boldsymbol{v}_i$, $(1 \leq i \leq n)$. Since W is closed under scalar multiplication, each of $a_i\boldsymbol{v}_i$, $(1 \leq i \leq n)$ lies in W for any choice of a_i, $(1 \leq i \leq n)$ in $\mathbb{R}$. But, then $a_i\boldsymbol{v}_i$, $(1 \leq i \leq n)$ lies in W, because W is closed under addition. This means that W contains every member of U, which proves (4).

Theorem 6. *The intersection of any number of subspaces of a vector space V of the chemical reaction* (1) *over the field* $\mathbb{R}$ *is a subspace of V.*

Proof. Let $\{W_i : i \quad \boldsymbol{I}\}$ be a collection of subspaces of V and let $W = \quad (W_i : i \quad \boldsymbol{I})$. Since each W_i is a subspace, then $\mathbf{0} \quad W_i,\ i \quad \boldsymbol{I}$. Thus $\mathbf{0} \quad W$. Assume $\boldsymbol{u}, \boldsymbol{v} \quad W$. Then, $\boldsymbol{u}, \boldsymbol{v} \quad W_i,\ i \quad \boldsymbol{I}$. Since each W_i is a subspace, then $(a\boldsymbol{u} + b\boldsymbol{v}) \quad W_i,\ i \quad \boldsymbol{I}$. Therefore $(a\boldsymbol{u} + b\boldsymbol{v}) \quad W$. Thus W is a subspace of V of the chemical reaction (1).

Theorem 7. *The hyperplanes* (1)*, obtained from the chemical reaction, in n unknowns* $x_1, x_2, \ldots, x_n$ *over the field* $\mathbb{R}$ *has a solution set W, which is a subspace of the vector space* $\mathbb{R}^n$*.*

Proof. The system (1) is equivalent to the matrix equation $\boldsymbol{Ax} = \mathbf{0}$. Since $\boldsymbol{A}\mathbf{0} = \mathbf{0}$, the zero vector $\mathbf{0} \quad W$. Assume $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in W, *i. e.*, $\boldsymbol{u}$ and $\boldsymbol{v}$ are solutions of the matrix equation $\boldsymbol{Ax} = \mathbf{0}$. Then $\boldsymbol{Au} = \mathbf{0}$ and $\boldsymbol{Av} = \mathbf{0}$. Therefore, $\ a, b \quad \mathbb{R}$, we have $\boldsymbol{A}(a\boldsymbol{u} + b\boldsymbol{v}) = a\boldsymbol{Au} + b\boldsymbol{Av} = a\mathbf{0} + b\mathbf{0} = \mathbf{0} + \mathbf{0} = \mathbf{0}$. Hence $a\boldsymbol{u} + b\boldsymbol{v}$ is a solution of the matrix equation $\boldsymbol{Ax} = \mathbf{0}$, *i. e.*, $a\boldsymbol{u} + b\boldsymbol{v} \quad W$. Thus W is a subspace of $\mathbb{R}^n$.

Proposition 8. *If W is a subspace of V of the chemical reaction* (1) *over the field* $\mathbb{R}$*, then span*$\{W\} = W$*.*

Proof. Since W is a subspace of V of the chemical reaction (1) over the field $\mathbb{R}$, W is closed under linear combinations. Hence span$\{W\} \quad W$. But $W \quad$ span$\{W\}$. Both inclusions yield span$\{W\} = W$.

The relationship between the two planes

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n + a = 0, \text{ and } b_1x_1 + b_2x_2 + \cdots + b_nx_n + b = 0,$$

can be described as follows:

1. intersecting if $a_1/b_1 \neq a_2/b_2 \neq \cdots \neq a_n/b_n$,
2. parallel if $a_1/b_1 = a_2/b_2 = \cdots = a_n/b_n \neq a/b$,
3. coincident if $a_1/b_1 = a_2/b_2 = \cdots = a_n/b_n = a/b$.

The angle $\sphericalangle\alpha(\boldsymbol{n}_1, \boldsymbol{n}_2)$ between two hyperplanes is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_1 = (a_1, a_2, \ldots, a_n) \text{ and } \boldsymbol{n}_2 = (b_1, b_2, \ldots, b_n) \qquad (5)$$

i.e.

$$\sphericalangle\alpha(\boldsymbol{n}_1, \boldsymbol{n}_2) = \arccos\left\{\left|\sum_{i=1}^{n} a_i b_i\right| / \left[\left(\sum_{i=1}^{n} a_i^2\right)^{1/2} \left(\sum_{i=1}^{n} b_i^2\right)^{1/2}\right]\right\}. \quad (6)$$

In the next section some very hard problems will be solved from the theory of balancing chemical reactions. Just, for that purpose was built a new n-dimensional geometrical method for balancing two generators aliphatic hydrocarbon chemical reactions. Here balanced reactions are completely new and according to our best knowledge for the first time they appear in scientific literature.

Main results

Problem 1

We shall balance the following aliphatic hydrocarbon chemical reaction

$$x_1\, C_2H_2 + x_2\, CH_4 + x_3\, C_2H_4 + x_4\, C_3H_4 + x_5\, C_2H_6 + x_6\, C_3H_6 + x_7\, C_4H_6 + x_8\, C_3H_8 + x_9\, C_4H_8 \rightarrow x_{10}\, C_5H_{10}. \quad (7)$$

Solution

According to the reaction (7), carbon and hydrogen atoms are disposed adequately on the following hyperplanes

$$\begin{aligned} 2x_1 + x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6 + 4x_7 + 3x_8 + 4x_9 &= 5x_{10}, \\ 2x_1 + 4x_2 + 4x_3 + 4x_4 + 6x_5 + 6x_6 + 6x_7 + 8x_8 + 8x_9 &= 10x_{10}, \end{aligned} \quad (8)$$

which intersection is

$$\begin{aligned} x_1 &= - 2x_3/3 - 4x_4/3 - x_5/3 - x_6 - 5x_7/3 - 2x_8/3 - 4x_9/3 + 5x_{10}/3, \\ x_2 &= - 2x_3/3 - x_4/3 - 4x_5/3 - x_6 - 2x_7/3 - 5x_8/3 - 4x_9/3 + 5x_{10}/3, \end{aligned} \quad (9)$$

where $x_i > 0$, $(3 \leq i \leq 10)$ are arbitrary real numbers. The intersection point has these coordinates

$$(- 2x_3/3 - 4x_4/3 - x_5/3 - x_6 - 5x_7/3 - 2x_8/3 - 4x_9/3 + 5x_{10}/3, - 2x_3/3 - x_4/3 - 4x_5/3 - x_6 - 2x_7/3 - 5x_8/3 - 4x_9/3 + 5x_{10}/3, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}), \quad (10)$$

where $x_i > 0$, $(3 \leq i \leq 10)$ are arbitrary real numbers.

The system (8) has two (nonzero) linear equations in ten unknowns; and hence it has $10 - 2 = 8$ free variables $x_i > 0$, $(3 \leq i \leq 10)$. Thus, the dimension of the solution space W of the system (8) is $\dim W = 8$. To obtain a basis for W, we set

$$\begin{gathered} x_3 = 1, x_4 = \cdots = x_{10} = 0, \\ x_3 = 0, x_4 = 1, x_5 = \cdots = x_{10} = 0, \\ x_3 = x_4 = 0, x_5 = 1, x_6 = \cdots = x_{10} = 0, \\ x_3 = \cdots = x_5 = 0, x_6 = 1, x_7 = \cdots = x_{10} = 0, \\ x_3 = \cdots = x_6 = 0, x_7 = 1, x_8 = \cdots = x_{10} = 0, \\ x_3 = \cdots = x_7 = 0, x_8 = 1, x_9 = x_{10} = 0, \end{gathered} \quad (11)$$

$$x_3 = \cdots = x_8 = 0,\ x_9 = 1,\ x_{10} = 0,$$
$$x_3 = \cdots = x_9 = 0,\ x_{10} = 1,$$
in the expression (10) to obtain the solutions

$$\begin{aligned} \boldsymbol{a}_1 &= (-2/3, -2/3, 1, 0, 0, 0, 0, 0, 0, 0),\\ \boldsymbol{a}_2 &= (-4/3, -1/3, 0, 1, 0, 0, 0, 0, 0, 0),\\ \boldsymbol{a}_3 &= (-1/3, -4/3, 0, 0, 1, 0, 0, 0, 0, 0),\\ \boldsymbol{a}_4 &= (-1, -1, 0, 0, 0, 1, 0, 0, 0, 0),\\ \boldsymbol{a}_5 &= (-5/3, -2/3, 0, 0, 0, 0, 1, 0, 0, 0),\\ \boldsymbol{a}_6 &= (-2/3, -5/3, 0, 0, 0, 0, 0, 1, 0, 0),\\ \boldsymbol{a}_7 &= (-4/3, -4/3, 0, 0, 0, 0, 0, 0, 1, 0),\\ \boldsymbol{a}_8 &= (5/3, 5/3, 0, 0, 0, 0, 0, 0, 0, 1). \end{aligned} \qquad (12)$$

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \boldsymbol{a}_4, \boldsymbol{a}_5, \boldsymbol{a}_6, \boldsymbol{a}_7, \boldsymbol{a}_8\}$ is a basis of the solution space W.

The angle $\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H)$ between carbon and hydrogen hyperplane (8) is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (2, 1, 2, 3, 2, 3, 4, 3, 4, -5) \text{ and } \boldsymbol{n}_H = (2, 4, 4, 4, 6, 6, 6, 8, 8, -10),$$

i.e.,

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos\{(2\times2 + 1\times4 + 2\times4 + 3\times4 + 2\times6 + 3\times6 + 4\times6 + 3\times8 + 4\times8 + 5\times10)/[(2^2 + 1^2 + 2^2 + 3^2 + 2^2 + 3^2 + 4^2 + 3^2 + 4^2 + 5^2)^{1/2}\,(2^2 + 4^2 + 4^2 + 4^2 + 6^2 + 6^2 + 6^2 + 8^2 + 8^2 + 10^2)^{1/2}]\}$$
$$= \arccos[188/(97\times388)^{1/2}] = \arccos(94/97) = 14.3°.$$

After substitution of the expressions (9) into (7), the balanced reaction (1) obtains its general form

$$\begin{aligned} &(-2x_3/3 - 4x_4/3 - x_5/3 - x_6 - 5x_7/3 - 2x_8/3 - 4x_9/3 + 5x_{10}/3)\, C_2H_2 \\ &+ (-2x_3/3 - x_4/3 - 4x_5/3 - x_6 - 2x_7/3 - 5x_8/3 - 4x_9/3 + 5x_{10}/3)\, CH_4 + x_3\, C_2H_4 \\ &+ x_4\, C_3H_4 + x_5\, C_2H_6 + x_6\, C_3H_6 + x_7\, C_4H_6 + x_8\, C_3H_8 + x_9\, C_4H_8 \rightarrow x_{10}\, C_5H_{10}, \end{aligned} \qquad (13)$$

where $x_i > 0$, $(3 \le i \le 10)$ are arbitrary real numbers.

Since the generators $x_1, x_2 > 0$, then for the general chemical reaction (13) holds this system of linear inequalities

$$\begin{aligned} &-2x_3/3 - 4x_4/3 - x_5/3 - x_6 - 5x_7/3 - 2x_8/3 - 4x_9/3 + 5x_{10}/3 > 0,\\ &-2x_3/3 - x_4/3 - 4x_5/3 - x_6 - 2x_7/3 - 5x_8/3 - 4x_9/3 + 5x_{10}/3 > 0. \end{aligned} \qquad (14)$$

From (8), one obtains the inequality

$$x_{10} > (4x_3 + 5x_4 + 5x_5 + 6x_6 + 7x_7 + 7x_8 + 8x_9)/10. \qquad (15)$$

Actually, the inequality (15) is a necessary and sufficient condition to hold the general reaction (13).

In order to determine a particular reaction of (13) we shall consider the following case.

For $x_3 = x_4 = \cdots = x_9 = 5$, from (15) one obtains $x_{10} = 22$. Now, from (13) immediately follows the particular reaction

$$5\,C_2H_2 + 5\,CH_4 + 15\,C_2H_4 + 15\,C_3H_4 + 15\,C_2H_6 + 15\,C_3H_6 + 15\,C_4H_6 + 15\,C_3H_8 + 15\,C_4H_8 \rightarrow 66\,C_5H_{10}. \quad (16)$$

Problem 2

Now, we shall balance this alkyne's chemical reaction

$$x_1\,C_3H_4 + x_2\,C_4H_6 + x_3\,C_5H_8 + x_4\,C_6H_{10} + x_5\,C_7H_{12} + x_6\,C_8H_{14} + x_7\,C_9H_{16} + x_8\,C_{10}H_{18} + x_9\,C_{11}H_{20} \rightarrow x_{10}\,C_{12}H_{22} + x_{11}\,C_2H_2. \quad (17)$$

Solution

From the above reaction adequately follow these carbon and hydrogen hyperplanes

$$\begin{aligned} 3x_1 + 4x_2 + 5x_3 + 6x_4 + 7x_5 + 8x_6 + 9x_7 + 10x_8 + 11x_9 &= 12x_{10} + 2x_{11}, \\ 4x_1 + 6x_2 + 8x_3 + 10x_4 + 12x_5 + 14x_6 + 16x_7 + 18x_8 + 20x_9 &= 22x_{10} + 2x_{11}, \end{aligned} \quad (18)$$

which intersection is

$$\begin{aligned} x_{10} &= (x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 + 6x_6 + 7x_7 + 8x_8 + 9x_9)/10, \\ x_{11} &= (9x_1 + 8x_2 + 7x_3 + 6x_4 + 5x_5 + 4x_6 + 3x_7 + 2x_8 + x_9)/10, \end{aligned} \quad (19)$$

where $x_i > 0$, $(1 \le i \le 9)$ are arbitrary real numbers. The intersection point has these coordinates

$$[x_1, x_2, x_3, \ldots, x_9, (x_1 + 2x_2 + 3x_3 + \cdots + 9x_9)/10, (9x_1 + 8x_2 + 7x_3 + 6x_4 + \cdots + 2x_8 + x_9)/10], \quad (20)$$

where $x_i > 0$, $(1 \le i \le 9)$ are arbitrary real numbers.

The system (18) has two (nonzero) linear equations in eleven unknowns; and hence it has 11 - 2 = 9 free variables $x_i > 0$, $(1 \le i \le 9)$. Thus, the dimension of the solution space W of the system (18) is dim $W = 9$. To obtain a basis for W, we set

$$\begin{gathered} x_1 = 1, x_2 = \cdots = x_9 = 0, \\ x_1 = 0, x_2 = 1, x_3 = \cdots = x_9 = 0, \\ x_1 = x_2 = 0, x_3 = 1, x_4 = \cdots = x_9 = 0, \\ x_1 = \cdots = x_3 = 0, x_4 = 1, x_5 = \cdots = x_9 = 0, \\ x_1 = \cdots = x_4 = 0, x_5 = 1, x_6 = \cdots = x_9 = 0, \\ x_1 = \cdots = x_5 = 0, x_6 = 1, x_7 = \cdots = x_9 = 0, \\ x_1 = \cdots = x_6 = 0, x_7 = 1, x_8 = x_9 = 0, \\ x_1 = \cdots = x_7 = 0, x_8 = 1, x_9 = 0, \\ x_1 = \cdots = x_8 = 0, x_9 = 1, \end{gathered} \quad (21)$$

in the expression (20) to obtain the solutions

$$\begin{aligned} \boldsymbol{a}_1 &= (1, 0, 0, 0, 0, 0, 0, 0, 0, 1/10, 9/10), \\ \boldsymbol{a}_2 &= (0, 1, 0, 0, 0, 0, 0, 0, 0, 2/10, 8/10), \\ \boldsymbol{a}_3 &= (0, 0, 1, 0, 0, 0, 0, 0, 0, 3/10, 7/10), \\ \boldsymbol{a}_4 &= (0, 0, 0, 1, 0, 0, 0, 0, 0, 4/10, 6/10), \\ \boldsymbol{a}_5 &= (0, 0, 0, 0, 1, 0, 0, 0, 0, 5/10, 5/10), \end{aligned} \quad (22)$$

$$\boldsymbol{a}_6 = (0, 0, 0, 0, 0, 1, 0, 0, 0, 6/10, 4/10),$$
$$\boldsymbol{a}_7 = (0, 0, 0, 0, 0, 0, 1, 0, 0, 7/10, 3/10),$$
$$\boldsymbol{a}_8 = (0, 0, 0, 0, 0, 0, 0, 1, 0, 8/10, 2/10),$$
$$\boldsymbol{a}_9 = (0, 0, 0, 0, 0, 0, 0, 0, 1, 9/10, 1/10).$$

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \boldsymbol{a}_4, \boldsymbol{a}_5, \boldsymbol{a}_6, \boldsymbol{a}_7, \boldsymbol{a}_8, \boldsymbol{a}_9\}$ is a basis of the solution space W.

The angle $\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H)$ between carbon and hydrogen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (3, 4, 5, 6, 7, 8, 9, 10, 11, - 12, - 2) \text{ and } \boldsymbol{n}_H = (4, 6, 8, 10, 12, 14, 16, 18, 20, - 22, - 2),$$

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos \{(2\times2 + 3\times4 + 4\times6 + 5\times8 + 6\times10 + 7\times12 + 8\times14 + 9\times16 + 10\times18 + 11\times20 + 12\times22)/[(2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2)^{1/2} (2^2 + 4^2 + 6^2 + 8^2 + 10^2 + 12^2 + 14^2 + 16^2 + 18^2 + 20^2 + 22^2)^{1/2}]\} = \arccos [12\times13/(2\times3\times23\times177)^{1/2}] = 3.5°.$$

After substitution of the generators (19) into (17), the chemical reaction (17) obtains its general form

$$x_1\, C_3H_4 + x_2\, C_4H_6 + x_3\, C_5H_8 + x_4\, C_6H_{10} + x_5\, C_7H_{12} + x_6\, C_8H_{14} + x_7\, C_9H_{16} + x_8\, C_{10}H_{18} + x_9\, C_{11}H_{20} \rightarrow [(x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 + 6x_6 + 7x_7 + 8x_8 + 9x_9)/10]\, C_{12}H_{22} + [(9x_1 + 8x_2 + 7x_3 + 6x_4 + 5x_5 + 4x_6 + 3x_7 + 2x_8 + x_9)/10]\, C_2H_2. \tag{23}$$

where $x_i > 0$, $(1 \leq i \leq 9)$ are arbitrary real numbers.

Example

Let's consider a particular reaction of (23). For $x_1 = x_2 = \cdots = x_9 = 1$ immediately from (23) follows balanced particular reaction

$$2\, C_3H_4 + 2\, C_4H_6 + 2\, C_5H_8 + 2\, C_6H_{10} + 2\, C_7H_{12} + 2\, C_8H_{14} + 2\, C_9H_{16} + 2\, C_{10}H_{18} + 2\, C_{11}H_{20} \rightarrow 9\, C_{12}H_{22} + 9\, C_2H_2. \tag{24}$$

Problem 3

The above alkyne's reaction (17) gives an opportunity for its consideration in more general form. Taking into account this fact, now we shall balance the general alkyne's chemical reaction

$$\sum_{i=1}^{n-2} x_i C_{i+2}H_{2i+2} \rightarrow x_{n-1}C_{n+1}H_{2n} + x_nC_2H_2, \ (n > 2). \tag{25}$$

Solution

The general alkyne's chemical reaction (25) reduces adequately to the following carbon and hydrogen hyperplanes

$$3x_1 + 4x_2 + 5x_3 + \cdots + nx_{n-2} = (n + 1)x_{n-1} + 2x_n, \quad (26)$$
$$4x_1 + 6x_2 + 8x_3 + \cdots + (2n - 2)x_{n-2} = 2nx_{n-1} + 2x_n,$$

which intersection is

$$x_{n-1} = [1/(n - 1)] \sum_{i=1}^{n-2} ix_i, \quad (27)$$

$$x_n = [1/(n - 1)] \sum_{i=1}^{n-2} (n - i - 1)x_i,$$

where $n > 2$ and $x_i > 0$, $(1 \leq i \leq n - 2)$ are arbitrary real numbers. The intersection point has these coordinates

$$\{x_1, x_2, x_3, \ldots, x_{n-2}, [x_1 + 2x_2 + 3x_3 + \cdots + (n - 2)x_{n-2}]/(n - 1), \quad (28)$$
$$[(n - 2)x_1 + (n - 3)x_2 + (n - 4)x_3 + \cdots + 2x_{n-3} + x_{n-2}]/(n - 1)\},$$

where $x_i > 0$, $(1 \leq i \leq n - 2)$ are arbitrary real numbers.

The system (26) has two (nonzero) linear equations in n unknowns; and hence it has $n - 2$ free variables $x_i > 0$, $(1 \leq i \leq n - 2)$. Thus, the dimension of the solution space W of the system (26) is dim $W = n - 2$. To obtain a basis for W, we set

$$x_1 = 1, x_2 = \cdots = x_{n-2} = 0,$$
$$x_1 = 0, x_2 = 1, x_3 = \cdots = x_{n-2} = 0,$$
$$x_1 = x_2 = 0, x_3 = 1, x_4 = \cdots = x_{n-2} = 0, \quad (29)$$
$$\vdots$$
$$x_1 = \cdots = x_{n-3} = 0, x_{n-2} = 1,$$

in the expression (28) to obtain the solutions

$$\boldsymbol{a}_1 = [1, 0, 0, \ldots, 0, 1/(n - 1), (n - 2)/(n - 1)],$$
$$\boldsymbol{a}_2 = [0, 1, 0, \ldots, 0, 2/(n - 1), (n - 3)/(n - 1)],$$
$$\boldsymbol{a}_3 = [0, 0, 1, \ldots, 0, 3/(n - 1), (n - 4)/(n - 1)], \quad (30)$$
$$\vdots$$
$$\boldsymbol{a}_{n-2} = [0, 0, 0, \ldots, 1, (n - 2)/(n - 1), 1/(n - 1)].$$

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \ldots, \boldsymbol{a}_{n-2}\}$ is a basis of the solution space W.

The angle $\measuredangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H)$ between carbon and hydrogen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (3, 4, 5, \ldots, n, - n - 1, - 2) \text{ and } \boldsymbol{n}_H = (4, 6, 8, \ldots, 2n - 2, - 2n, - 2)$$

$$\measuredangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos\{|2\times2 + 3\times4 + 4\times6 + 5\times8 + \cdots + n\times(2n - 2) + (n + 1)\times2n|/$$
$$[(2^2 + 3^2 + 4^2 + \cdots + n^2 + (n + 1)^2)^{1/2} (2^2 + 4^2 + 6^2 + \cdots + (2n - 2)^2 + (2n)^2)^{1/2}]\}.$$

Since

$$2\times2 + 3\times4 + 4\times6 + 5\times8 + \cdots + n\times(2n - 2) + (n + 1)\times2n = 2n(n + 1)(n + 2)/3,$$
$$2^2 + 3^2 + 4^2 + 5^2 + \cdots + n^2 + (n + 1)^2 = n(2n^2 + 9n + 13)/6$$

and

$$2^2 + 4^2 + 6^2 + 8^2 + \cdots + (2n - 2)^2 + (2n)^2 = 2n(n + 1)(2n + 1)/3,$$

then

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos\{2(n + 1)(n + 2)/[(n + 1)(2n + 1)(2n^2 + 9n + 13)]^{1/2}\}.$$

According to (27) and (25), balanced alkyne's chemical reaction obtains this general form

$$(n - 1)\sum_{i=1}^{n-2} x_i C_{i+2}H_{2i+2} \rightarrow \Big(\sum_{i=1}^{n-2} ix_i\Big)C_{n+1}H_{2n} + \Big[\sum_{i=1}^{n-2}(n - i - 1)x_i\Big]C_2H_2,\ (n > 2). \qquad (31)$$

where $x_i > 0$, $(1 \le i \le n - 2)$ are arbitrary real numbers.[1)]

Example

Now, we shall consider a particular case of (31). For $x_1 = x_2 = \cdots = x_{n-2} = 1$, the reaction (31) transforms into following balanced particular reaction[2)]

$$2\sum_{i=1}^{n-2} C_{i+2}H_{2i+2} \rightarrow (n - 2)(C_{n+1}H_{2n} + C_2H_2),\ (n > 2). \qquad (32)$$

Problem 4

Like an interesting reaction, we shall balance this alkane's chemical reaction

$$x_1\,C_2H_6 + x_2\,C_3H_8 + x_3\,C_4H_{10} + x_4\,C_5H_{12} + x_5\,C_6H_{14} + x_6\,C_7H_{16} \qquad (33)$$
$$+ x_7\,C_8H_{18} + x_8\,C_9H_{20} + x_9\,C_{10}H_{22} + x_{10}\,C_{11}H_{24} \rightarrow x_{11}\,C_{12}H_{26} + x_{12}\,CH_4.$$

Solution

From the above alkane's chemical reaction (33) follows these hyperplanes

$$2x_1 + 3x_2 + 4x_3 + 5x_4 + 6x_5 + 7x_6 + 8x_7 + 9x_8 + 10x_9 + 11x_{10} = 12x_{11} + x_{12}, \qquad (34)$$
$$6x_1 + 8x_2 + 10x_3 + 12x_4 + 14x_5 + 16x_6 + 18x_7 + 20x_8 + 22x_9 + 24x_{10} = 26x_{11} + 4x_{12},$$

which intersection is

$$x_{11} = (x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 + 6x_6 + 7x_7 + 8x_8 + 9x_9 + 10x_{10})/11, \qquad (35)$$
$$x_{12} = (10x_1 + 9x_2 + 8x_3 + 7x_4 + 6x_5 + 5x_6 + 4x_7 + 3x_8 + 2x_9 + x_{10})/11,$$

where $x_i > 0$, $(1 \le i \le 10)$ are arbitrary real numbers. The intersection point has these coordinates

$$[x_1, x_2, x_3, \ldots, x_9, x_{10}, (x_1 + 2x_2 + 3x_3 + \cdots + 10x_{10})/11, \qquad (36)$$
$$(10x_1 + 9x_2 + 8x_3 + 7x_4 + \cdots + 2x_9 + x_{10})/11],$$

where $x_i > 0$, $(1 \le i \le 10)$ are arbitrary real numbers.

The system (34) has two (nonzero) linear equations in twelve unknowns; and hence it has 12 - 2 = 10 free variables $x_i > 0$, $(1 \le i \le 10)$. Thus, the dimension of the solution space W of the system (34) is dim $W = 10$. To obtain a basis for W, we set

$$\begin{gathered} x_1 = 1, x_2 = \cdots = x_{10} = 0, \\ x_1 = 0, x_2 = 1, x_3 = \cdots = x_{10} = 0, \\ x_1 = x_2 = 0, x_3 = 1, x_4 = \cdots = x_{10} = 0, \\ x_1 = \cdots = x_3 = 0, x_4 = 1, x_5 = \cdots = x_{10} = 0, \\ x_1 = \cdots = x_4 = 0, x_5 = 1, x_6 = \cdots = x_{10} = 0, \\ x_1 = \cdots = x_5 = 0, x_6 = 1, x_7 = \cdots = x_{10} = 0, \\ x_1 = \cdots = x_6 = 0, x_7 = 1, x_8 = \cdots = x_{10} = 0, \\ x_1 = \cdots = x_7 = 0, x_8 = 1, x_9 = 0, \\ x_1 = \cdots = x_8 = 0, x_9 = 1, x_{10} = 0, \\ x_1 = \cdots = x_9 = 0, x_{10} = 1, \end{gathered} \tag{37}$$

in the expression (30) to obtain the solutions

$$\begin{aligned} \boldsymbol{a}_1 &= (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/11, 10/11), \\ \boldsymbol{a}_2 &= (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2/11, 9/11), \\ \boldsymbol{a}_3 &= (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 3/11, 8/11), \\ \boldsymbol{a}_4 &= (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 4/11, 7/11), \\ \boldsymbol{a}_5 &= (0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 5/11, 6/11), \\ \boldsymbol{a}_6 &= (0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 6/11, 5/11), \\ \boldsymbol{a}_7 &= (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 7/11, 4/11), \\ \boldsymbol{a}_8 &= (0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 8/11, 3/11), \\ \boldsymbol{a}_9 &= (0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 9/11, 2/11), \\ \boldsymbol{a}_{10} &= (0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 10/11, 1/11). \end{aligned} \tag{38}$$

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \boldsymbol{a}_4, \boldsymbol{a}_5, \boldsymbol{a}_6, \boldsymbol{a}_7, \boldsymbol{a}_8, \boldsymbol{a}_9, \boldsymbol{a}_{10}\}$ is a basis of the solution space W.

The angle $\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H)$ between carbon and hydrogen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (2, 3, 4, 5, \ldots, 11, -12, -1) \text{ and } \boldsymbol{n}_H = (6, 8, 10, 12, \ldots, 24, -26, -4)$$

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos \{(1\times4 + 2\times6 + 3\times8 + 4\times10 + 6\times10 + \cdots + 12\times26)/$$
$$[(1 + 2^2 + 3^2 + 4^2 + \cdots + 12^2)^{1/2} (4^2 + 6^2 + 8^2 + 10^2 + \cdots + 26^2)^{1/2}]\}$$
$$= \arccos \{2\times13\times14/[5(13\times409)^{1/2}]\} = 3.26°.$$

After substitution of the generators (35) into (33), the chemical reaction (33) obtains its general form

$$\begin{aligned} &x_1\, C_2H_6 + x_2\, C_3H_8 + x_3\, C_4H_{10} + x_4\, C_5H_{12} + x_5\, C_6H_{14} + x_6\, C_7H_{16} \\ &\quad + x_7\, C_8H_{18} + x_8\, C_9H_{20} + x_9\, C_{10}H_{22} + x_{10}\, C_{11}H_{24} \end{aligned} \tag{39}$$

$\rightarrow [(x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 + 6x_6 + 7x_7 + 8x_8 + 9x_9 + 10x_{10})/11]\ C_{12}H_{26}$
$+ [(10x_1 + 9x_2 + 8x_3 + 7x_4 + 6x_5 + 5x_6 + 4x_7 + 3x_8 + 2x_9 + x_{10})/11]\ CH_4.$
where $x_i > 0$, $(1 \le i \le 10)$ are arbitrary real numbers.

Example

Next, we shall consider a particular reaction of (39). For $x_1 = x_2 = \cdots = x_{10} = 1$ immediately from (39) follows balanced particular reaction

$$C_2H_6 + C_3H_8 + C_4H_{10} + C_5H_{12} + C_6H_{14} + C_7H_{16} + C_8H_{18} + C_9H_{20} + C_{10}H_{22} + C_{11}H_{24} \rightarrow 5\ C_{12}H_{26} + 5\ CH_4. \qquad (40)$$

Problem 5

According to the last problem, now arises a need to balance the general alkane's chemical reaction

$$\sum_{i=1}^{n-2} x_i C_{i+1}H_{2i+4} \rightarrow x_{n-1}C_nH_{2n+2} + x_nCH_4,\ (n > 2). \qquad (41)$$

Solution

The general alkane's chemical reaction (41) reduces to the hyperplanes

$$2x_1 + 3x_2 + 4x_3 + \cdots + (n - 1)\ x_{n-2} = nx_{n-1} + x_n, \qquad (42)$$
$$6x_1 + 8x_2 + 10x_3 + \cdots + 2nx_{n-2} = (2n + 2)x_{n-1} + 4x_n,$$

which intersection is given by (27). The intersection point has coordinates (28).

The system (42) has two (nonzero) linear equations in n unknowns; and hence it has $n - 2$ free variables $x_i > 0$, $(1 \le i \le n - 2)$. Thus, the dimension of the solution space W of the system (42) is dim $W = n - 2$. To obtain a basis for W, we set (29) in (28) to obtain solutions (30).

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \ldots, \boldsymbol{a}_{n-2}\}$ is a basis of the solution space W.

The angle $\measuredangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H)$ between carbon and hydrogen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (2, 3, 4, 5, \ldots, (n - 1), -n, -1) \text{ and } \boldsymbol{n}_H = (6, 8, 10, 12, \ldots, 2n, -(2n + 2), -4)$$

$$\measuredangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos |1\times4 + 2\times6 + 3\times8 + 4\times10 + \cdots + (n - 1)\times2n + n\times(2n + 2)| / [(1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2)^{1/2} (4^2 + 6^2 + \cdots + (2n + 2)^2)^{1/2}].$$

Since

$$1\times4 + 2\times6 + 3\times8 + 4\times10 + \cdots + (n - 1)\times2n + n\times(2n + 2) = 2n(n + 1)(n + 2)/3,$$
$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$$

and

$$4^2 + 6^2 + 8^2 + 10^2 + \cdots + (2n + 2)^2 = 2n(2n^2 + 9n + 13)/3,$$

then

$$\measuredangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos \{2(n + 1)^{1/2}(n + 2)/[(2n + 1)(2n^2 + 9n + 13)]^{1/2}\}.$$

According to (27) and (41), balanced alkane's chemical reaction obtains this general form

$$(n - 1)\sum_{i=1}^{n-2} x_i C_{i+1}H_{2i+4} \rightarrow \left(\sum_{i=1}^{n-2} ix_i\right)C_nH_{2n+2} + \left[\sum_{i=1}^{n-2} (n - i - 1)x_i\right]CH_4, (n > 2). \quad (43)$$

where $x_i > 0$, $(1 \le i \le n - 2)$ are arbitrary real numbers.[3)]

Example

Let's consider a particular case of (43). For $x_1 = x_2 = \cdots = x_{n-2} = 1$, the reaction (43) transforms into following balanced particular reaction[4)]

$$2\sum_{i=1}^{n-2} C_{i+1}H_{2i+4} \rightarrow (n - 2)(C_nH_{2n+2} + CH_4), (n > 2). \quad (44)$$

Problem 6

Next, we shall balance the general alkene's chemical reaction

$$\sum_{i=1}^{n-1} x_i C_{i+1}H_{2i+2} \rightarrow x_n C_{n+1}H_{2n+2}, (n > 1). \quad (45)$$

Solution

Since the carbon and hydrogen atoms are disposed on the coincident hyperplanes, then the above alkene's chemical reaction (45) reduces to this linear equation

$$2x_1 + 3x_2 + 4x_3 + \cdots + nx_{n-1} = (n + 1)x_n, \quad (46)$$

which general solution is

$$x_n = [1/(n + 1)] \sum_{i=1}^{n-1} (i + 1)x_i, (n > 1) \quad (47)$$

where $x_i > 0$, $(1 \le i \le n - 1)$ are arbitrary real numbers. The intersection point has these coordinates

$$\{x_1, x_2, x_3, \ldots, x_{n-1}, [2x_1 + 3x_2\ 4x_3 + \cdots + nx_{n-1}]/(n + 1)\}, \quad (48)$$

where $x_i > 0$, $(1 \le i \le n - 2)$ are arbitrary real numbers.

The reaction (45) reduces to one (nonzero) linear equations in n unknowns; and hence it has $n - 1$ free variables $x_i > 0$, $(1 \le i \le n - 1)$. Thus, the dimension of the solution space W of (46) is dim $W = n - 1$. To obtain a basis for W, we set

$$\begin{gathered} x_1 = 1, x_2 = \cdots = x_{n-1} = 0, \\ x_1 = 0, x_2 = 1, x_3 = \cdots = x_{n-1} = 0, \\ x_1 = x_2 = 0, x_3 = 1, x_4 = \cdots = x_{n-1} = 0, \\ \vdots \end{gathered} \quad (49)$$

$$x_1 = \cdots = x_{n-2} = 0,\ x_{n-1} = 1,$$

in the expression (48) to obtain the solutions

$$\begin{aligned} \boldsymbol{a}_1 &= [1, 0, 0, \ldots, 0, 2/(n+1)], \\ \boldsymbol{a}_2 &= [0, 1, 0, \ldots, 0, 3/(n+1)], \\ \boldsymbol{a}_3 &= [0, 0, 1, \ldots, 0, 4/(n+1)], \\ &\vdots \\ \boldsymbol{a}_{n-1} &= [0, 0, 0, \ldots, 1, n/(n+1)]. \end{aligned} \tag{50}$$

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \ldots, \boldsymbol{a}_{n-1}\}$ is a basis of the solution space W.

After substitution of the generator (47) into (45), balanced alkene's chemical reaction obtains this general form

$$(n+1)\sum_{i=1}^{n-1} x_i C_{i+1}H_{2i+2} \rightarrow \Big[\sum_{i=1}^{n-1} (i+1)x_i\Big] C_{n+1}H_{2n+2},\ (n > 1) \tag{51}$$

where $x_i > 0$, $(1 \leq i \leq n - 1)$ are arbitrary real numbers.

Example

Now, we shall consider a particular case of (51). For $x_1 = x_2 = \cdots = x_{n-1} = 1$, the reaction (51) transforms into following balanced particular reaction

$$(2n+2)\sum_{i=1}^{n-1} C_{i+1}H_{2i+2} \rightarrow (n-1)(n+2)C_{n+1}H_{2n+2},\ (n > 1). \tag{52}$$

Problem 7

Next, the general alcohol's chemical reaction will be considered

$$\sum_{i=1}^{n-2} x_i C_{i+1}H_{2i+4}O \rightarrow x_{n-1}C_nH_{2n+2}O + x_nCH_4O,\ (n > 2). \tag{53}$$

Solution

The general alcohol's chemical reaction (53) reduces to the following hyperplanes

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 + \cdots + (n-1)x_{n-2} &= nx_{n-1} + x_n, \\ 3x_1 + 4x_2 + 5x_3 + \cdots + nx_{n-2} &= (n+1)x_{n-1} + 2x_n, \\ x_1 + x_2 + x_3 + \cdots + x_{n-2} &= x_{n-1} + x_n, \end{aligned} \tag{54}$$

which intersection is given by (27). The intersection point has coordinates (28). The system (54) has two (nonzero) linear equations in n unknowns; and hence it has $n - 2$ free variables $x_i > 0$, $(1 \leq i \leq n - 2)$. Thus, the dimension of the solution space W of the system (54) is dim $W = n - 2$. To obtain a basis for W, we set (29) in the expression (28) to obtain the solutions (30). The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \ldots, \boldsymbol{a}_{n-2}\}$ is a basis of the solution space W.

The angle $\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_O)$ between carbon and oxygen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (2, 3, 4, \ldots, n - 1, - n, - 1) \text{ and } \boldsymbol{n}_O = (1, 1, 1, \ldots, 1, - 1, - 1)$$

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_O) = \arccos \{|1\times1 + 2\times1 + 3\times1 + 4\times1 + \cdots + (n - 1)\times1 + n\times1|/ [(1^2 + 1^2 + 1^2 + \cdots + 1^2 + 1^2)^{1/2} (1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n - 1)^2 + n^2)^{1/2}]\}.$$

Since

$$1\times1 + 2\times1 + 3\times1 + 4\times1 + \cdots + (n - 1)\times1 + n\times1 = n(n + 1)/2,$$

$$1^2 + 1^2 + 1^2 + 1^2 + \cdots + 1^2 + 1^2 = n,$$

and

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n - 1)^2 + n^2 = n(n + 1)(2n + 1)/6,$$

then

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_O) = \arccos\{[3(n + 1)/2(2n + 1)]^{1/2}\}.$$

The angle $\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H)$ between carbon and hydrogen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_C = (2, 3, 4, \ldots, n - 1, - n, - 1),\ \boldsymbol{n}_H = (3, 4, 5, \ldots, n, - n - 1, - 2)$$

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos \{|1\times2 + 2\times3 + 3\times4 + 4\times5 + \cdots + (n - 1)\times n + n\times(n + 1)|/ [(1^2 + 2^2 + 3^2 + \cdots + (n - 1)^2 + n^2)^{1/2} (2^2 + 3^2 + 4^2 + \cdots + n^2 + (n + 1)^2)^{1/2}]\}.$$

Since

$$1\times2 + 2\times3 + 3\times4 + 4\times5 + \cdots + (n - 1)\times n + n\times(n + 1) = n(n + 1)/2,$$

$$1^2 + 2^2 + 3^2 + \cdots + (n - 1)^2 + n^2 = n(n + 1)(2n + 1)/6,$$

and

$$2^2 + 3^2 + 4^2 + \cdots + n^2 + (n + 1)^2 = n(2n^2 + 9n + 13)/6,$$

then

$$\sphericalangle\alpha(\boldsymbol{n}_C, \boldsymbol{n}_H) = \arccos\{[3(n + 1)/(2n + 1)(2n^2 + 9n + 13)]^{1/2}\}.$$

The angle $\sphericalangle\alpha(\boldsymbol{n}_O, \boldsymbol{n}_H)$ between oxygen and hydrogen hyperplane is equal to the acute angle determined by the normal vectors of the planes

$$\boldsymbol{n}_O = (1, 1, 1, \ldots, 1, - 1, - 1),\ \boldsymbol{n}_H = (3, 4, 5, \ldots, n, - n - 1, - 2)$$

$$\sphericalangle\alpha(\boldsymbol{n}_O, \boldsymbol{n}_H) = \arccos \{|1\times2 + 1\times3 + 1\times4 + 1\times5 + \cdots + 1\times n + 1\times(n + 1)|/ [(1^2 + 1^2 + 1^2 + \cdots + 1^2 + 1^2)^{1/2} (2^2 + 3^2 + 4^2 + \cdots + n^2 + (n + 1)^2)^{1/2}]\}.$$

Since

$$1\times2 + 1\times3 + 1\times4 + 1\times5 + \cdots + 1\times n + 1\times(n + 1) = n(n + 3)/2,$$

$$1^2 + 1^2 + 1^2 + \cdots + 1^2 + 1^2 = n,$$

and

$$2^2 + 3^2 + 4^2 + \cdots + n^2 + (n + 1)^2 = n(2n^2 + 9n + 13)/6,$$

then

$$\sphericalangle\alpha(\boldsymbol{n}_O, \boldsymbol{n}_H) = \arccos\{[3(n + 3)^2/2(2n^2 + 9n + 13)]^{1/2}\}.$$

According to (27) and (53), balanced alcohol's reaction obtains this general form

$$(n - 1)\sum_{i=1}^{n-2} x_i C_{i+1}H_{2i+4}O \rightarrow \left(\sum_{i=1}^{n-2} ix_i\right)C_nH_{2n+2}O + \left[\sum_{i=1}^{n-2} (n - i - 1)x_i\right]CH_4O, (n > 2) \quad (55)$$

where $x_i > 0$, $(1 \le i \le n - 2)$ are arbitrary real numbers.

Example

Let's consider a particular case of (55). For $x_1 = x_2 = \cdots = x_{n-2} = 1$, the reaction (55) becomes

$$2\sum_{i=1}^{n-2} C_{i+1}H_{2i+4}O \rightarrow (n - 2)(C_nH_{2n+2}O + CH_4O), (n > 2). \quad (56)$$

Discussion

Presently in chemistry and mathematics, there are several formal mathematical methods for balancing chemical reactions, which work succesfully for chemical reactions possess atoms with fractional and integer oxidation numbers. These methods are founded by virtue of generalized matrix inverses and all of them need higher level of algebraic knowledge for their application. Just it was a stumbling block for chemists to use these methods for their daily purposes. In order to be avoid that awkward position, the author created this formal geometrical method for balancing continuum chemical reactions, with an intention to adapt a new contemporary mathematical method according to chemists' requirements.

By the way, this geometrical method reduces any chemical reaction to a set of hyperplanes of its atoms. Intersection of the hyperplanes is a hyperline, where lie all required reaction coefficients. In order to be verified its power and supremacy it was applied on several continuum classes organic reactions, such that obtained results showed that it works perfectly.

Conclusion

In this article are balanced only continuum class organic chemical reactions, such those of aliphatic hydrocarbon chemical reaction. Among considered organic reactions were: alkyne's general and its particular chemical reactions, alkane's general and its particular chemical reactions, alkene's general and its particular chemical reaction, and alcohol's general and its particular chemical reaction. All chemical reactions looked as elementary two and three atom molecular reactions, but they were very hard to balance. By this method the author proved again that balancing chemical reactions does not have anything with chemistry, because it is a pure mathematical issue.

The strengths of the geometrical method are: (1) This method provides an alternative approach for balancing continuum chemical reactions. By this method is showed that algebraic methods can be substituted by geometrical methods; (2) Since this method is well formalized, it belongs to the class of consistent methods for balancing chemical reaction; (3) This method showed that any chemical reactions can be treated as *n*-dimensional geometrical entity; (4) In fact, here-offered geometrical method simplifies mathematical operations provided by the previous well-known matrix methods and is very easily acceptable for daily practice. The geometrical method has this advantage, because it fits for all continuum chemical reactions, which previously were balanced only by the methods of generalized matrix inverses; (5) For determination of intersection point of hyperplanes any method for solution of system of linear equations can be used; (6) By this method the general form of the balanced chemical reaction much faster than by other matrix methods can be determined; (7) From the general balanced reaction the other particular and sub-particular reactions can be determined; (8) By this method, the angle $\measuredangle\alpha(\boldsymbol{n}_1, \boldsymbol{n}_2)$ between atom hyperplanes can be determined very easily; (9) The geometrical method provides the dimension of the solution space; (10) Also, by this method a basis of the solution space can be determined; (11) Necessary and sufficient conditions for which some reaction holds can be determined by this method too. These conditions determine the reaction interval of its possibility; (12) This method gives an opportunity to be extended with other numerical calculations necessary for continuum reactions; (13) Here offered geometrical method represents a well basis for building a software package.

The weak sides of the geometrical method are: (14) By this method the minimal reaction coefficients cannot be determined; (15) Also, this method cannot recognize when chemical reaction reduces to one generator reaction; (16) It cannot predict quantitative relations among reaction coefficients; (17) This method cannot arrange molecules disposition; (18) The geometrical method cannot be predicted reaction stability.

This method wild opens the doors in chemistry and mathematics too, for a new research of continuum chemical reactions, which unfortunately today cannot be balanced by usage of computer, because there is not such method. Here developed geometrical method is a big challenge for researchers to extend and adapt it for a computer application. Sure that it is not easy and simple job, but it deserves to be realized as soon as possible.

NOTES

1. In the reaction (31), if one substitutes $n = 11$, then it transforms into (23).
2. The alkyne's reaction (32), for $n = 11$ becomes sub-particular reaction (24).
3. In the reaction (43), if one substitutes $n = 12$, then it transforms into (39).
4. The alkane's reaction (44), for $n = 12$ becomes sub-particular reaction (40).

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